Entry Barriers in a Two-Sector economy with Perfect and Imperfect Financial Markets

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Abstract

In this paper, I analyze the impact of entry barriers on a two-sector economy with near-perfect and imperfect financial markets. The literature suggests that higher barriers to entry would hurt the economy through occupational and factor misallocation. However, a separate analysis of economies with nearly perfect and imperfect financial markets shows that these results only hold for the economies with imperfect financial structures. In the economies with near-perfect financial markets, the entry barriers have almost no impact or may positively impact output or total factor productivity (TFP). This study shows that higher entry costs would hurt the productivity of the sector with high concentration, i.e., with large-scale firms, and would benefit the more competitive sector, i.e., with many small firms. As a result, the entry barriers might help or hurt economies depending on their sector/industry structure. To analyze the dynamics of entry barriers and their impact on TFP, I develop an entrepreneurship model in continuous time with two sectors in the presence of both financial and physical frictions. My analysis suggests that higher entry barriers would help the economies with a relatively high share of the small-scaled sector and vice versa.

1 Introduction

Explaining the productivity gap across countries has taken lots of attention in macroeconomics literature. Misallocation of resources is identified as one of

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the most important sources of these productivity differences across countries. See Banerjee and Duflo (2005), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009) for good discussions on resource (mis)allocation and total factor productivity.

The literature has explored three main channels through which misallocation affects productivity: the barriers to entry, which could be due to regulations or entry costs; factor adjustment costs such as investment or hiring/firing costs, and; financial constraints, which means how easy it is for entrepreneurs and firms to access financing.

In the macro development literature, entry barriers are considered an important source of misallocation that can explain the TFP differences between countries. The main channel is that a high entry cost would deter an entrepreneur from starting her business, which could be for two reasons: The entrepreneur does not have the money or thinks the entry cost is too much for the level of business she has in mind. This paper's main focus is analyzing the barriers to entry while interacting with financial and investment friction. More specifically, I want to explore the impact of entry costs on aggregate outcomes under different financial and investment structures. Financing frictions help us distinguish between the economies with perfect and imperfect financial markets and see the differences in the impact of entry barriers between them.

My analysis focuses on two important channels through which the entry barriers impact TFP: Selection by wealth, and selection by productivity. The first channel creates misallocation and keeps productive but poor entrepreneurs from entering the market. This channel depends heavily on the financial markets and is of less issue in the economies with good financial markets. The impact of the second channel on productivity is vague, and its interaction with investment friction and sectoral build of the economy determines the direction of change.

With imperfect financial markets, we have the standard mechanism explained in the literature. That is, entry costs will hurt the economy through occupational and factor misallocation. For example, a highly productive but poor individual won't be able to pay the entry cost and start her business because of her limited access to financing. On the other hand, a wealthy individual who is less productive can actually enter the business by paying the entry cost out of her pocket. This will create the occupational misallocation. Now imagine, the poor individual somehow manages to pay the entry cost and start the business. Obviously, the amount of capital that she can start her business with would be very low and not optimal because she has spent most of her savings to pay for the entry cost. The wealthy individual, on the other hand, can start business with the optimal amount of the capital. This is the factor misallocation channel.

The mechanism explained above does not fully apply to the perfect financial markets. The reason is that the productive but poor individual can now finance most of her entry cost as well as the capital she needs to start her business. As more productive individuals enter the economy and the prices (including interest rate and wages) change, the opportunity cost of entry will be high for unproductive but wealthy individuals. As a result, they will be lending their money to the productive entrepreneurs. This will weaken both the occupational and factor misallocation channels substantially.

If an economy with perfect financial markets is populated with many small-scaled firms, and there's an adjustment cost of investment, then higher entry costs may increase the productivity. First reason is similar to the argument above, which says the entrants now will be more productive. That is, the highly productive entrants will make the productivity go up on average which will create a tough competition for unproductive incumbents. The second reason is that, when entry cost increases, because of the adjustment cost of investment, only the very productive ones will find it cost-effective to adjust their capital. The unproductive ones will not be able to adjust their capital until the opportunity cost will be so high that will force them sell their capital and exit. This will create more room for entrepreneurs with higher productivity as the leaving entrepreneurs will lend their money to entering ones.

