Heterogeneous Households, Mortgage Debt and House Prices over the Great Recession *

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Abstract

This paper studies the U.S. housing market in the Great Recession. I build a quantitative general equilibrium model with heterogeneous households and two sectors. Households face portfolio problems that involve selecting the stock of housing, mortgage debts and non-housing assets. The real house price is endogenous and households have the options to default on mortgage debt. At the cross-sectional level, the model reproduces the distribution of housing and non-housing wealth in the data. I find that a negative productivity shock alone can generate a real economic recession and a housing downturn with decline in real house price, but fails to generate contraction in mortgage debt. Then I experiment the model with a permanent tightening of financial condition and the same productivity shock. The countercyclical financial condition explains the substantial decline in mortgage debt and aggravates the decrease of real house price. Households deleverage immediately when the financial market is tightened. Under normal financial conditions, households take advantage of high housing return by taking large leverage and insure themselves against default risk by holding financial assets to smooth consumption. When the financial condition is tighter, households find it costly to take mortgage debts to save financial assets. Thus they avoid large interest payments and default risk by sharply reducing leverage in the financial shock.

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1 Introduction

Housing has been an important source of business cycle fluctuations. From 2002 to 2006, the U.S. housing market experienced a rapid increase in housing production and housing price, which contributed to the economic boom from 2004-2007. The housing market collapsed in 2006 and the sharp decline in housing price has led to the so called Great Recession. Figure 1-2 shows the data facts on the housing variables and the business cycle components of main macroeconomic variables in the Great Recession. First of all, the housing market experienced a long depression. From 2006Q3-2011Q4, real house price and real housing wealth declined about 30%, residential construction decreased 70% and real mortgage debt reduced about 19%. Secondly, the housing downturn was accompanied by a severe contraction in real economic activity. Specifically, real output was 5% lower than its 2007Q4 level in early 2009. Hours dropped 10% and consumption decreased 4.5% from 2008Q2 to 2009Q2. Finally, household leverage in the housing market dropped 17% between 2009Q1 and 2013Q1 as total mortgage debt fell sharply.[1]

How can we understand the recessions in the housing market and the real economy? Does the financial shock contribute to the decreases of mortgage debts and house prices? To answer these questions, I build a quantitative general equilibrium model with indefinitely lived heterogeneous households and two sectors. In each period, households solve consumption, labor and portfolio problems that involve selecting the stock of housing, mortgage debts and non-housing/financial assets to maximize expected life time value. Houses are risky assets that are exposed to idiosyncratic depreciation shock while financial assets are risk free. Houses can serve as collateral to borrow mortgage. Nevertheless, households have the option to default on mortgage debt at the cost of having their houses foreclosed. Financial intermediaries issue mortgage and price mortgage in the way such that household default risk is fully reflected. In order to endogenize real house price, this paper considers productions in two sectors: a consumption good sector and a housing good sector. At the cross-sectional level, the model reproduces the distribution of housing and non-housing wealth.

I find that a negative productivity shock alone can generate a real economic recession and a housing contraction with the declines in housing construction, housing wealth and real house price. However, the productivity shock fails to explain the reduction of mortgage debt. To understand the behavior of mortgage debts in the data, I keep the productivity shock and introduce a financial change by raising bank's cost of issuing mortgage permanently to mimic the Great Recession. Comparing this two-shock experiment to the pure productivity shock experiment, I find that the tighter financial condition is fully responsible for the substantial

[1] Leverage in this paper is defined as the ratio of mortgage debt and housing wealth.

decline in mortgage debt and also aggravates the decrease in real house price. Foreclosure rate increases initially and drops down later as households deleverage once the financial change is triggered. Quantitatively, the model can explain the entire declines in real mortgage debt and leverage, 1.3% of the reduction in real house price, 9% of the drop in real housing wealth and 17% of the decrease in housing construction in the Great Recession.

The model has four key ingredients. Firstly, when a household default the only punishment is to lose the ownership of the house. There is no recourse state and no transaction cost in housing purchase and selling. Also, households are not discriminated in the financial markets after they default. As a result, the default option is chosen if and only if the housing asset is underwater, i.e. the housing value is smaller than the mortgage loan value. This "ruthless default" rule simplifies mortgage pricing and reduces the state space so that the computational difficulty of solving the model is greatly reduced. Secondly, houses can be rented out for rental income but housing assets are risky as they are exposed to idiosyncratic depreciation shocks.[2] Since housing asset is risky, households also save using risk free financial assets which have lower returns.[3] Thirdly, issuing mortgage is costly so that the banks lose an additional r_w units of real resource per unit of mortgage debt issued. This assumption breaks up households' indeterminacy between saving and borrowing. Finally, I model two production sectors to endogenize the real house price.

In the Great Recession Experiment with both negative financial and productivity shocks, the reduction in productivity is responsible for the real economic recession and the decline in housing construction. Housing wealth shrinks as households who experience large reductions in their labor income demand less housing assets. Real house price decreases in the shock due to two reasons. On the one hand, the smaller productivity reduces consumption good output more than housing output. Real house price, which is the relative price of houses with consumption good as the numeraire, declines as consumption good becomes more valuable. On the other hand, the tighter financial condition reduces households' demands for mortgage debts and thus lowers the real interest rate. With smaller capital rental rate in the housing sector, real house price falls down further. Therefore, the tighter financial condition leads to further decline in real housing price.[4]

Given tighter housing financial condition, households find it optimal to sharply decrease leverage and mortgage debts. To understand the substantial decline in mortgage debt and the mechanism of the household deleveraging process, it is important to first clarify why

[2]Depreciation shock is i.i.d drawn from an invariant distribution. In this paper housing depreciation simply means that housing assets wear out physically.

[3]Rental income is the only source of housing return in the steady state as house price is stable. In the model benchmark economy, rental income generates a housing return that is higher than the risk free interest rate.

[4]Specifically, the drop in housing productivity accounts for 40% of the decrease in real house price while the tighter financial condition is responsible for the rest of the 60% decrease in the model.

households save using risk free financial assets and risky housing assets simultaneously. Although houses are subject to random idiosyncratic depreciation shocks, households want to obtain the high return from owning houses but also try to insure themselves against the idiosyncratic depreciation shock. When households are hit by big depreciation shocks so that their houses are underwater, they default and their net worth only depends on their holdings of financial assets, which cannot be seized by the banks.[5] To maintain consumption at a level above the labor income, households thus also hold risk free financial assets even if they have lower return than housing assets. In the steady state, households borrow mortgage not only to fund the purchase of housing but also to save risk free financial assets. The benefit of borrowing to save is the increase in household value from consumption smoothing. The cost of borrowing to save is the decrease in value caused by higher default risk and larger interest payments. However, when financial condition becomes tighter, households find the cost of borrowing to save has greatly increased. Thus in the financial crisis households avoid large interest payments and default risk by sharply reducing leverage and mortgage debts. Accordingly, households' demand for financial assets also decline.

In the new steady state with tighter housing finance, macroeconomic variables such as output, consumption and investment remain roughly the same as in the benchmark economy. In contrast, the housing market has several major changes. Specifically, outstanding mortgage debt falls 23%, homeownership rate decreases 0.6%, foreclosure rate drops 9.5% and housing price decreases 0.11%. In addition, the tighter financial condition also leads to a 0.13% increase in wealth inequality.[6]

Literature Review First of all, this paper is related to business cycle models with home production. Leading examples are Davis and Heathcote (2005), Iacoviello and Neri (2010), and Greenwood and Hercovitz (1991). These papers study multi-sector productions and can match housing investment well. However, they do not distinguish owning and renting, do not model household heterogeneity in housing and wealth, and thus cannot match the housing and non-housing wealth distribution in the data.

This paper is closely related to papers that study the housing market with heterogeneous agents, endogenous default and exogenous house prices such as Chatterjee and Eygungor (2010), Corbae and Quintin (2012), and Jeske, Krueger and Mitman (2011). Jeske et al (2011) builds a heterogeneous agents model with endogenous mortgage default options to study the macroeconomic and distributional impact of the subsidy from Government Sponsored Enterprises. They find that eliminating the subsidy leads to substantial reduction of mortgage origination and increases aggregate welfare. Their insightful paper provides a

[5]There is no recourse state in this model.

[6]Wealth inequality is measured by the wealth Gini coefficient. Wealth is in terms of household net worth in this paper.

useful framework on housing and mortgage market with collateralized default and mortgage pricing. This model builds on Jeske et al (2010) but significantly differs from their work in three respects. Firstly, the real house price is endogenous in this model while it is held constant in theirs. Secondly, they study an endowment economy, while I model two sectors that produce consumption goods and housing goods respectively. This setting enables me to study the housing market in a general equilibrium framework. Thirdly, although the household heterogeneity in this paper shares many elements in common with theirs, household heterogeneity in this paper is different and richer in that households also value leisure.

Finally, this paper is related with literature that studies the impact of housing market over business cycle with heterogeneous agents. A leading example is Iacoviello and Pavan (2013). They study housing and mortgage debt activities over the business cycle and find that higher individual income risk and lower down payments can explain the reduced volatility of housing investment, the reduced procyclicality of debt and part of the reduced volatility of GDP. Similar to what they have done, this paper can generate decreases of housing demand and mortgage debt in the experiment with financial shocks. In this case, this paper is complementary to Iacoviello and Pavan (2013). However, two limitations in Iacoviello and Pavan (2013) are fixed in this paper. Firstly, they study the life cycle of housing and mortgage debt with exogenous housing prices. Secondly, they do not model mortgage default options. In contrast, this paper allows households to default on mortgage debt and endogenizes real house price. As a result, this paper can study the implication of real and financial shocks on house price and generate the ups and downs of housing foreclosure rate. In addition, the interaction between life cycle, risk and housing demand are the key elements in Iacoviello and Pavan (2013), but they are not the focus of this paper. To the best of my knowledge, this is the first paper that has studied the housing market with endogenous housing price and mortgage default options under the heterogeneous agent framework.

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 discusses parameterization. Section 4 summarizes the steady state results. Section 5 considers the transitional dynamics of a pure technology shock. Section 6 presents the results of the permanent financial change following a negative productivity shock. Section 7 concludes.

2 The Model

2.1 Heterogeneity and Demographics

There is a continuum of households in the economy that are indexed by $i \in [0, 1]$. Each household is endowed with one unit of time to divide between labor and leisure. Households live indefinitely and have idiosyncratic labor productivity ϵ. In the economy, households save using two kinds of assets. Firstly, households can hold risk-free non-housing/financial asset a which earns risk-free interest rate r per unit of assets saved. Secondly, households can purchase perfectly divisible housing asset h. However, houses are risky assets as they are subject to idiosyncratic housing depreciation shocks.[7] Let δ' denote the housing depreciation shock tomorrow. Depreciation δ' is an independent draw across time for every household from the continuously differentiable cumulative distribution function $F(\delta')$, $\delta' \in [\underline{\delta}, 1]$ with $\underline{\delta} \leq 0$.

There is a competitive housing rental market where households can trade housing services. One unit of housing asset generates one unit of housing service. A house purchased at the beginning of a period can be rented out immediately and thus generate rental income in the same period as the purchase. Short selling of risk free non-housing assets and houses are prohibited.

Households can use housing assets as collateral to take mortgage issued by the bank. Let m' denote the size of the mortgage, and p_m denote the mortgage price. A household that enters the next period with (h', m') has the option to default on his mortgage payment after observing the housing price p'. If he chooses to default, the punishment is losing the ownership of the house to the banks. A defaulted household is not punished in any other form in the financial market. There is no recourse state and no transaction cost in housing purchases and sales. Given these assumptions, a household chooses to default if and only if his housing asset is underwater, i.e. housing value is smaller than the mortgage payment. That is,

$$p'(1 - \delta')h' < m' \tag{1}$$

Equation (1) is the household default decision rule. It implies that the ex-ante default probability at the origination of the mortgage prior to observing the depreciation only depends on the size of the mortgage m' and housing value $p'h'$.[8] Thus mortgage price p_m is simply a function of (m', h'). It also implies that the cutoff housing depreciation rate at which a household is indifferent between defaulting and repaying is $\delta^* = \max\left\{\underline{\delta}, 1 - \frac{m'}{p'h'}\right\}$.

[7] Housing depreciation in this paper means that housing assets wear out physically.
[8] See Jeske et al (2011).