But, if the economy with perfect financial markets is populated with few large-scaled firms, a higher entry cost would decrease the productivity. To see how let's consider the extreme case, a pure monopoly. A monopolist is already highly protected against competitors, and when entry cost increases even more, it becomes even harder for entrants to get in. A monopoly also implies a great amount of capital which requires lots of resources. Even in a nearly perfect financial market, lender needs to account for depreciation of the capital as well as the irreversibility component of the adjustment cost. That introduces some frictions to financing, which, along with the increase in the entry cost, will make it almost impossible for potential entrants to start business.

Now consider an economy with perfect financial markets that has two sectors, one populated with large-scaled firms and the other with small-scaled firms. In such a case, the impact of entry cost on aggregate outcomes will depend on the sectoral structure of this economy meaning which sector is dominant and how large- or small-scaled (or how capital-intensive) the sectors are. It will also depend on the costs of adjusting capital in that economy.

There's also another channel to see the impact of entry costs on aggregate outcomes. If we think of the entry cost as a sunk cost to the producer and the economy, higher entry costs hurt the economy in the intensive margin (more is paid by the entrants) but help it in the extensive margin (decrease in the number of entrants). This sunk cost is lost to the economy which could be used as capital in production or as consumption which would increase the welfare in the economy. Therefore, an analysis of elasticity of entry with respect to entry cost might be helpful.

All being said, analysis of the impact of entry costs on aggregate outcomes requires inclusion of all the features mentioned above. That's the reason I build an entrepreneurship model with two sectors (a large-scaled and a small-scaled) and with investment adjustment costs including both convex and non-convex components. The model also includes financial frictions in the form of collateral constraint. We abstract from labor adjustment costs to avoid complication of the model. Also, Shaker-Akhtekhane (2018) shows that inclusion of capital adjustment costs is sufficient in explaining capital misallocation channel which is of greater importance in this paper.

2 Related Literature

This paper is tied with three related branches of macroeconomics literature: entry barriers, financial development, and adjustment costs.

Djankov et al (2002) is one of the main papers to analyze the impact of entry barriers on a macro scale. They create entry cost data using startup costs from 85 countries, and show that there's no good in having high entry barriers, but maybe for politicians. They, however, do not control for the effects of financial structure of the countries, which is included and is of great importance in my paper.

Barseghyan (2008) shows a reverse relation between entry costs and TFP, and Barseghyan and DiCeico (2011) have a similar model with industry structure indicating the same results. One main difference between this paper and theirs is the timing of the models. Barseghyan and DiCeico (2011) following Hopenhayn (1992) and Hopenhayn and Rogerson (1993) assume that entrants pay the entry cost before observing their productivity, but I will take the reverse order similar to Buera et al (2011). That is, an entrepreneur knows the quality of her idea before deciding to operate or not. This simple change would make huge difference in the results. Another difference with my work is that they do not consider the capital adjustment cost and financial frictions in their model.

Poschke (2010) uses technology choice in his model and explains the productivity differences between Euro economies and the U.S. through the lens of entry cost. He also builds on Hopenhayn (1992), and introduces technology choice as well as differentiated products as in Blanchard and Giavazzi (2003). We introduce two sectors in this paper and don't have differentiated intermediate products to avoid computational burden. I also couldn't think of a channel through which adding that feature would enrich my model. Moscoso-Boedo and Mukoyama (2012) analyze the impact of entry regulations and fixed firing costs in the form of taxes on productivity and try to explain the cross-country income differences. Their analysis suggests that entry costs will reduce the productivity by making firms grow inefficiently large. A main difference with the present paper is that they abstract from financial frictions. Another difference is the timing as explained above.

From the perspective of financial friction, this paper is closely related to Buera, Kaboski and Shin (2011). They do a quantitative steady state analysis trying to explain the relationship between aggregate/sectoral TFP and financial development. They don't have the entry cost as well as the capital adjustment costs. As a result, they don't have the exit friction which is important to analyze the exit channel caused by investment adjustment costs. They also have weak occupational switch channel which I strengthen through inclusion of entry costs and investment adjustment costs. This will help me analyze the impact of misallocation on aggregate outcomes in both intensive (entrepreneurs with non-optimal amount of capital) and extensive (productive agents not starting their own businesses or got stuck in the wrong sector) margins. Buera and Shin (2013) do a similar exercise on the impact of financial frictions on TFP, but their main focus is on transition dynamics, which I abstracted from in this paper, but such an analysis can be done in another related work.

In a related work, Moll (2014) develops a model to analyze the impact of financial frictions on productivity through capital misallocation. He studies transition dynamics with focus on the persistence of the productivity shocks which is shown to be of substantial importance. My model contains the shocks' persistence element for both sectors. But, as mentioned earlier, I will look at the transition dynamics in a separate work. Midrigan and Xu (2014) use plant level data to analyze the impact of financial frictions on TFP. They consider two channels, occupational misallocation and capital misallocation channels. Their findings emphasize the importance of the former channel. They look at the formal (modern) and informal (traditional) sectors of Korea, Columbia and China, but they abstract from capital adjustment costs in their model that has entry into the formal (modern) sector. Although the focus of my paper is on the impact of entry costs, I have included both channels through the inclusion of multiple frictions mentioned earlier. This gives my model enough richness to evaluate the impact of other frictions on TFP as well. There are many related works that look into the importance of financial systems. See Jeong and Townsend (2007), Cole, Greenwood and Sanchez (2012), Buera (2009), Amaral and Quintin (2010) just to name few.

There's also a large literature on the impact of physical frictions on economic aggregates. Hopenhayn and Rogerson (1993), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) are the leading works that explain the impact of misallocation caused by factor adjustment costs on productivity and aggregate outcomes. Some other related papers are Bartelsman et al (2013), Asker et al (2014), Guner et al (2008) and Decker et al (2018). My work can capture a more extensive area since it includes financial frictions in addition to entry costs and physical adjustment frictions. However, the focus of this paper is different than those discussed above.

The use of continuous time methods in solving heterogeneous agent models is becoming more and more popular because of its advantages in numerical solution and tractability of the models in continuous time. See Achdou et al (2017) and references therein for the solution of heterogeneous agent models in continuous time. I have extended the finite difference technique used in Achdou et al (2017) to deal with more than two continuous states as well as the non-convexities that arise in the solution of my model.

One main contribution of this paper is introducing a mechanism to analyze the impact of entry barriers on cross-country productivity and output. My mechanism differs between the economies with perfect financial markets and those with imperfect financial markets, because the entry barriers act differently in each of these economies. I also use sectoral structure of the economies as well as the costs of adjusting capital to introduce different channels to explain the impact of entry costs on productivity and output.

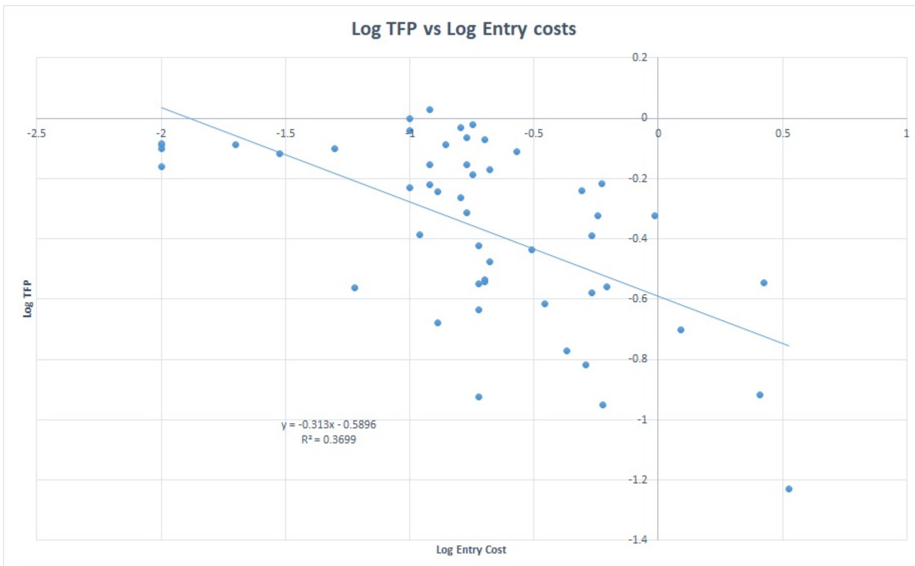
Another major contribution is creating a model that has both financial frictions and physical frictions in a two sector model. Presence of investment adjustment costs in the form of quadratic and partial irreversibility will generate occupational misallocation because of costly occupational switch and exit decisions. Also having the entry costs along with the frictions is another main feature of my model. The entry cost used in this paper is proportional to the scale with which the entrepreneur wants to start her business.[1] Including all these features makes my model very rich but these models become enormously complicated to solve. But the value of having such a model is that it can be used to answer several related questions too. For example, we can evaluate the importance of financial frictions in a two-sector environment with entry/exit and adjustment costs, or we might be interested in analyzing the effect of capital adjustment costs on aggregate outcomes. Either of these questions are important and have been studied in the literature, so it makes my modeling environment even more interesting.