2.2 Households problem

Let x denote household net worth which is the real value of all assets brought into the period after the housing depreciation shock is materialized. Households thus have two individual state variables (x, ϵ). Let $\mu(x, \epsilon)$ denote households distribution over individual state variables (x, ϵ). Then aggregate variables are (z, μ)[9]. Since the main interest of the paper is the stationary economy and perfect foresight transitions, the dependence of prices on (z, μ) are left implicit. In each period, households maximize discounted expected lifetime value from consumption, leisure and housing service taking real interest rate r, real wage rate w, real rental price p_s and mortgage price $p_m(\cdot, \cdot)$ as given. That is, households solve the following problem

$$V(x, \epsilon) = \max_{c,s,a',h',m',n} u(c, s, 1-n) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) \int_{\underline{\delta}}^{1} V(x', \epsilon') dF(\delta') \tag{2}$$

subject to

$$c + p_s s + a' + ph' - m' p_m(m', h') = w\epsilon n + x + p_s h' \tag{3}$$

$$a', h', m' \geq 0, \quad 0 \leq n < 1 \tag{4}$$

where net worth $x' = (1 + r')a' + \max\{0, p'(1 - \delta')h' - m'\}$

Equation (3) is the household budget constraint. The R.H.S. of equation (3) denotes resources available to the household within the period including the labor income and net worth. Since the timing is that houses purchased this period can be rented out immediately, household rental income $p_s h'$ also shows up as part of the household resources within the period. The L.H.S of equation (3) is the household allocation of resources among consumption, housing service and asset portfolio which involves selecting financial asset, housing asset and mortgage debt so that household expected life time value is maximized.

Future net worth x' is consisted of financial asset income and home equity. If future housing value after the realization of housing depreciation is larger than the mortgage debt, home equity is positive and equals $p'(1-\delta')h' - m'$. In this case, households repay the debts. Otherwise, household home equity is zero and net worth $x' = (1 + r')a'$ as households choose the default options and have their houses foreclosed.[10]

[9] z denotes aggregate productivity, which is introduced in the next subsection in detail.

[10] In reality, it is possible that home equity become negative and households do not trigger default for various reasons. For example, there are penalties on household credit report if they default. Homeownership itself might be valuable to the households and it is costly to find and move to new places. In the model, existing assumptions eliminate these possibilities and default is chosen iff housing asset is underwater. Thus, home equity is nonnegative.

2.3 The Banking Sector

Assume that banks are perfectly competitive and have the technology to convert risk free assets into productive capital without any cost. At the beginning of each period, banks take deposits of financial assets from households, lend capital to housing production sector and issue mortgages. Following Jeske et al (2011), I assume that issuing mortgage is costly so that banks have to lose an additional r_w units of real resources per unit of mortgage issued. r_w characterizes the screening, monitoring, administrative as well as maintenance costs associated with each unit of mortgage. Thus the effective cost of issuing a unit of mortgage equals $r + r_w$ and banks discount the expected payments received next period at $\frac{1}{1+r+r_w}$.[11] When households default, banks seize the after depreciation housing value. However, bank foreclosure process is costly and only recovers a fraction $\theta \in [0, 1]$ of the collateral value.

Banks take into account that households might default on the mortgage payments next period. Therefore, mortgage price is such that each mortgage contract compensates for the expected loss in the case of default.

$$m'p_m(m', h') = \frac{1}{1 + r + r_w} \left\{ m'F(\delta^*) + \theta p'h' \int_{\delta^*}^{1} (1 - \delta')dF(\delta') \right\} \tag{5}$$

where $\delta^* = \max \left\{ \underline{\delta}, 1 - \frac{m'}{p'h'} \right\}$ is the cutoff housing depreciation rate at which a household is indifferent between defaulting and repaying.

In equation (5), $m'p_m(m', h')$ is the actual units of consumption that a household obtains when he takes a mortgage of size m' and buys a house of size h'. The right hand side is the expected discounted revenue that banks receive next period from (m', h'). With probability $F(\delta^*)$ household receives a housing depreciation shock δ' that is lower than the threshold depreciation δ^* so that repaying mortgage is optimal. With probability $1 - F(\delta^*)$ households default and banks liquidize the house after a costly foreclosure process which only recovers θ fraction of the after depreciation housing value.

2.4 Representative Production Sectors

There are two representative production sectors in the economy, a consumption good sector and a housing good sector. Assume that labor is perfectly mobile and aggregate productivity z is the same across sectors.

[11] r_w is paid when the mortgage is repaid. When households default on mortgage payment, they also default on the mortgage issuance cost.

The consumption good sector produces consumption goods using labor according to production technology $Y_c = zN_c$. Consumption good sector solves the following problem

$$\max_{N_c \geq 0} \{zN_c - wN_c\} \tag{6}$$

The housing sector produces new homes using capital and labor according to production technology $I_h = (zN_h)^{1-\nu} K^{\nu}$. Let δ_k denote capital depreciation and p be the real housing price with consumption good as the numeraire. Home producers solve the following problem

$$\max_{K, N_h \geq 0} \left\{p\,(zN_h)^{1-\nu} K^{\nu} - (r + \delta_k)K - wN_h\right\} \tag{7}$$

The above two static maximization problems imply that profits are maximized by choosing K, N_h, N_c, such that

$$w = z \tag{8}$$

$$r = p\nu K^{\nu-1} (zN_h)^{1-\nu} - \delta_k \tag{9}$$

$$w = p(1-\nu)z^{1-\nu} K^{\nu} N_h^{-\nu} \tag{10}$$

2.5 General Equilibrium

A recursive competitive equilibrium consists of a set of functions

$$(p, p_s, p_m, r, w, V, c, s, n, a', h', m', N_c, N_h, K, \mu) \tag{11}$$

that satisfies the following conditions.

(i) Given prices p, p_s, p_m, r and w, the value function V solves (2) and c, s, n, a', m', h' are the associated policy functions

(ii) Given prices, policies N_c solves the consumption good production problem and N_h, K solves the housing production problem

(iii) Given $p_m(\cdot,\cdot)$, financial intermediaries break even for all (m', h')

(iv) Consumption good market clears

$$\int c\,d\mu + I = Y_c \tag{12}$$

where capital investment $I = \bar{K}' - (1-\delta_k)\bar{K}$, where $\bar{K}$ is aggregate capital stock this period and $\bar{K}'$ is aggregate capital stock next period.

(v) Housing rental market clears

$$\int s d\mu = \int h' d\mu \tag{13}$$

(vi) Labor market clears

$$N_c + N_h = \int (\epsilon n)\, d\mu \tag{14}$$

(vii) Asset market clears

$$\int a' d\mu = \int p_m(m', h') m' d\mu + \bar{K}' \tag{15}$$

(viii) Capital market clears

$$K = \bar{K} \tag{16}$$

(ix) Housing market clears

$$\int h' d\mu = I_h + H \tag{17}$$

where I_h is the newly built houses this period and H is the effective aggregate housing stock after depreciation and foreclosure.

(x) The evolution of household distribution over individual variables, $\mu(x, \epsilon)$, is consistent.

3 Parameterization

One period in the model is a quarter. Table 1 lists the parameters that are adopted exogenously from data. Suppose the idiosyncratic labor productivity ϵ follows a log AR(1) process

$$log\epsilon_{t+1} = \rho_\epsilon log\epsilon_t + (1 - \rho_\epsilon^2)\eta_{\epsilon,t}, \quad \eta_\epsilon \sim N(0, \sigma_\eta^2) \tag{18}$$

I follow Jeske et al (2011) to set the persistence of labor productivity $\rho_\epsilon = 0.98$ and the standard deviation $\sigma_\eta = 0.3$, which stand in line with empirical literature on labor productivity and a vast literature on the nature and specification of the household income process. The estimates of the average default loss is 22% in Pennington-Cross (2004) using national data. I let $\theta = 0.78$ to be consistent with Pennington-Cross (2004). I follow Jeske et al (2011) to set the CRRA parameter $\sigma = 3.9$.

To generate realistic housing foreclosure in the steady state of the model, the housing depreciation shock $F(\delta)$ is assumed to be a Pareto distribution with probability density function

$$f(\delta) = \frac{1}{\sigma_\delta}\left(1 + \frac{\gamma(\delta - \underline{\delta})}{\sigma_\delta}\right)^{(-\frac{1}{\gamma}-1)} \tag{19}$$

I calibrate the three parameters γ, $\underline{\delta}$ and σ_δ by targeting three moments in the data: mortgage foreclosure rate, mean depreciation of residential fixed assets and the standard deviation of housing prices. According to the National Delinquency Survey from Mortgage Banker Association (MBA(2006)), the average quarterly foreclosure rate of all mortgage loans is about 0.43% from 2002Q1 to 2006Q4 . The mean depreciation for residential housing is calculated as the consumption of fixed capital in housing sector divided by the total capital stock of residential housing. The data on the consumption of fixed capital in housing sector is taken from Table 7.4.5 of National Income and Product Account (NIPA), and the capital stock of residential housing is taken from Fixed Asset Table 1.1.[12] My estimation of the quarterly mean depreciation for residential housing is 1.4%.[13] The standard deviation of housing value is obtained by utilizing the state volatility parameter from the Federal Housing Finance Agency (FHFA or OFHEO). The state volatility parameter, which is measured using sales prices only, reflects the standard deviation of housing price growth after four quarters from 1991Q1 to 2013Q2. According to the FHFA, the standard deviation of housing prices in the 51 states of the United States varies from 6-9% and has a mean value of 7.8%. Therefore, I choose the volatility target to be 7.8%.

Household receives utility from consumption c, housing service s and leisure $1-n$. The momentary utility function is

$$u(c, s, 1-n) = \frac{\left(c^{\tau_1} s^{\tau-\tau_1}(1-n)^{1-\tau}\right)^{1-\sigma} - 1}{1-\sigma} \tag{20}$$

I choose parameter τ endogenously so that households in the model on average work one-third of their time. τ_1 is chosen so that the share of housing in total consumption expenditure is 14.4% , which is measured using the annual data from 1969 to 2001 (NIPA Table 2.4.5). As shown in Figure 1, real mortgage debt is about 0.42 times as large as the housing wealth from 1969Q1 to 2002Q4. The mortgage administration cost $r_w = 0.0008$ is endogenously determined so that the aggregate leverage ratio in the steady state hits this target. The time discount factor $\beta = 0.954$ is endogenously pinned down to generate an annual interest rate of 5% in the steady state.

On the production side, I set parameter $(1-\nu) = 0.87$ to match the labor's share in construction sector. The average labor's share in construction sector from 1987 to 2002 is

[12] Table 7.4.5 published by BEA June 25, 2010

[13] This estimation stands in line with Macro-Housing literature such as Jeske et al (2011) and Iacoviello and Pavan (2013).

Table 1: Exogenously Adopted Parameters

	Interpretation	Value	Source
δ_k	capital depreciation	0.017	U.S. data
ρ_ϵ	productivity persistence	0.98	Jeske et al (2011)
σ_ϵ	productivity variance	0.30	Jeske et al (2011)
ν	capital's share in housing	0.13	GDP-by-Industry
θ	foreclosure technology	0.78	Pennington and Cross (2004)
σ	CRRA parameter	3.9	Jeske et al (2011)

Table 2: Endogenously calibrated parameters and data moments

	Target Moment	Model	Target	Data Source
β	Risk free rate	1.25	1.25	U.S. data
τ	Average labor hours	0.33	0.33	U.S. data
τ_1	Consumption's share	0.86	0.86	NIPA
r_w	Aggregate leverage	0.41	0.42	Flow of Funds
γ	Foreclosure rate	0.42%	0.43%	MBA(2006)
σ_δ	House value volatility	0.07	0.08	OFHEO HPI data
$\underline{\delta}$	Average housing depreciation	1.3%	1.4%	NIPA

measured to be 0.87 using the method and data source provided in Davis and Heathcote (2005).[14] I choose capital depreciation $\delta_k = 0.017$ to be consistent with Khan and Thomas (2007). I assume that aggregate productivity is the same in both consumption good sector and home production sector. Aggregate productivity z follows a log AR(1) process

$$log(z_{t+1}) = \rho_z log(z_t) + \zeta_t, \quad \zeta_t \sim N(0, \sigma_\zeta^2) \tag{21}$$

where $\rho_z = 0.95$ as in Bloom et al (2011) and $\sigma_\zeta = 0.0072$ as in King and Rebelo (2000).