I use two different criteria to distinguish the sectors: Different fixed per period costs as well as different factor shares. Buera et al (2011) only use fixed per period costs to differentiate between sectors. Midrigan and Xu (2014) use different production functions where one sector (traditional) only uses labor and the other sector (modern) uses both labor and capital. They also use a per period fixed cost in the modern sector but the nature of this shock is more individual-based rather than sector based because the cost is proportional to the idiosyncratic shock of the individuals. This choice will be explained in the next section.

There's also a main technical contribution which is related to solving the model in continuous time. I take advantage of time efficiency and tractability of solving the model in continuous time. I have extended the solution method for continuous time heterogeneous agent models with many states as well as those with non-convexities. To deal with many states, I have extended the creation of the sparse matrices that solve the HJBVI and KFE to any N-dimensional states. I have also introduced a new upwind method that uses two step backward and forward differences. This is improving the convergence of the model significantly. This extension can be of great importance in solving related heterogeneous agent models in continuous time because it introduces a new tool that would enable us to incorporate higher moments in Hamilton-Jacobi-Bellman (HJB) and Kolmogorov Forward equation (KFE) and solve them using finite difference method.

[1] The related literature mainly uses a fixed entry cost.

Figure 1: Cross country TFP and start-up costs



3 Facts from Data

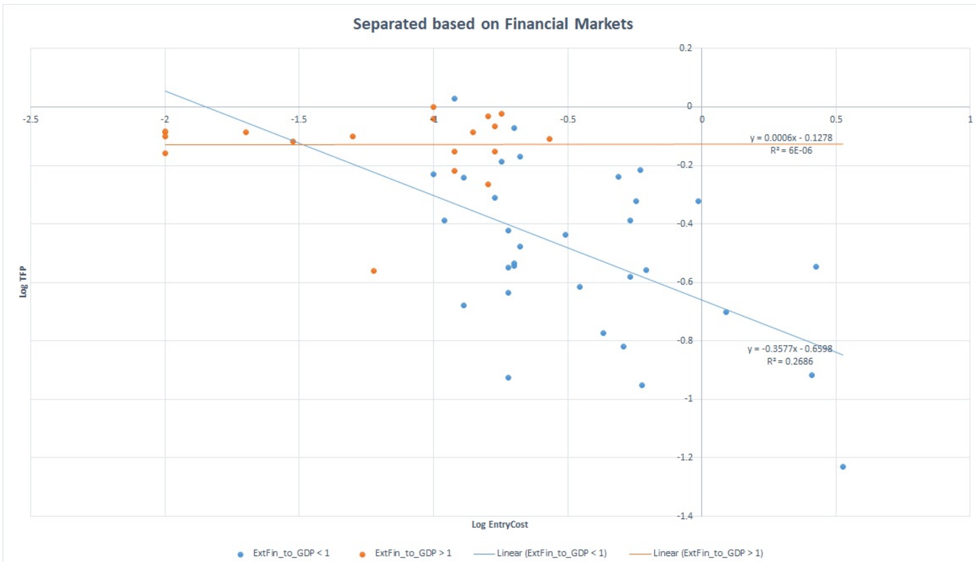
3.1 Entry Costs and Productivity

The links between entry barriers and aggregate outcomes are studied in the literature, and there's a consensus on the negative impact of entry costs on productivity and output. The main channels introduced in the literature are occupational and factor misallocation. I use the cross-country data on startup costs constructed by Djankov et al (2002), which is for 1999 and combine it with the productivity data for the same year from Penn World Table.[2] Following Barseghyan and DeCeico (2009) I will use both variables in logs. This combined data set will provide a negative correlation of about -63%, which is considerably large. This relation is depicted in Figure 1.

However, it can be argued that such a relationship cannot be the same for the economies with the perfect and imperfect financial markets. Most of the channels explored in the literature are consistent with financially imperfect markets. For instance, a high entry cost in a less financially developed country may keep a productive but poor individual out of business as she

[2]All productivity measures are relative to the U.S.