[14] I abstract land as a production factor in the housing sector. I estimate the capital output ratio in the construction sector to be about 0.16, which is trivial. Thus it is more appropriate to interpret the capital here as the combination of capital and land since physical capital itself is almost neglible.

4 The Steady State

In this section, I illustrate the steady state properties. Figure 3 plots the mortgage price function $p_m(m, h)$ provided by the banking sector as described in equation (5). Since $F(\delta)$ is a continuous differentiable distribution, $p_m(m, h)$ is also continuous and differentiable in m and h. As shown in Figure 3, mortgage price is higher when a larger house h is pledged as collateral, holding mortgage m constant. Given the housing asset h, mortgage price decreases as mortgage debt m increases.

Actually, mortgage price is simply determined by household leverage. Let $\iota = \frac{m}{ph}$ denote the leverage ratio, then equation (5) can be rewritten as[15]

$$p_m(\iota) = \frac{1}{1 + r + r_w} \left\{ F(1 - \iota) + \frac{\theta}{\iota} \int_{1-\iota}^{1} (1 - \delta) dF(\delta) \right\} \tag{22}$$

Taking derivative with respect to ι, one can find that $p'_m(\iota) < 0$. Thus mortgage price is monotonically decreasing in leverage. Intuitively, a larger leverage implies a higher probability of default as the threshold depreciaiton rate is lower. Therefore, banks require higher interest compensation from households for the higher risk they take. When a household takes larger mortgage and/or buy smaller houses, he chooses higher leverage which yields a smaller mortgage price according to equation (22). Moreover, leverage and mortgage decision is equivalent with the definition of ι, given h and p.

Figure 4 plots the value function over net worth and labor productivity. Household value is higher the larger his net worth and/or labor productivity. Let $g' = a' + (p - p_s)h' - m'p_m(h, m)$, then g can be interpreted as net saving.[16] By solving a consumption-savings problem, I find that the net saving policy is linear and increasing in net worth and labor productivity, which is shown in Figure 5.

Figure 6 shows the housing decision as a function of net worth and labor productivity. Larger net worth and labor productivity means more resource is available to households to allocate between different assets and households find it optimal to buy a larger house. Although households can obtain housing service from renting, households demand risky housing assets because they carry higher expected return than the financial assets. Specifically, the expected return to housing investment comes from two sources: the implicit rental income and the potential appreciation in home value. Since house price is constant in the steady state, the unique source of return for housing investment in the steady state is the rental

[15] Proposition 2 in Jeske et al (2011) implies that it is never optimal for households to choose leverage $\iota > 1 - \underline{\delta}$ in equilibrium. Thus the threshold depreciation $\delta^* = 1 - \iota$ without loss of generality.

[16] With this definition, the household problem can be transformed into a consumption-savings problem which is available in the appendix.

income.[17]

Figure 7 shows that household leverage decreases monotonically as net worth and/or labor productivity increases. Leverage is high (at close to 68%) to households with little wealth. Leverage then drops quickly as net worth increases until it reaches around 38%. After that, leverage no longer declines because households start to increase holdings of risk free non-housing assets, as can be seen from Figure 8.

Households save more risk free financial assets as net worth increases, but decreases holdings of financial assets when labor productivity is larger. The reason is because households with little wealth or higher productivity expect to finance their current and future consumption primarily using labor income. In contrast, high wealth and low productivity household expect to finance current and future consumption primarily from capital income. Thus high wealth and low productivity households tend to increase the share of safe assets in their portfolio.

In the steady state, households buy house, save low-interest bearing financial assets, and borrow high-interest mortgages simultaneously. The reason is because households want to take advantage of the high expected return from owning houses but also try to insure themselves against the adverse idiosyncratic depreciation shock. When a household gets hit by a large depreciation shock so that his house is underwater, he defaults and his net worth only depends on how much financial asset he owns, which is $(1+r)a$. To maintain a level of consumption above labor income, he finds it optimal to hold risk-free financial asset, a.

In addition, the mortgage debts that households borrow are only partly used to fund the purchase of houses. Actually, part of the debts is used to save risk free assets in the steady state. The benefit of borrowing to save is the increase in household value from consumption smoothing with risk free financial asset. Since the financial assets are not seized by the banks when households default, accumulating financial assets enables them to maintain consumption at a higher level than the labor income. On the other hand, the cost of borrowing to save is the decrease in household value due to larger default risk and net interest payment. When the financial condition is such that the benefit of borrowing to save is larger than the cost, households borrow mortgage debts to increase their holdings of risk free financial assets.[18]

In the steady state, the model reproduces a housing foreclosure rate of 0.42% which is consistent with the data. Specifically, households who have their houses foreclosed are mostly those with little net worth, because they are the high leverage takers at each labor

[17] Given the expected housing depreciation is 1.4%, p equals 0.93 and p_s equals 0.0275 in the steady state, the expected housing return is obviously higher than the risk free interest rate which is equal to 1.25%.

[18] Mian and Sufi (2011) has documented that borrowed funds based on home equity are used for increasing consumption.

Table 3: Steady State Numerical Results

Variable	Interpretation	Value
percent of hhs with $h' > 0$		97.5%
percent of hhs with $h' > s$	homeownership	48.3%
Wealth Gini	wealth inequality	0.505
$pH/(4 \times GDP)$	housing wealth	1.27
Non-housing asset	non-housing asset share	33%
$p \times I_h/GDP$	housing investment share	7.8%
$K_h/(4 \times GDP)$	Business capital in housing sector	0.086

productivity level.

The model reproduces the U.S. wealth distribution in general. Wealth in the model is defined as household net worth. Diaz-Gimenez et al (1997) reported that the Gini coefficient of wealth is 0.78 in the 1992 SCF. The wealth Gini coefficient in the steady state of this model is 0.505, which is close to that in the data. Jeske et al (2011) obtains a Gini coefficient 0.46 in their steady state. Given that this paper shares many elements in heterogeneity with theirs, this model fits the data a little better in terms of wealth inequality. Iacoviello and Pavan (2013) obtains a Gini coefficient equals to 0.73 in their steady state with two discount factors and 0.53 with a single discount factor.

In the steady state, housing wealth takes up 67% of total household assets in the steady state, which is consistent with the data as housing wealth takes up almost half of the national wealth in the United States from 1952-2008. Moreover, housing wealth is 1.27 times that of real GDP in the benchmark economy which is close to 1.1 times in the data from 1969 to 2007. In the steady state, about 97.5% of households owns strictly positive housing assets and 48.3% of households owns larger houses than the amount of housing services they actually consume. Since housing is perfectly divisible in the model, I regard the "percent of households with $h' > s$" as the best proxy of homeownership rate in the model.[19] Accordingly, homeownership rate in the steady state is close to the data which is equal to 64% on average from 1994 to 2007.

[19] In this case, homeownership does not correspond to the traditional concept of owner-occupation. Instead homeownership here means that households' holdings of housing assets can fully satisfy their demands for housing services. This definition is consistent with Henderson and Ionnides (1983). Under the assumption that housing asset is perfect divisible, this definition is the best proxy to homeownership in the data.

5 Negative productivity shock

In this section, I present the results of the benchmark economy with a persistent negative productivity shock. In the first period, productivity drops one standard deviation (2.3%) in both housing and consumption good sectors in the first period and recovers gradually afterwards according to equation (21).

Figure 10-12 shows the transitional paths of main economic variables understudy. When productivity shock hits the economy at $t = 1$, the marginal productivity of capital (MPK) decreases which leads to an initial decline in real interest rates. Production sectors demand less labor as the marginal product of labor (MPL) decreases at the shock. The decreases in productivity and labor input together contribute to a 3.6% initial decline in real aggregate output. Output and interest rate then recovers little by little as productivity increases over time. The transitional path of real wage coincides with the path of productivity because $w = z$ in each period.

Real housing price drops 0.13% initially at the shock. It then increase little by little to 0.1% higher than that in the steady state as the housing productivity recovers over time.[20] Since the decline of productivity reduces the output of consumption good more than housing good, real housing price falls off as consumption good becomes more valuable. Note that households are forward looking in this model since future net worth depends on the realized interest rate and housing price next period. When household solves the portfolio problem, he looks at both current and future prices. Given the transitional paths of the house price and the real interest rate, household observes that home value appreciates in the first few periods, so he selects larger leverage to take advantage of the higher housing return.

Aggregate net saving drops initially at the shock. Specifically, it is households with high productivity and low wealth experience the largest decline in net saving. Since the high productivity and low wealth people work more hours in the steady state, the large declines in wage and labor hours decreases their labor income sharply. To smooth consumption, they have to significantly reduce their net savings. In contrast, people with the lowest productivity and high net worth supply very little labor in the steady state. As a result, the large drop in wage has little impact over their labor income. Given that housing price is growing after the initial decline, they actually increase their net savings slightly. Aggregate net saving declines further after the first period since households need to smooth consumption but labor income only recovers slowly.

[20] The decline in real housing price is relatively small compare to the size of the shock because aggregate productivity falls in both consumption good and housing good sectors. If the shock only happened in consumption good sector, then the real house price would drop about 2% in the first period. However, housing output would increase initially in this case as productivity in the housing sector is relatively higher.

Figure 11 shows the aggregate leverage derived using aggregate mortgage debt divided by aggregate housing wealth. Aggregate leverage rises up 3.1% initially and later falls down gradually. The movements of leverage at the individual level are consistent with the ups and downs of the aggregate leverage. As shown in Figure 12, household leverage policy shifts up in period 1. Suppose a household with the lowest labor productivity is at point A in Figure 12 in the steady state, house price appreciation moves him up to point B which corresponds to a higher leverage assuming his net saving does not change. Nevertheless, lower wage reduces his labor income. To smooth consumption, he decreases net saving. Thus on the graph he moves up further from point B to point C which corresponds to a smaller saving and even higher leverage. Therefore, leverage at the individual level rises up initially and there is no household deleverage process in the negative productivity shock.

Leverage falls down gradually after the increase in the first period because the increment of housing price in a period matters. If housing price appreciates greatly next period, it is optimal for households to increase leverage/mortgage debt significantly this period. If the increment of housing price next period is small, households only increase leverage/mortgage debt slightly. Given that the increments of home price decreases period by period in Figure 11, leverage/mortgage debt thus falls down gradually.

Aggregate financial assets rise up initially at the shock. As explained in the previous section, households hold risk free financial assets in order to smooth consumption because housing asset is risky and high depreciation shocks might trigger default. The increase in leverage ι implies larger default risk which is equal to $1-F(1-\iota)$ as the threshold depreciation rate is equal to $1-\iota$. Since households take higher leverage at the shock, they also increase the holding of risk free financial assets to insure themselves against higher default risk.

Aggregate housing demand declines about 0.4% at the shock and continues to decrease for about 25 periods. Nevertheless, housing demand exhibits rich heterogeneity at the micro-level. Firstly, households with little wealth and high labor productivity decrease housing demand. They belong to the group of households that experiences the largest decline in labor income. To smooth consumption, they have to decrease net savings. Since they finance consumption primarily from labor income and save little risk free assets in the steady state, housing wealth takes up a very large share in their asset portfolio. Therefore, the reduction in net saving is achieved by decreasing holdings of housing assets. Secondly, households with low productivity and high wealth, who supply little labor in the steady state, increase housing demand to take advantage of the housing price appreciation because their labor income are almost unaffected by the large drop in wage. However, the share of housing assets in their portfolio decreases because their default risk increase and they insure themselves against higher default risk by holding more safe assets. Since most people in the economy belongs to the first group, aggregate housing demand declines when the shock hits. Housing demand declines further for about 25 periods for two reasons. Firstly, household labor income recovers gradually over time. Secondly, the return to housing investment decreases as the increment of house price is falling.

Aggregate labor supply drops initially and recovers slowly as productivity increases. Since the substitution effect is dominating, households supply smaller amounts of labor although they are poorer in the shock. Total labor input in the consumption good sector decreases initially as the marginal productivity of labor, which is equal to aggregate productivity z, declines at the shock. Given perfect mobility in the labor market and the large drop in real wage, the labor input in the housing sector is determined by the tradeoff of two forces. On the one hand, marginal productivity of labor decreases and thus home builders should require smaller labor. On the other hand, wage rate declines sharply (relative to that in the standard one sector model) so that it is optimal to increase labor input because labor is much cheaper relative to capital. The tradeoff of the two forces is that the labor input increases slightly in period one. Labor input in the housing sector declines later as capital depreciates and wage recovers.

Capital stock decreases gradually as the marginal productivity of capital in the housing sector falls at the shock. Aggregate housing service declines when the shock hits because the reduction in the labor income leads to a large initial decline in total household consumption expenditure. Therefore, housing service expenditure follows to decline as it takes up a fixed share (85.6%) in total household consumption expenditure. Aggregate demand for housing service declines further after period 1 as rental price increases faster than the recovery of housing service expenditure.

Housing investment, I_h, slumps due to the decrease in productivity and the distressed housing demand. Lower MPK and MPL makes home builders disinvest in capital and demand lower labor hours. The recovery in housing investment is slow and later than the recovery in the consumption good sector. The reason is because only the housing good sector produces with capital.

Foreclosure rate increases 0.14% in the first period when the productivity shock hits because there is an initial decrease in real housing price. However, foreclosure rate jumps up further in the second period as households take larger leverage to take advantage of the house price appreciation. Foreclosure rate remains high for several periods and comes down eventually as household leverage falls off.

In summary, a persistent negative productivity shock alone can generate a persistent economic recession with declines of housing demand,housing investment and house price, but it fails to create decreases in mortgage debts, leverage and foreclosure rate. To explain the fluctuations of housing variables understudy, I raise bank's cost of issuing mortgage permanently to create a financial change similar to that in the Great Recession.

6 The Great Recession Experiment

Economic booms and busts are closely related with changes in housing financial conditions as the Great Recession is caused by the financial innovations in the mortgage market. The housing finance has been tightened since 2008 as most financial institutions raise their down payment requirements and the mortgage backed securities have been restricted thereafter.

To mimic the environment in the Great Recession, I raise banks' cost of issuing mortgage permanently, and at the same time let aggregate productivity decrease one standard deviation (2.3%) as in section 5. The permanent change in housing finance and the decline in productivity thus trigger an economic transition until the economy reaches the new steady state. Specifically, the mortgage administration fee r_w is raised permanently from 0.0008 in the benchmark economy to 0.0012 in period 1.[21] The increase in r_w captures the increased cost of financial intermediation and the permanent structural change in mortgage finance. I call this two-shock experiment the Great Recession (GR) Experiment.

6.1 Transitional Dynamics of the Great Recession Experiment

With both negative financial and productivity shocks, the economy enters into a deep recession immediately. Specifically, the marginal productivity of labor (MPL) decreases at the shock which leads to the decline in real wage and the smaller demand for labor in the production sectors. The reductions in productivity and labor input together contribute to a 3.4% initial decline in real aggregate output. As productivity increases over time, aggregate output and interest rate recovers little by little. Since the impulse response of main economic variables are similar to that in the pure productivity shock, the decrease in productivity is responsible for the contractions of real output, consumption, investment and housing construction.

Before discussing the transition of the housing market, it is worth noticing that the real interest rate decreases more in the GR experiment than its initial decline in the pure

[21] I raise r_w to 0.0012 so that the initial decline in mortgage debt is about 16%. Given that the decrease in real output in the Great Recession Experiment is 3.4% which is about 68% as large as that in the data, the 16% initial decline in mortgage debts can fully account for the around 20% decrease of mortgage debts in the data.

productivity shock. This is because the tighter financial condition makes borrowing more costly, thus households' demands for loans to finance the purchase of houses decrease.

The impulse responses of the housing variables differ from that of a pure productivity shock in several dimensions. First of all, housing price declines 0.25% at the shock which is more significant than that in the pure productivity shock.[22] The decrease in house price is because of two reasons. On the one hand, the decrease of productivity reduces the consumption good production more than the housing production. Real housing price falls off as consumption good becomes more valuable. On the other hand, the tighter financial condition reduces households' demands for mortgage debts and thus lowers real interest rate. With smaller capital rental rate in the housing sector, real house price falls down further.

Secondly, leverage/mortgage debt slumps as soon as the financial transition is triggered. As shown in Figure 13, aggregate leverage falls 16% at the shock and continues to decline thereafter. Consistent with the movements of aggregate leverage, leverage at the household level also falls when the financial transition starts. As shown in Figure 15, household leverage policy curve shift down greatly in period 1. To understand the shifts of leverage policy, suppose a household with the highest labor productivity and high net worth is at point A' in the steady state. The tighter financial condition makes him move down to point B' which corresponds to a lower leverage, assuming his net saving does not change. Nevertheless, lower wage rate lead to a significant decrease in his labor income. To smooth consumption, he decreases net saving. Thus on the graph he moves from point B' to point C', which corresponds to a smaller saving and about the same level of leverage as B'. Nevertheless, for households with the highest productivity and the smallest net worth, their leverage policy curve remains basically unchanged. However, they experience the largest decrease in labor income which makes them to significantly reduce net savings. Thus they end up taking larger leverage than in the steady state. Since few households are in the second group that increases leverage, the aggregate economy experiences a deleveraging process in the Great Recession Experiment.

The reason that households deleverage in the shock is because larger interest payment dampens households' incentive of borrowing mortgage to save financial assets. Since households borrow at the mortgage interest rate and earns interest payments by saving financial assets at the risk free interest rate, the parameter r_w which is the difference between risk free interest rate and risk free mortgage interest rate determines whether households are willing

[22]The decline of quarterly real house price varies from 0.5% to 0.9% in all post-war recessions except the 2000-2001 recession and the Great Recession. The real house price increased in the 2000-2001 recession. The collapse of the housing price in the Great Recession might because that the pre-crisis housing price has severely deviated from the fundamental, i.e. the high housing price before the crisis is a bubble. Since this paper does not generate a price bubble in the steady state, it is reasonable that real housing price does not experience big slump in this two-shock experiment. See footnote 20.

to borrow debts to accumulate risk free financial assets.[23] On the one hand, the benefit of borrowing to save is the increase in household value from consumption smoothing with risk free financial asset. Since the risk free financial asset is not seized by the bank when household default, accumulating risk free asset enables them to maintain consumption at a higher level than the labor income.

On the other hand, the cost of borrowing to save, which is the net interest payment that household pay out, increases as r_w becomes larger. In the benchmark economy, the gap between the risk free interest rate and risk free mortgage rate is relatively small so that the cost of borrowing to save is smaller than the benefit. In this case, households find it optimal to borrow large amount of mortgages to fund housing purchases as well as to increase the holdings of financial assets. With a tighter financial condition that increases the gap between borrowing and saving interest rates, households find the cost of borrowing to save becomes higher than its benefit. Thus households take a smaller leverage/mortgage and use most of their borrowings to fund the purchase of houses after the financial change has taken place. Therefore, the financial change explains the sharp declines in aggregate leverage, aggregate mortgage debt and aggregate non-housing financial asset. This is the mechanism of the household deleveraging process.

Thirdly, aggregate housing demand decreases at the shock and continues to fall for 20 periods. Aggregate housing demand declines because the low wealth high labor productivity households experience large drops in their labor income. To smooth consumption, they decrease net savings by demanding smaller houses which are their main saving instruments. However, the decline in housing demand is slightly smaller than that in the pure productivity shock. Since only part of the household borrowing is used to purchase housing assets in the steady state, the large decline of mortgage debt turns out to have little impact on aggregate housing demand as the part of the mortgage debt that households use to fund the purchase of houses is not affected. With tighter financial conditions, households borrow mortgage debts primarily for purchasing houses.

Foreclosure rate rises up 0.29% in the first period due to the decrease in real house price. Foreclosure rate experiences large drop since period 2 as households deleverage as soon as the financial change takes place in period 1.[24] Since the financial change is permanent which makes the leverage remain low, the foreclosure rate continues to stay at a smaller level than that in the benchmark economy.

In summary, the tightened financial condition is fully responsible for the substantial de-

[23] The risk free interest rate $= r$ while the risk free mortgage rate $= r + r_w$

[24] The increase in foreclosure rate in this experiment is much smaller compare to that in the data because the decrease in real house price is small in the model. The raise of the foreclosure rate is also very transitory because mortgage is only a one-period debt. Extending the mortgage debt to a long-term contract can generate more persistent increase in foreclosure.

cline in mortgage debt and part of the decrease in housing price. In contrast, the reduction in aggregate productivity can explain the drop in housing investment as well as the decreases in aggregate output, consumption, business investment and hours. Given that the initial decline in output is about 68% as large as that in the Great Recession, the two-shock experiment can account for the entire decline in mortgage debts and leverage, 1.3% of the decline in real house price, 9% of the drop in real housing wealth and 17% of the decrease in housing investment.

6.2 The new steady state with tighter housing finance

This subsection discusses the properties of the economy in the new steady state with tighter housing finance condition ($r_w = 0.0012$). As shown in table 4, aggregate output, consumption, investment, housing demand is roughly the same as that in the benchmark economy with the tighter housing finance condition. On the production side, variables such as capital, labor input and housing investment share barely change with the tighter finance condition. The equilibrium real wage and real rental price also change little. In contrast, the risk free interest rate has reduced from 1.25% to 1.225%. Thus the effective mortgage interest rate actually declines after the financial change. The large drop of the outstanding mortgage debts does not conflict with the smaller risk free mortgage interest rate because it is the difference between the risk free interest rate and the risk free mortgage rate rather than the absolute cost of mortgage borrowing that matters critically in household borrowing and saving decisions.

Several major changes take place in the housing market. Aggregate mortgage debt falls to around 77% of its benchmark value, foreclosure rate drops from 0.42% to 0.38%. Homeownership rate, which is represented by the percent of households with $h' > s$, decreases from 48.3% to 48.1%. If I use the percent of households with $h > 0$ to characterize homeownership rate, homeownership rate is slightly larger with tighter housing finance. Non-housing asset now takes up 28.3% of net household wealth, which is much smaller compare to 32.9% in benchmark economy. In addition, tighter housing finance leads to larger wealth inequality as the Gini coefficient is 0.13% higher than that in the benchmark economy. The financial change tends to have little impact on aggregate housing demand and real house price only falls 0.11%. Finally, the tighter borrowing in the mortgage market leads to a large increase in home equity as shown in Figure 16. The increase in home equity implies that a larger share of the household borrowing is spent on purchasing houses in the new steady state.

Table 4: Numerical Results of Higher Financial Intermediation Cost

Variable	Interpretation	Benchmark value ($r_w = 0.0008$)	High financial cost ($r_w = 0.0012$)
r	real interest rate	1.25%	1.225%
w	real wage rate	1.0	1.0
p	real housing price	0.931	0.930
p_s	real rental price	0.0275	0.0276
H	housing stock	1.833	1.828
Ml	mortgage loan	0.693	0.542
Default rate	foreclosure	0.42%	0.38%
Mean net worth	Mean net worth	1.80	1.80
K	capital	0.116	0.121
N	labor	0.33	0.33
C	consumption	0.307	0.307
Y	output	0.336	0.336
I	business investment	0.002	0.002
I_h	housing investment	0.0283	0.0293
$\mu(h' > 0)$		97.5%	97.8%
$\mu(h' > s)$	homeownership	48.3%	48.1%
Wealth Gini	wealth inequality	0.5047	0.5053
$pH/(4 \times GDP)$	housing wealth to output	1.27	1.27
Non-housing asset	non-housing asset share	32.9%	28.3%
pI_h/GDP	housing investment share	7.8%	8.1%
$K_h/(4 \times GDP)$	business capital in housing sector	0.086	0.09

7 Concluding remarks

This paper develops a dynamic stochastic general equilibrium model with heterogeneous households and two sectors to study the housing market in the Great Recession and explore whether the financial shock has contributed to the large decreases in real mortgage debts and house prices. I have calibrated the model to reproduce the housing and non-housing wealth distribution in the data. The resulting economy is characterized by the household behavior of saving using risk free financial assets, risky housing assets as well as borrowing using mortgage debts at the same time.

Two experiments have been conducted in order to understand the housing and real economic contractions in the Great Recession. Comparing the results in the two experiments, I have come to the following conclusions. Firstly, the decline of productivity is responsible for the real economic recession and the reduction of housing construction. Housing wealth shrinks as households who experience large declines in their labor income demand less housing assets. Secondly, both the reduction of aggregate productivity and the tighter financial markets contribute to the decline in real house price. On the one hand, although the reduction of house price is alleviated by the smaller productivity in the housing sector, the falling productivity in the consumption good sector makes consumption goods more rare and thus decreases the real house price. One the other hand, the tighter financial condition aggravates the decrease of real house price. Given the falling productivity in the mortgage issuance sector, households demand smaller mortgage debts which lowers the equilibrium interest rate. With smaller capital rental price in the housing sector, the real housing price declines further.