Figure 2: Cross country TFP and start-up costs, separated by financial development



does not have the funds needed to pay for the entry costs and there's very limited access to financing. Even if she manages somehow to start her business, she will most probably be undercapitalized. These are the occupational and capital misallocation channels studied in the literature. But as we can see, none of these arguments will hold in a financially developed economies because the entrepreneur can now borrow almost all she needs to start the business.

To see the point made above, I have used cross-country data on external finance to GDP which is an indicator of the level of financial development. I use the same data as in Buera et al (2011) to separate countries into two groups: Financially perfect and imperfect economies. I have used a very simple criterion to make this distinction. I regard the countries with the ratio of external finance to GDP greater than one as financially perfect, and those with that ratio less than one as financially imperfect economies. Figure 2 shows the relationship between entry cost and TFP separated by these two groups of economies.

As we can see in Figure 2, the countries with imperfect financial markets exhibit the same pattern as described in the literature. However, for the countries with perfect financial markets the figure shows no evidence on the impact of entry costs on TFP. This means that we need a different mechanism to explain the impact of entry costs in financially developed economies.

In order to understand the relationship explained above in financially developed economies, I've looked at the cross-sector data on financing and entry barriers in the U.S. I have separated the U.S. economy into two main categories: the sectors with high financing needs (mainly large-scaled), and those with little financing needs (mainly small-scaled). This differentiation is also consistent with Buera et al (2011). I've used data on median startup costs as well as the funding source taken from U.S. Census Bureau's 2016 annual survey of entrepreneurs and data on sector level multifactor productivity taken from Bureau of Labor Statistics. As another proxy for productivity I have used total revenue divided by payroll also taken from Bureau of Labor Statistics. I have found the results shown in Table 1.[3]

The results provided in Table 1 also justify the elements included in my model. As we can see the sectors matter as well as the sectoral structure of the economy, which means which sector plays a dominant role in that economy. Since, as discussed in Buera et al (2011), the large-scaled sector is more capital intensive than the small-scaled one, it is important to include capital adjustment costs in the model to capture the full effect of changes in startup costs.

3.2 Differences Between the Sectors

I differentiate the sectors using different per period fixed costs as well as different factor shares of production. There are several criteria used in the literature to differentiate among sectors in multi-sector models. Here, I will discuss a few related works and provide my reasoning on why I use this specification in my model.

[3] The sector-level data are very limited which may cast shadow on the analysis. Looking at the industry-level data would be ideal to make my point, but currently I don't have such data. To partially overcome this issue, motivated by a similar mechanism, I have used the same cross-country data used above, but this time focused on perfect financial markets. I have separated those economies using their share of service sector in total value added (the data is taken from OECD Data). Regression results (Log[TFP] on Log[startup cost]) using these subgroups are very similar to the results obtained from the U.S. sectoral analysis. That is, we observe a positive slope for the economies with greater share of service sector and a negative slope for those with smaller share of services in the total value added.

Table 1: Impact of start-up costs, different sectors

	Low financing (small-scaled)	High financing (large-scaled)
Log(MFP) vs Log(startup cost)		
Correlation	0.4698	-0.3812
Regression Coeff (β_1)	3.3732 (2.588)	-4.0423 (4.002)
R-squared	0.2207	0.1453
TR/Payroll vs Startup cost		
Correlation	0.9377	-0.1908
Regression Coeff (β_1)	5.7438 (0.869)	-0.7909 (1.661)
R-squared	0.8792	0.0364

Note: The regression on the top panel is $Log(MFP) = \beta_0 + \beta_1 Log(startup\ cost) + u$, and the regression on the bottom panel is $(Revenue/Payroll) = \beta_0 + \beta_1(startup\ cost) + u$.

Erosa and Cabrillana (2008) as well as Buera, Kaboski and Shin (2011) use per period fixed costs of production to differentiate between the sectors. In the later, the manufacturing sector produces both consumption and investment goods, and the service sector produces only consumption goods. As a result, the service sector is strongly connected to consumption sector, and manufacturing sector to investment sector. As discussed in Valentinyi and Herrendorf (2008), the factor shares are very similar in manufacturing and services, but are not so when we define our sectors as investment and consumption. Hence, it might be safer to approach the model with sectors having different factor shares. Even if we assume that the sectors have the same factor shares, their returns-to-scales might be very different mainly because of differences in size.