Thirdly, the tightening of the financial condition is fully responsible for the substantial decrease of mortgage debt as the cost of borrowing mortgage to save financial assets have greatly increased. Under normal financial conditions, households take large mortgage debts to take advantage of high housing return and insure themselves against high default risk by holding financial assets to smooth consumption. When the financial condition is tighter so that it is costly to borrow mortgage to save financial assets, households sharply decrease leverage and their holdings of mortgage debts to avoid large interest payments and default risk.

Although this is the first paper that has studied the housing market with endogenous real housing price and mortgage default options under a heterogeneous agent framework, this paper can still be strengthened in three respects. Firstly, the increase of foreclosure rate in the Great Recession experiment is relatively transitory because mortgage is only a one-period debt in the model. The foreclosure boom could be more persistent and pronounced as in the data if the mortgage contracts are long-term. Secondly, the consumption good sector is relatively labor intensive, which results in a large investment volatility. This problem can be fixed by adding model elements that give households additional incentives to save risk free financial assets besides the incentives of using financial assets to insure themselves against default risk and labor productivity risk. Accordingly, the economy could generate enough capital to allocate between the consumption good sector and the housing sector. In this case, the high investment volatility would also disappear. Thirdly, the decline in the real house price in the model is comparable to the reductions of house price in most U.S. post war recessions, but the decrease is relatively shallow compares to the collapse of the house price in the Great Recession. One way to fit the large decrease of real house price is to generate a housing price bubble in the steady state. Considering the complexity of the current model, I leave these to future research.

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Appendices

A Data

I generate Figure 1 using the following data between 1969Q1 and 2013Q1. Housing wealth is the market value of the household owner-occupied real estate including vacant land and mobile homes from the Flow of Funds Accounts of United States. Data on units of new residential construction started is from the U.S. Census Bureau between 1969M1 to 2013M3. I convert the monthly data into quarterly data by taking sum of the monthly data in a quarter. The house price index is from the Freddie Mac which is available between 1975M1 and 2013M3. I obtain the quarterly price index by taking average of the monthly data in a quarter. Mortgage debt is obtained from the Board of Governors historical database. Housing foreclosure rate is reported in the National Delinquency Survey from 2004 to 2012 by the Mortgage Banker Association. Leverage is measured using real mortgage debt divided by real housing wealth. All real variables are obtained using GDP deflator which is measured using Gross Domestic Product (billions of dollars) from Table 1.1.5 divided by Real Gross Domestic Product (billions of chained 2005 dollars) from Table 1.1.6 of the Bureau of Economic Analysis.

For data that generates Figure 2, output is the Real Gross Domestic Product (billions of chained 2005 dollars) from Table 1.1.6 of the BEA. Consumption is Personal Consumption Expenditures less durable goods from Table 1.1.5 of the BEA. Business investment is the sum of durable goods and private nonresidential fixed investment from Table 1.1.5. Residential investment is also from Table 1.1.5. The real values of consumption and investment are calculated using the GDP deflator. Real GDP, consumption, business investment and residential investment are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600 from 1969Q1 to 2013Q1. Data on unemployment rate is from the Bureau of Labor Statistics. The Solow Residual is measured using data on private capital and hours. Private capital is the sum of private fixed assets and consumer durables from Fixed Asset Table 1.1 of the BEA. Hours data are those constructed by Cociuba, Prescott and Ueberfeldt (2012).

B Solution Methods

B.1 Consumption-Savings Problem

Following Jeske, Krueger and Mitman (2011), household's problem in this model can be transformed into the following consumption-savings problem

$$v(x, \epsilon) = \max_{c, g', n} u(c, 1-n; p_s) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon)\omega(g', \epsilon') \tag{23}$$

subject to

$$c + g' = w\epsilon n + x \tag{24}$$

$$0 \leq n < 1, \quad g' \geq 0 \tag{25}$$

where $\omega(g', \epsilon')$ is given by

$$\omega(g', \epsilon') = \max_{h', m', a' \geq 0} \int_{\underline{\delta}}^{1} v(x', \epsilon') dF(\delta') \tag{26}$$

s.t.

$$x' = (1 + r')a' + \max\{0, p'(1-\delta')h' - m'\} \tag{27}$$

$$g' = a' + (p - p_s)h' - m' p_m(m', h') \tag{28}$$

In order to solve the consumption-savings problem, I need to solve $\omega(g', \epsilon')$ first. I call this optimization problem that solves $\omega(g', \epsilon')$ the household portfolio problem. The portfolio problem is represented by equations (26)-(28) which involves selecting houses, financial assets and mortgage debt to maximize expected value.

B.2 The Steady State

1. Build grid points for individual labor productivity, net worth x and net saving g. I use 21 log-spaced points over $[0, 8]$ for g, 25 log-spaced points over $[0, 8]$ for x. I approximate the continuous AR(1) process for labor productivity with a 5 state Markov chain using the procedure in Tauchen and Hussey (1991).

2. Set guesses of real interest rate r and real rental price p_s.

3. Given r and p_s, do the following to find the value function and optimal decision rules:

(i) Set a guess for value function $v(x, \epsilon)$

(ii) Solve the household portfolio problem and find $\omega(g, \epsilon)$ for each (g, ϵ) on the grid.

(iii) Solve the consumption-savings problem represented by equation (23)-(25) to update value function and find the optimal net saving g^* for each (x, ϵ) on the grid.

(iv) Given $g^*(x, \epsilon)$, find the optimal asset decision rules $h^*(x, \epsilon)$, $m^*(x, \epsilon)$ and $a^*(x, \epsilon)$ using the decision rules solved in (ii).

4. Compute the steady state distribution of individuals over net worth, idiosyncratic productivity and default choice $\mu(x, \epsilon, d)$.

(i) I use 500 evenly spaced grid points over $[0, 8]$ for x and the same grid points chosen in step 1 for ϵ.

(ii) Approximate the housing, asset, mortgage decision rules solved in Step (3) using splines. For households starting with (x, ϵ, d), use the decision rules to determine the cutoff depreciation rate $\delta^* = 1 - \frac{m'}{p'h'}$. Then $1 - F(\delta^*)$ fraction out of $\mu(x, \epsilon, d)$ defaults and $F(\delta^*)$ fraction repay.

(iii) For people starting with (x, ϵ, d), they have the same δ^* and decision rules, but they might end up with different net worth next period since housing depreciation δ draw is idiosyncratic. I simulate δ draws for households at (x, ϵ, d) using Pareto distribution and calculate their future net worth x' correspondingly.

(iv) Suppose future net worth is such that $x_j \leq x' \leq x_{j+1}$ for a individual at (x, ϵ, d), where x_j and x_{j+1} are two adjacent net worth grid points. Then update μ assuming that individuals at (x, ϵ, d) move to (x_j, ϵ', d) with probability $\varphi \pi_\epsilon(\epsilon'|\epsilon)$ and to (x_{j+1}, ϵ', d) with probability $(1 - \varphi)\pi_\epsilon(\epsilon'|\epsilon)$, where $\varphi = \frac{x_{j+1} - x}{x_{j+1} - x_j}$. Update μ until it converges. The resulting μ is the fixed point of household distribution $\bar{\mu}$

5. With $\bar{\mu}$, calculate aggregates in the economy and solve the production problems. Then check market clearing conditions. If rental market and consumption good market do not clear, update guesses of r and p_s and come back to step 3.

B.3 Transitional Dynamics

The transitional dynamics is solved following three steps:

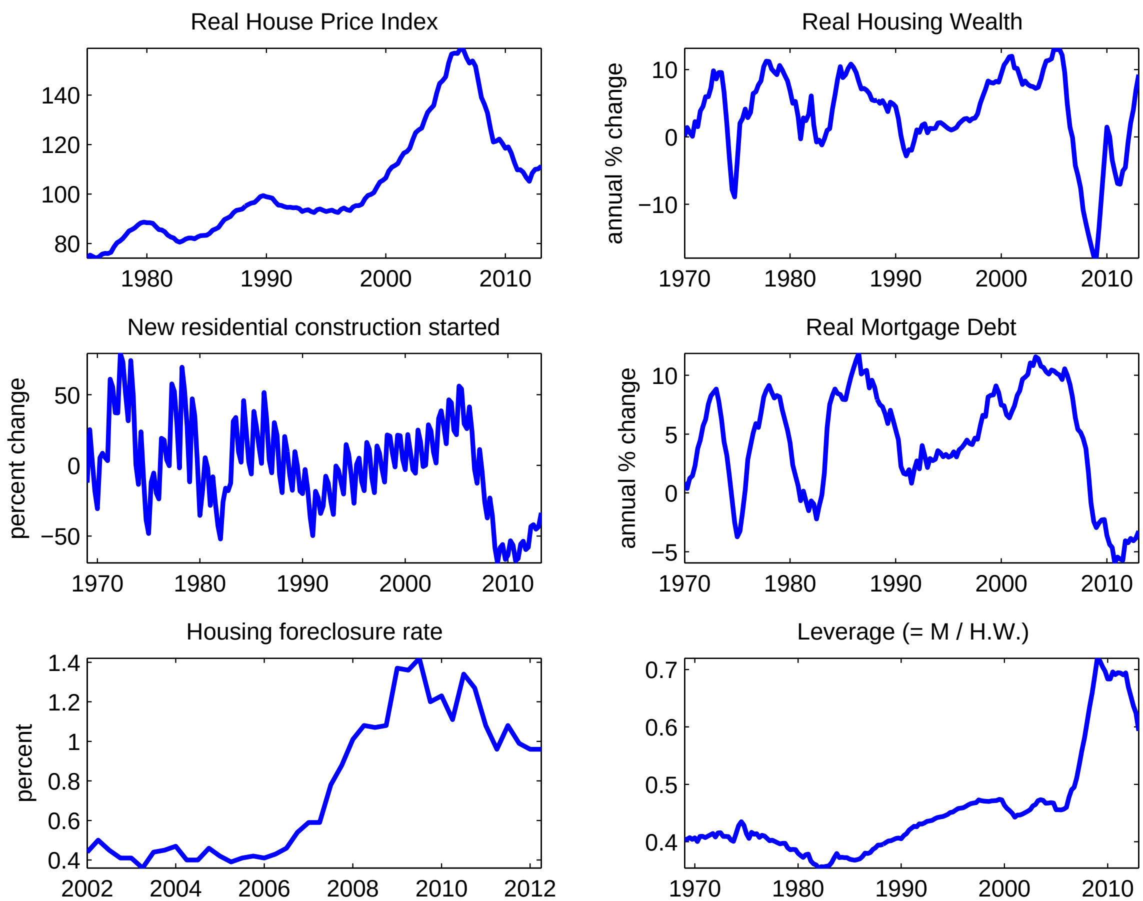
(1) Start with steady state value function and solve the value function backwardly from $t = T - 1$ to $t = 1$

(2) Start with steady state distribution, update the household distribution forwardly from $t = 1$ to $t = T$

(3) Check the market clearing conditions in each period to update interest rate and rental price.

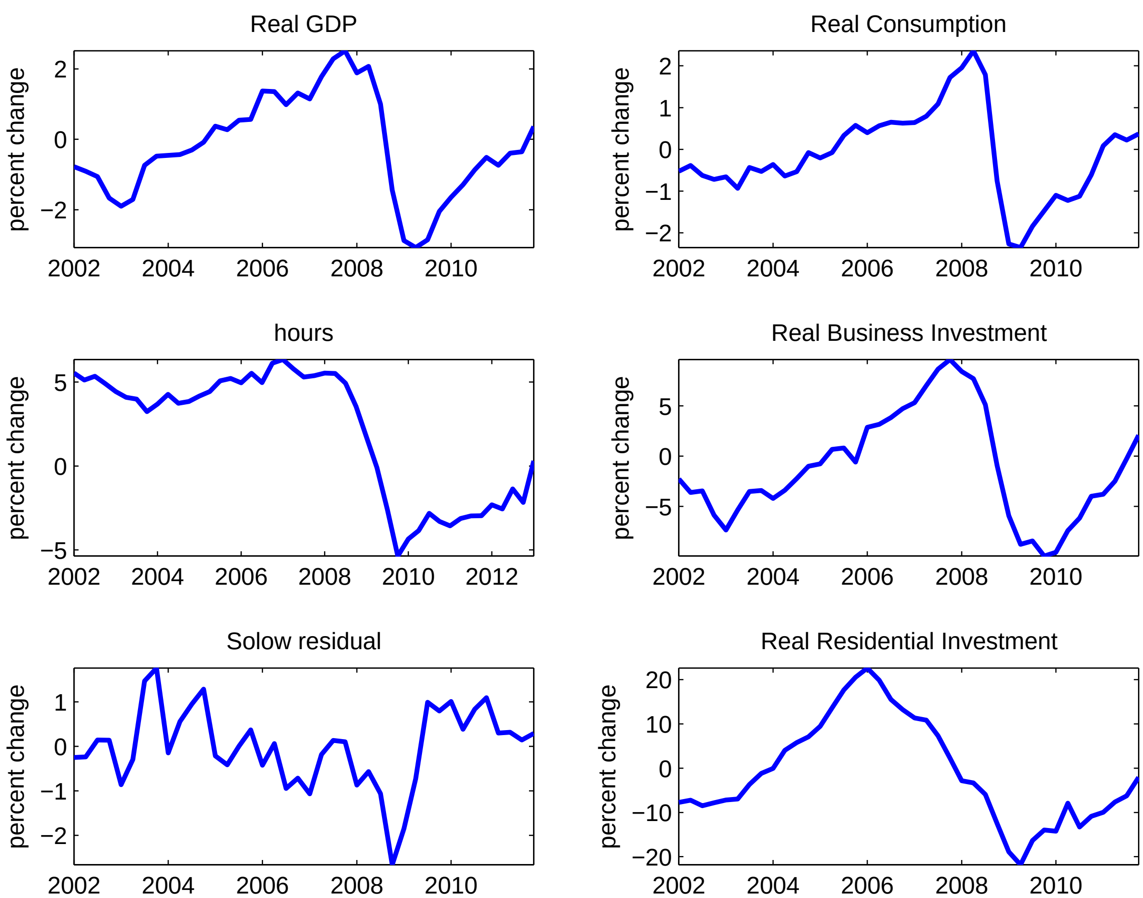
The length of time $T = 150$ in the negative productivity shock and $T = 300$ in the Great Recession Experiment. Since households in this model are forward looking, they look at current as well as future prices to make decisions. When the period t prices are updated, they have impact over the markets in period $t-1$ and t. Therefore, I use a small parameter 0.001 to update prices. The precision for value function $= 1.0e-4$. Precision for household distribution $= 1.0e-6$. Precision for transitional dynamics $= 1.5e-3$. All other precisions $= 1.0e-6$.

Figure 1: Housing Market Facts



Data Source: Freddie Mac, Board of Governors, Flow of Funds, U.S. Census Bureau, M.B.A

Figure 2: Macroeconomic Facts in the Great Recession



Data Source: NIPA tables, Fixe Asset tables and CPU (2012)

Figure 3: Mortgage Price Schedule

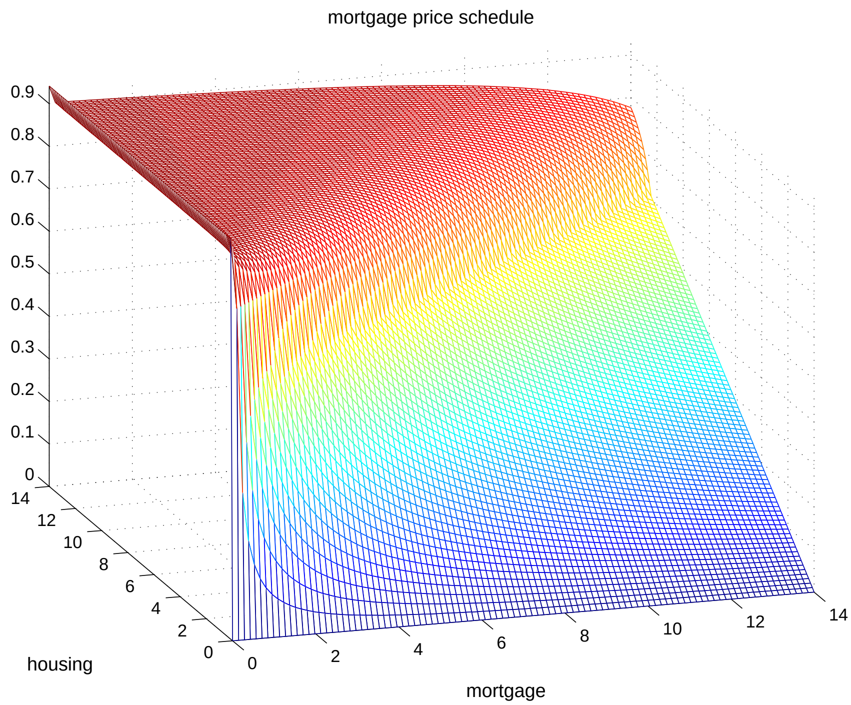


Figure 4: Household Value Function

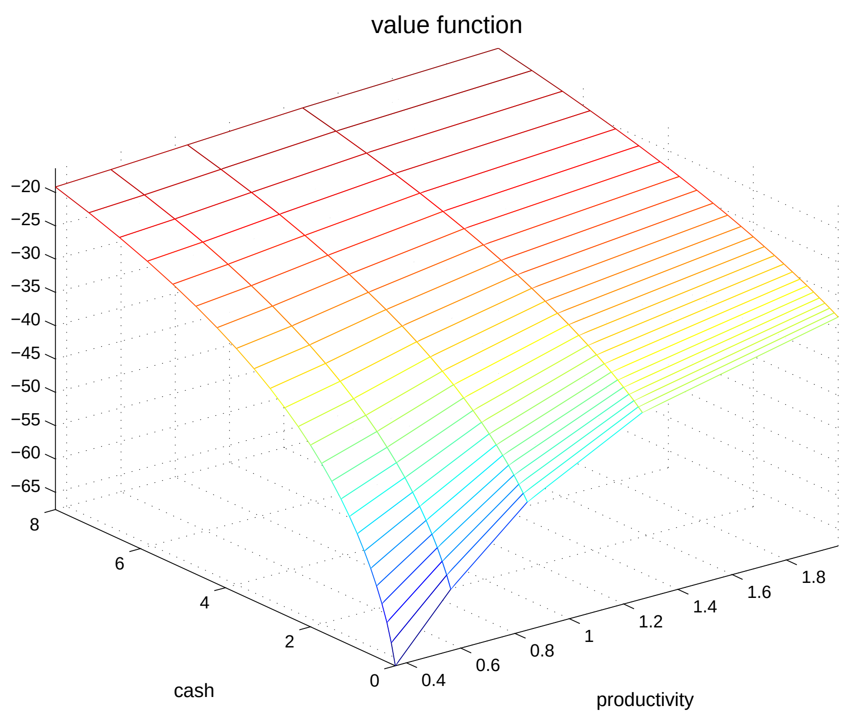


Figure 5: Net Saving Policy

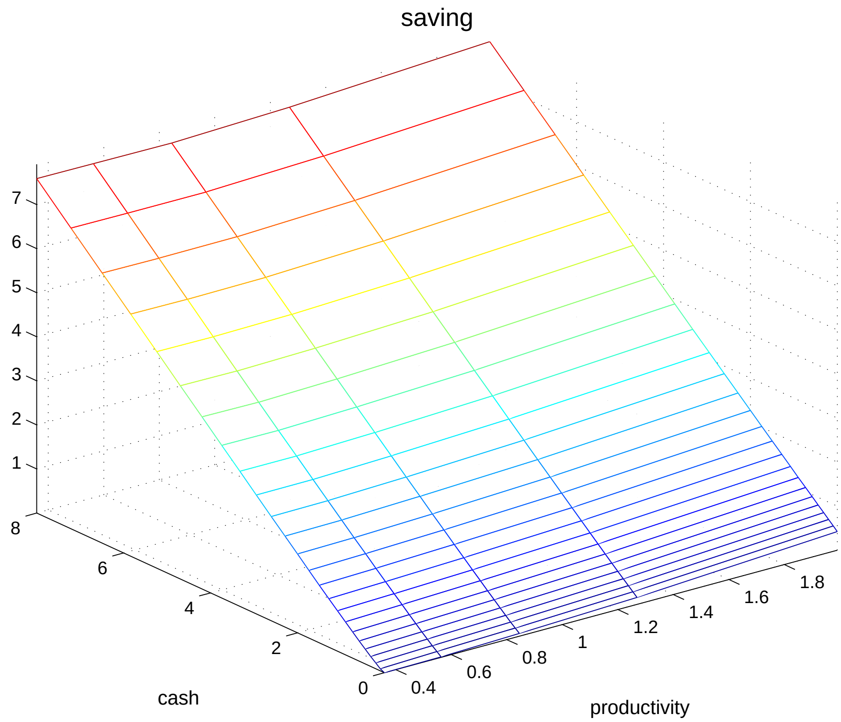


Figure 6: Housing Asset Policy Function

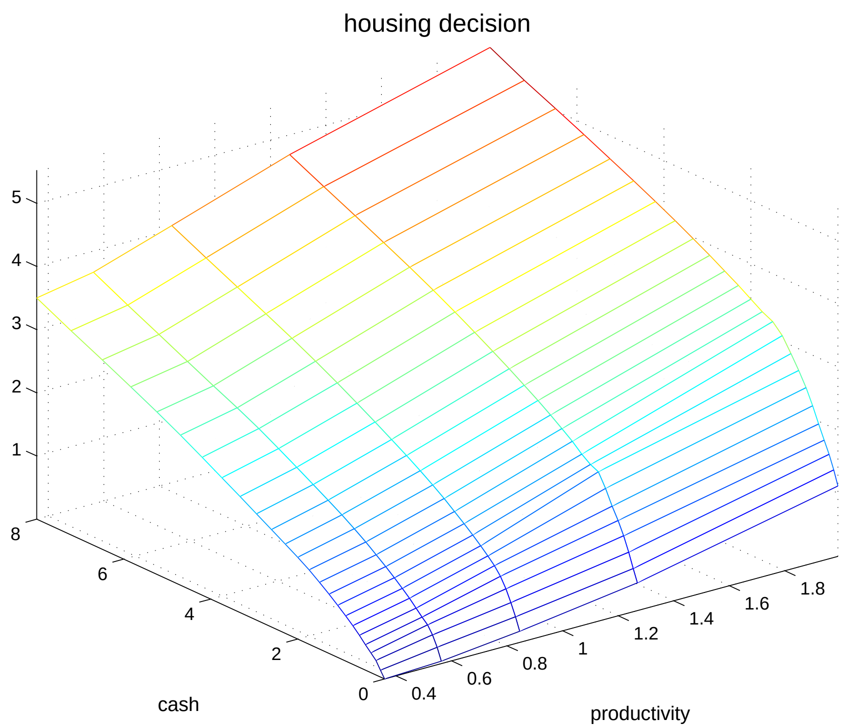


Figure 7: Household Leverage Policy, l=m/(ph)