In Midrigan and Xu (2014) the traditional sector only uses labor and the modern sector uses both labor and capital. Therefore, the factor shares are inherently different between the sectors. Since, I don't have an informal sector, it is reasonable to assume that sectors use capital and labor in production.

Moreover, in order to explain cross-country differences, the models need to be applicable to different countries. As a result, we need to differentiate using factor shares as well as the per period fixed costs. A lot of studies assume the same factor shares across countries following Kaldor's (1961) stylized facts. But, there are some empirical evidence suggesting that factor shares actually differ across countries, see Zuleta (2007) and Sturgill (2008). Also, it is shown in Pinheiro and Yang (2018) that U.S. and other developed countries have very different trends of labor share of output for manufacturing and services. Therefore, using different factor shares for different sectors might improve our cross-country evaluation.

There is also a study by Castro, Clementi and MacDonald (2009) who differentiate sectors using volatility of idiosyncratic productivity shocks. This implies that their manufacturing sector is riskier than services. This can be easily implemented in my model, with just a change in my volatility parameter in either sector, but I will abstract from this to avoid complicating the interpretation of my results.

4 Model

In this section, I explain the environment of the model as well as its formulation in continuous time.

4.1 Outline of the Model

I build and entrepreneurship model with two sectors, similar to Buera et all (2011) and Midrigan and Xu (2014). The model has financial and capital adjustment frictions as well as entry costs. I abstract from labor adjustment costs because Shaker-Akhtekhane (2018) shows that, having labor frictions doesn't make the model any richer when trying to analyze the impact of capital frictions (and capital misallocation) on aggregate outcomes.

Sectors: As explained before, the sectors will be differentiated through a per period fixed cost and factor shares in the production function. One of the sectors is assumed to have high financing needs (large-scaled) and the other have little financing needs (small-scaled). Following Buera et al (2011), I will call the large-scaled and small-scaled sectors Manufacturing (M) and Services (S), respectively. The output of the service sector will only be used for consumption, but the output of the manufacturing sector can be used for consumption or investment.

Financial Frictions: Financial frictions take the form of collateral constraint. I employ the notion commonly used in the literature that the entrepreneurs can only borrow up to an amount proportional to their capital stock. This gives the borrowing constraint, $b \geq -\phi k$, where k is the capital stock and b is deposits (debt if negative) of the entrepreneur. Parameter ϕ governs the degree of financial development as in the standard models with financial frictions. ϕ ranges from 0 (no financial markets) to 1 (perfect financial markets).

Adjustment Costs: Firms or entrepreneurs running them face adjustment costs when buying or selling capital. As discussed in Cooper and Haltiwanger (2006), having both convex and non-convex components of adjustment costs fits data the best. To comply with this, I use an adjustment cost consisting of a quadratic component as well as partial irreversibility.

Entry Costs: The entry cost I used in my paper is proportional to the scale with which the entrepreneur wants to start her business. Mckenzie and Woodruff (2006) use start-up firm's data from Mexico and show that for very small businesses the entry costs are very low. Consistent with their finding, I have defined the entry cost as a fraction of the capital that the entrepreneur starts production. This form of the setup cost is more intuitive as well. For example, an entrepreneur starting a small startup firm will have much less setup cost than the one starting a relatively large business. However, implementation of a fixed entry cost takes only an easy parameter adjustment in my model, and I will look into this as a robustness check.

4.2 Agents

There are a measure of N infinitely lived individuals in the economy. Individuals in the economy receive a pair of productivity shocks related to each sector, S and M. After observing the shock pair (z_S, z_M), an individual chooses to either work for a wage, w, or become an entrepreneur and start her business in either sector. Entrepreneurs buy (own) capital and hire labor accordingly to start production. The output produced by an entrepreneur operating in sector $j \in \{S, M\}$ is given by:

$$z_j f_j(k, l) = z_j k^{\alpha_j} l^{\theta_j}, \qquad \alpha_j + \theta_j < 1.$$