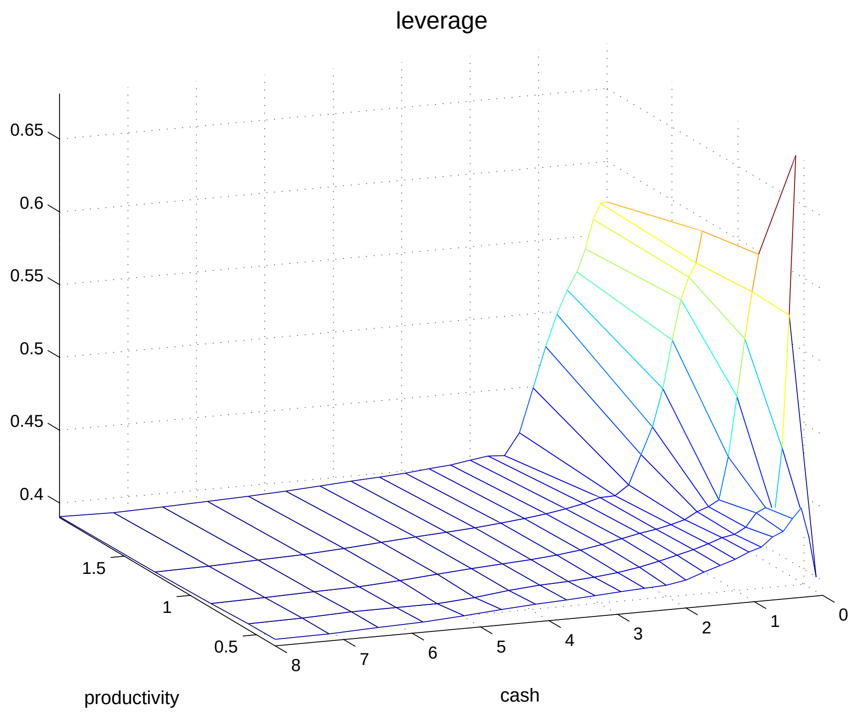


Figure 8: Financial Asset Policy

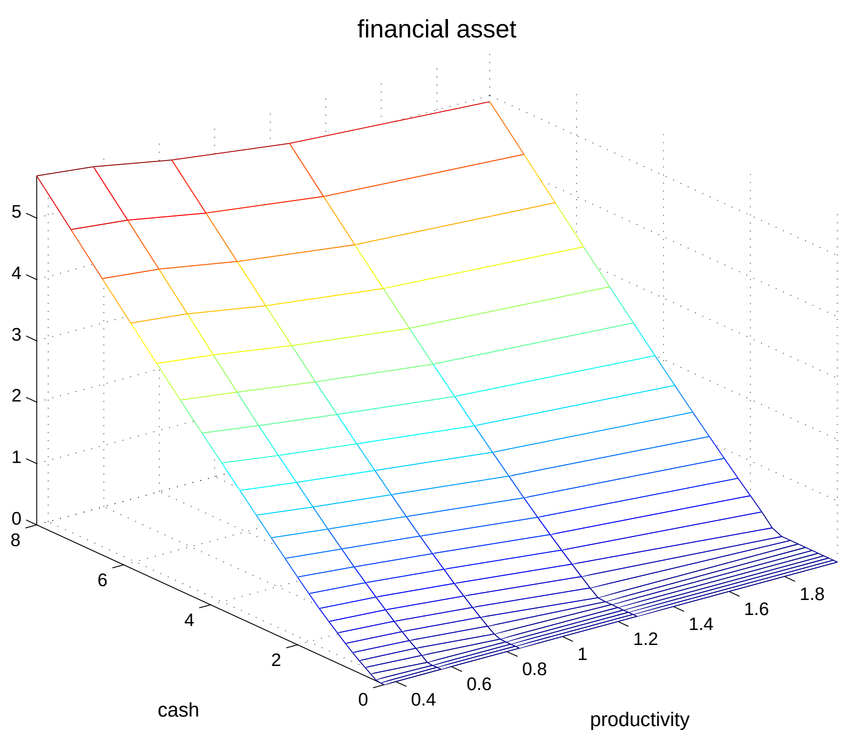


Figure 9: Labor Supply Policy

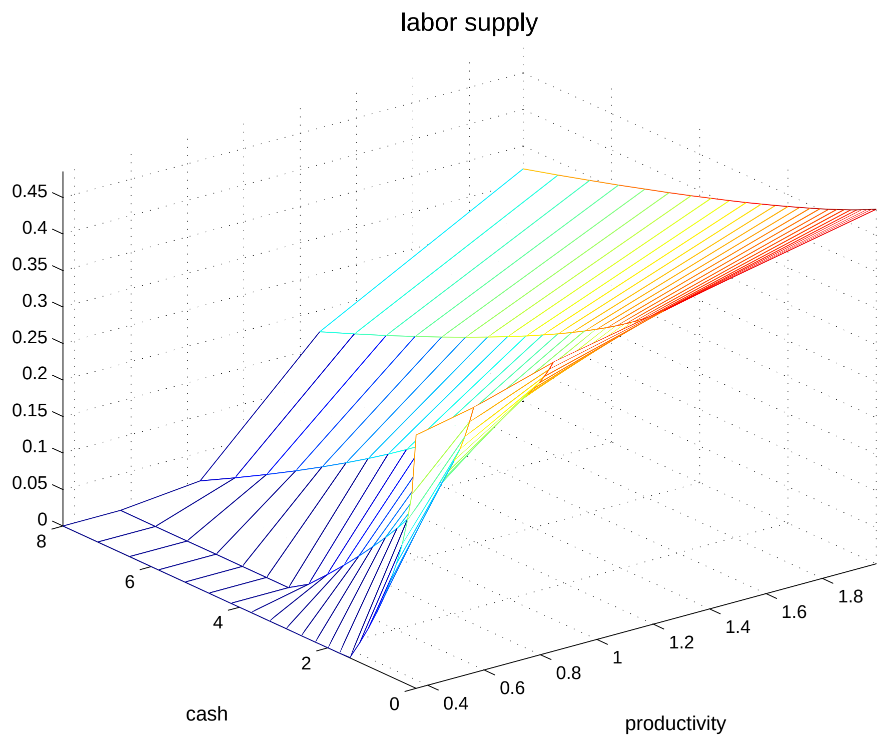


Figure 10: Negative Productivity Shock

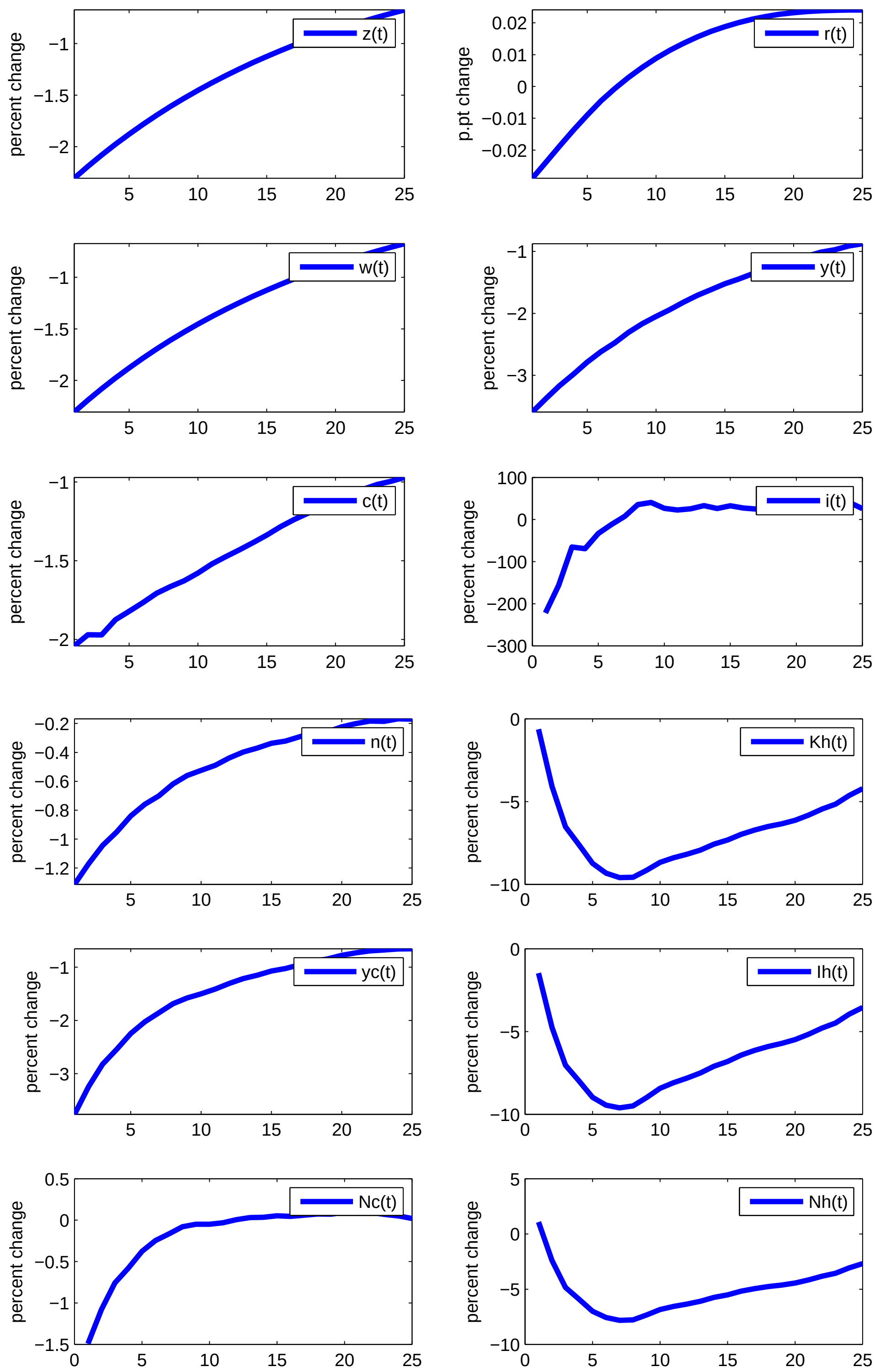


Figure 11: Negative Productivity Shock: Housing Market Variables

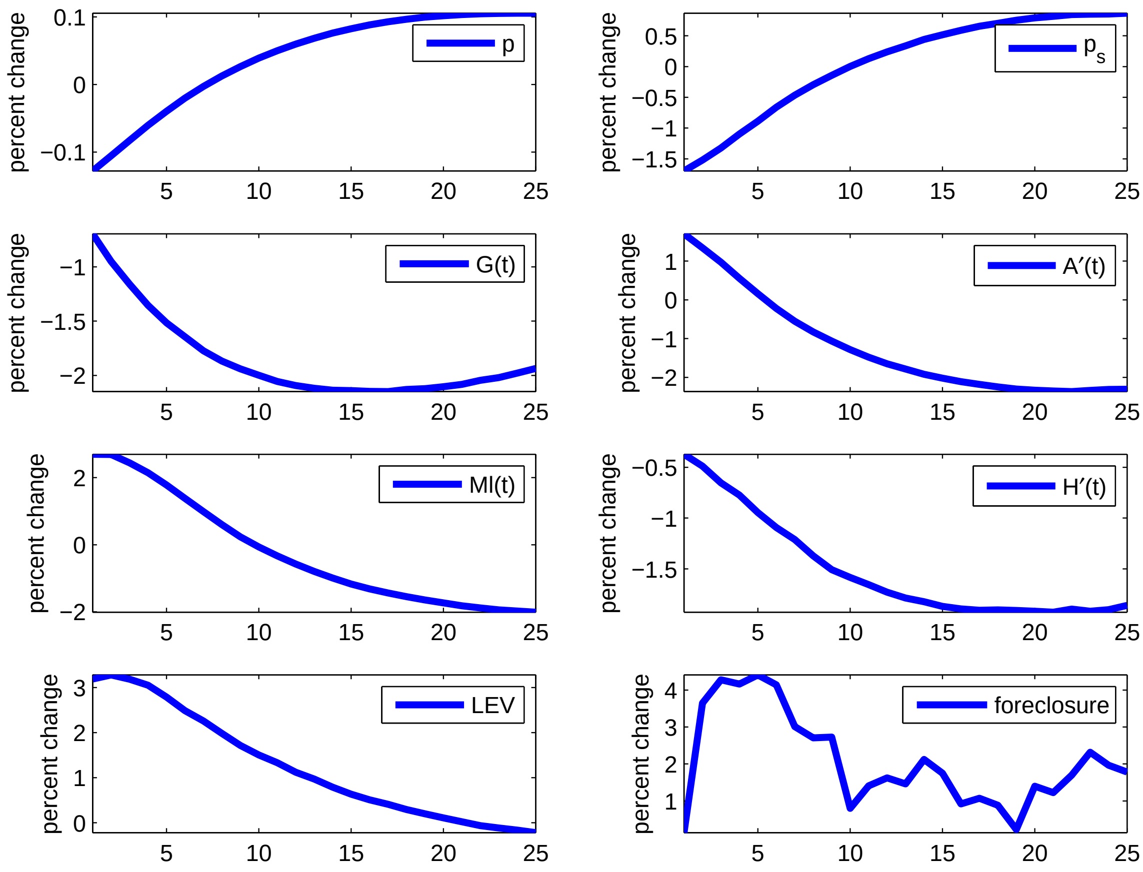


Figure 12: Shifts of Household Leverage Policy in the Productivity Shock

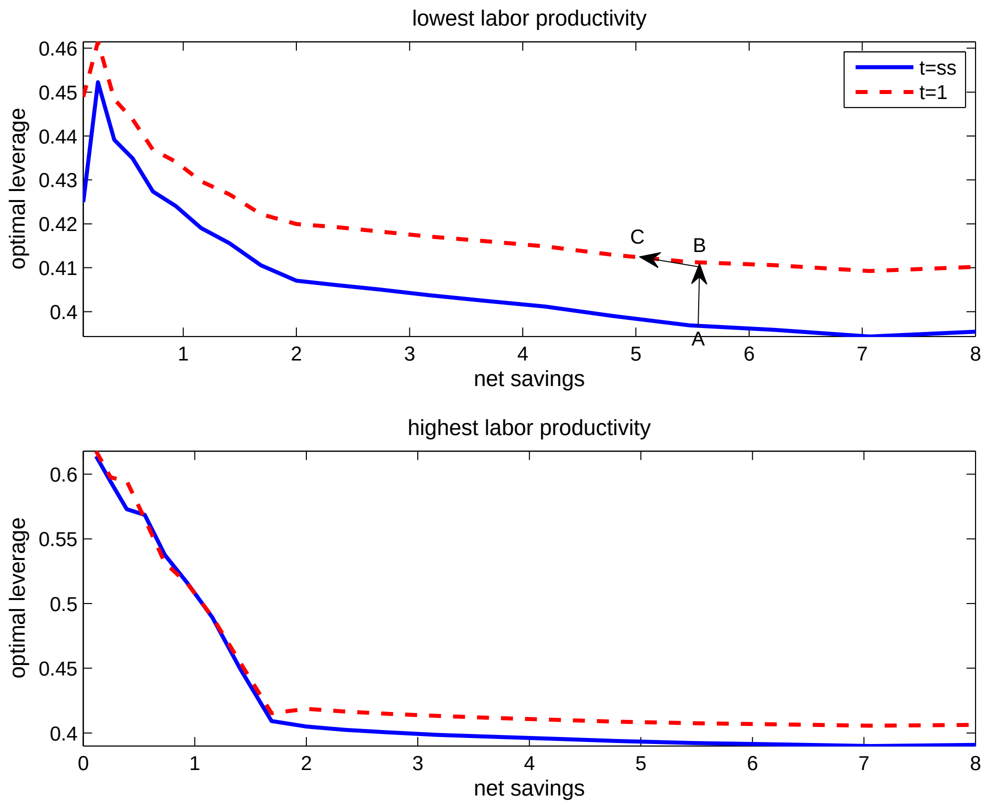


Figure 13: The Great Recession Experiment

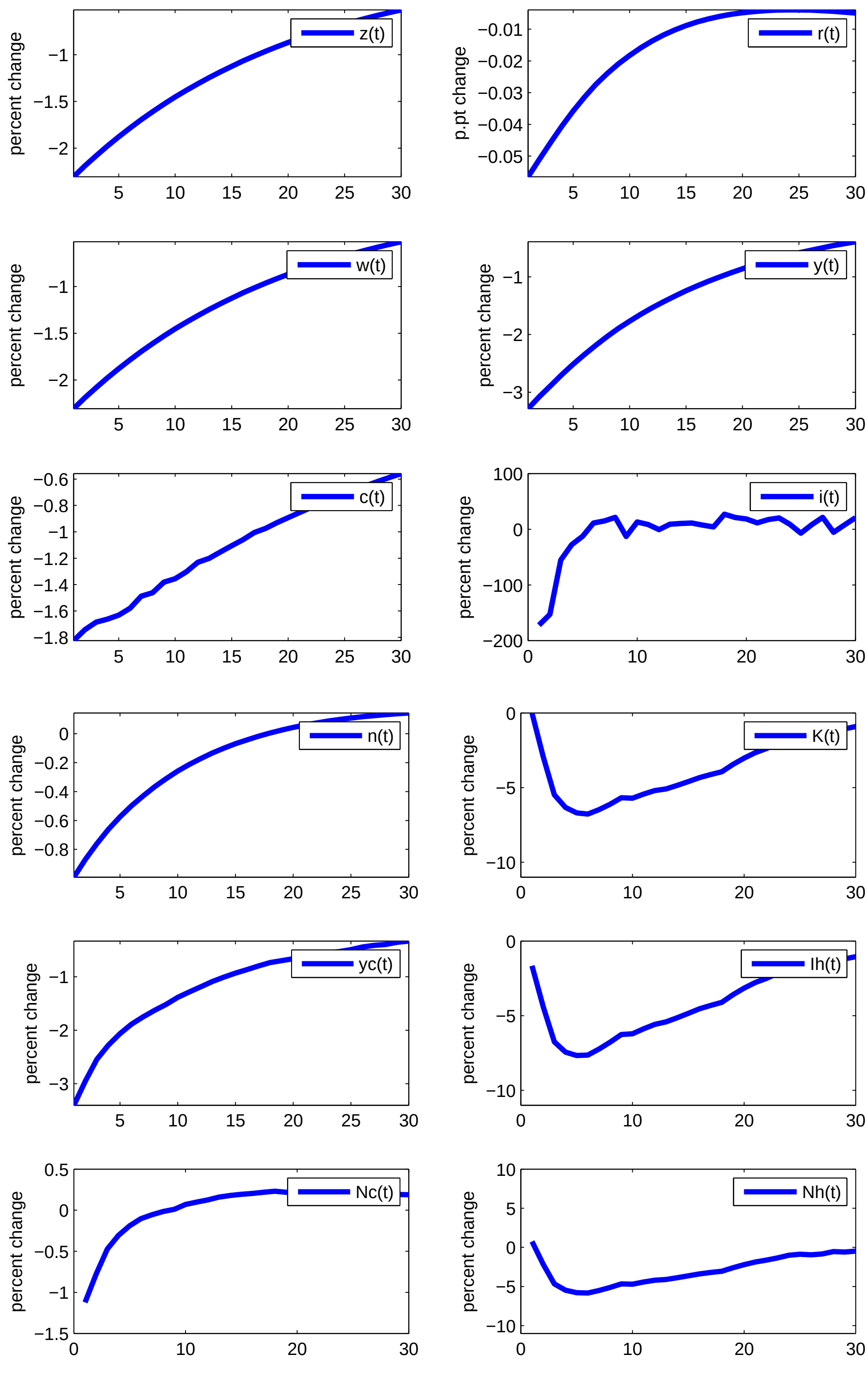


Figure 14: The Great Recession Experiment: Housing Market Variables

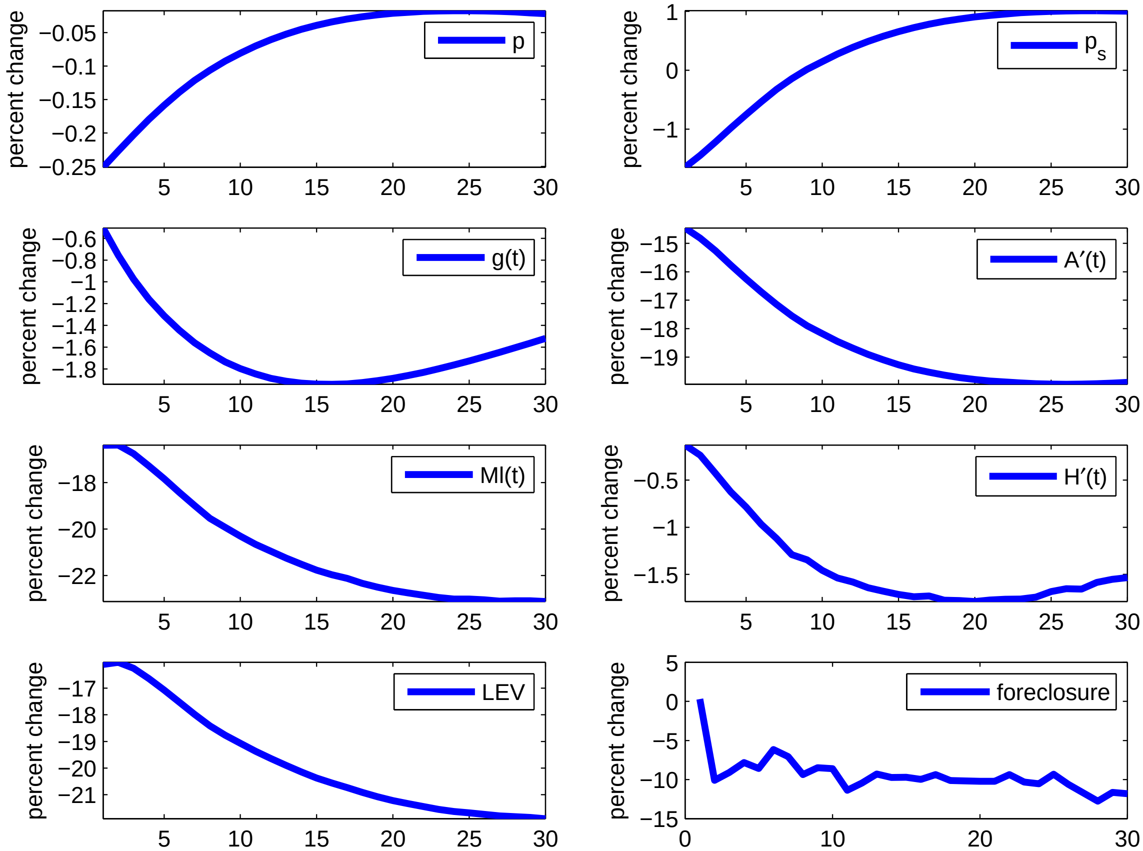


Figure 15: Shifts of Household Leverage Policy in the Great Recession Experiment

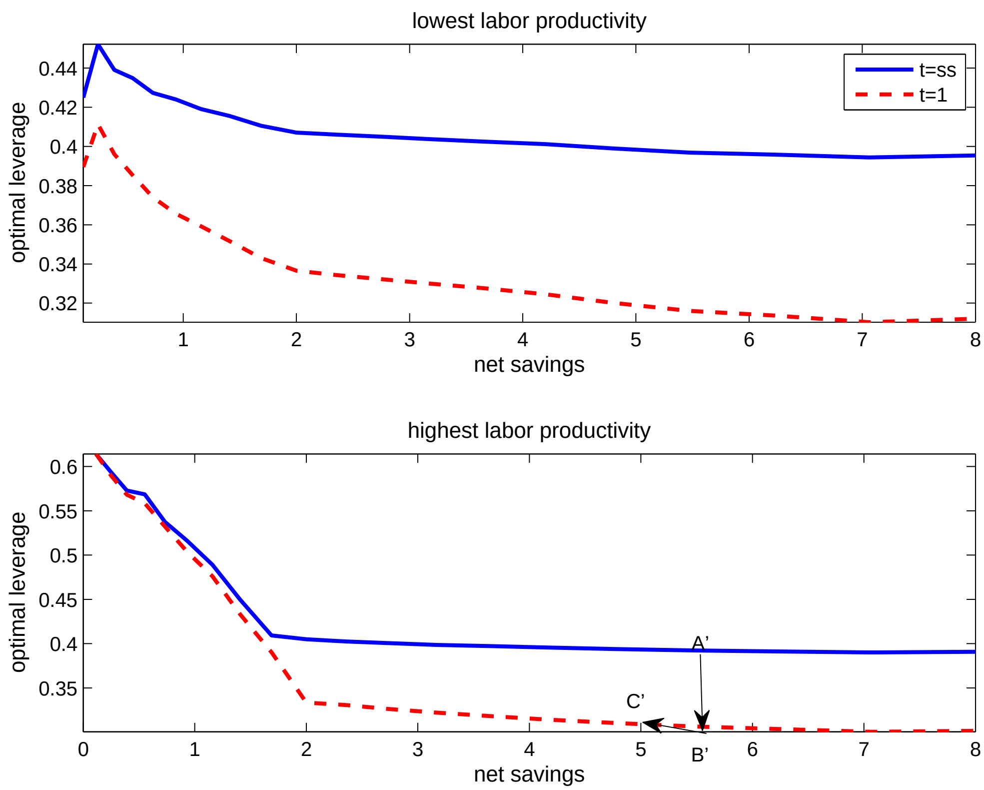


Figure 16: Home Equity with the Tighter Financial Condition