The price of goods in sector j is denoted by p_j. Every period, the entrepreneurs incur a sector-specific per period fixed cost, κ_j. They buy or adjust capital to their needs and hire labor accordingly at rate w. The capital depreciates at rate δ. Therefore, entrepreneurs' profits in sector j is equal to:

$$\pi_j = p_j z_j f_j(k,l) - wl - \delta k - p_j \kappa_j.$$

There's an entry cost proportional to the amount of capital that an entrant starts production with. Investment (disinvestment) at every period is subject to adjustment costs. The entrepreneurs will face different adjustment costs depending on whether they stay in the same sector or switch to the other sector. A general form of adjustment cost is given by:

$$C^{adj} = \begin{cases} C^{ENT} i, & \text{For entrants} \\ (C^P + C^{ENT})k + C^{ENT} i, & \text{For switching entrepreneur} \\ C^Q k(\frac{i}{k})^2 + C^P(-i)1_{(i<0)}, & \text{For continuing entrepreneur} \end{cases}$$

where C^{ENT} is the entry cost parameter, and C^P and C^Q are partial irreversibility and quadratic parameters of capital adjustment cost. Note that there's an implied assumption about the switching entrepreneurs costs. I assume that capital is not convertible from one sector to another. That is, in order to switch from one sector to another, an entrepreneur must sell her current capital and then buy new capital and pay the entry cost to start producing in the other sector. This assumption adds some extra occupational frictions and makes it harder to switch occupations.

Entrepreneurs' access to capital is constrained by their wealth. Individuals in the economy can lend at rate r and borrow at rate R through competitive financial intermediaries.[4] I denote the individuals' deposits by b which can also be negative for the net borrowers. Since the entrepreneurs own their capital, their wealth (denoted by ϖ) at the beginning of each period will be equal to: $\varpi = b + (1 - C_P)k$. In order to buy the capital they need, entrepreneurs can borrow from financial intermediaries subject to a borrowing limit given by $b \geq -\phi k$. Here, ϕ indicates perfectness of the financial markets, or perfectness of enforceability of financial contracts.[5] Financial intermediaries zero profit condition implies that $R = r$.

[4]Note that wage workers can only participate by lending their savings, and only entrepreneurs can borrow subject to their borrowing constraint.

[5]Borrowing and lending occur within a period, and individuals' wealth is always nonnegative.

Individuals maximize their lifetime consumption stream of both service and manufacturing goods. Following Buera et al (2011), the expected utility is given by the following:

$$U(c_S, c_M) = E_0 \int_{t=0}^{\infty} e^{-\rho t} u(c_{S,t}, c_{M,t}) dt$$

where

$$u(c_{S,t}, c_{M,t}) = \frac{1}{\sigma} \left(\psi c_{S,t}^{\epsilon} + (1-\psi) c_{M,t}^{\epsilon} \right)^{\frac{\sigma}{\epsilon}}.$$

Here ρ is the discount factor, $(1-\sigma)$ is the CRRA, $\frac{1}{1-\epsilon}$ is the inter-temporal elasticity of substitution between service and manufacturing goods, and parameter ψ controls the share of service goods in overall consumption expenditure.

4.3 Formulation of the Model

Before formulating the model, I need to carefully define the state variables. Because of the existence of possible correlated states in the model, I need to define the state variables in a way that leaves no inconsistent or unattainable area in the state space. For example, if I use capital and deposits as my state variables and use regular equidistant (or any type) grids, there will be a combination of capital and deposits where the entrepreneur has very little (almost no) capital but a large negative amount of deposits which is only meant for those entrepreneurs with large amount of capital. This combination contradicts the rule for borrowing limit, and the decisions made by the corresponding entrepreneurs will distort the whole transition matrix used in solving the linear complementarity problem (LCP).

I use capital stock, k, and the ratio of deposits to wealth, denoted by $a = \frac{b}{\varpi}$ as my endogenous state variables. Using the definition of ϕ and wealth, the state variable a varies in a fixed range which is given by:

$$(\underline{a}, \bar{a}) = \left(\frac{\phi}{\phi - (1 - C^P)}, 1 \right)$$

.

4.3.1 Wage Workers' Problem

Assumption 1: There is a storage cost of keeping capital every period, denoted by C^S, which is high enough that no wage worker finds it optimal to hold on her capital with the hope of using it in the near future.

This assumption makes sure that when shutting down and exiting the production, an entrepreneur liquidates all of her capital. It can be formally stated as the following proposition.

Proposition 1: Wage workers holding on no capital is equivalent to the following:

$$C^S > 1 + C^{ENT} - (1+r)(1-C^P).$$

See the Appendix for a proof. □

Based on the assumption stated above, capital is not a state variable for the wage workers. The only endogenous state for wage workers is their deposits, b, which is consistent with the states of entrepreneurs and can be easily translated into capital, k, and deposits-to-wealth ratio, a, upon entry to either sector. The evolution of the wage worker's assets is give by the following equation:

$$\dot{b} = w + r.b - p_S c_S - p_M c_M.$$

This simply means that next period deposits is equal to current deposits plus interest earned earned on them and wages minus consumption expenditures. Finally, given wages, interest rates and prices, the wage worker will solve the following problem:

$$V^W(b, z_S, z_M) = \max_{\tau, c_S, c_M} E_0 \int_0^\tau e^{-\rho t} u(c_{S,t}, c_{M,t}) dt + e^{-\rho \tau} max\{V^{*S}, V^{*M}\}$$

Subject to:

$$\begin{aligned} \dot{b} &= w + rb - \mathbf{p.c} \\ dz_{S,t} &= \mu(z_{S,t})dt + \sigma(z_{S,t})dW_t \\ dz_{M,t} &= \mu(z_{M,t})dt + \sigma(z_{M,t})dW_t \\ b &\geq 0 \end{aligned}$$

where $\mathbf{p.c} = p_S c_S + p_M c_M$, and for $j \in \{S, M\}$, V^{*j} is the value of entering sector j.

4.3.2 Entrepreneurs' Problem

Entrepreneurs' endogenous states are capital and deposits-to-wealth ratio, which are related through deposits, adjustment costs, and financial development parameter. Similar to the wage worker, an entrepreneur's deposits (debts) evolve according to the following equation:

$$\dot{b} = \pi^j - C^{adj} + r.b - p_S c_S - p_M c_M, \quad j \in \{S, M\}.$$

This means that next period deposits (or debts if negative) is equal to current deposits plus interest earned (or paid) on them and profits minus next periods investment and the corresponding adjustment costs and consumption expenditures. But we are interested in evolution of a.

Proposition 2: The evolution of deposits-to-wealth ratio, a is given by the following:

$$\dot{a} = \frac{(1-a)^2}{(1-C^P)k}(\pi^j - C^{adj} - \mathbf{p}.\mathbf{c}) + a(1-a)(r + \delta - \frac{i}{k}) \tag{4.1}$$

See the Appendix for a proof. □

Given wage, interest rates and prices, an entrepreneur operating in sector $j \in \{S, M\}$ will solve the following problem:

$$V^j(a, k, z_S, z_M) = \max_{\tau, c_S, c_M} E_0 \int_0^\tau e^{-\rho t} u(c_{S,t}, c_{M,t}) dt + e^{-\rho \tau} max\{V^{*W}, V^{*-j}\}$$

Subject to:

$$\begin{aligned}
\dot{a} &= \frac{(1-a)^2}{(1-C^P)k}(\pi^j - C^{adj} - \mathbf{p}.\mathbf{c}) + a(1-a)(r + \delta - \frac{i}{k}) \\
\dot{k} &= i - \delta k \\
dz_{S,t} &= \mu(z_{S,t})dt + \sigma(z_{S,t})dW_t \\
dz_{M,t} &= \mu(z_{M,t})dt + \sigma(z_{M,t})dW_t \\
\pi_j &= p_j z_j f_j(k, l) - wl - \delta k - p_j \kappa_j \\
C^{adj} &= C^Q k(\frac{i}{k})^2 + C^P(-i)1_{(i<0)} \\
a &\geq \frac{\phi}{\phi - (1 - C^P)} \quad \text{and} \quad a \leq 1.
\end{aligned}$$

where V^{*W} is the value associated with exit decision, and V^{*-j} is the value of switching from sector j to the other sector.

4.4 Value Functions

We solve for value functions using the Hamilton-Jacobi-Bellman (HJB) equations. Since the individuals can make decisions on entry, exit or switching between sectors, it becomes a stopping time problem. To solve these problems we make some modifications on the main problem and obtain Hamilton-Jacobi-Bellman variational inequality (HJBVI). See Benjamin Moll's (2016) notes on stopping time problems and their solution. Also see Shaker-Akhtekhane (2017).

For wage workers the HJB has the following form:

$$\begin{aligned}\rho V^W(b, z_S, z_M, t) &= \max_{c_S, c_M} u(c_S, c_M)\\ &+ \frac{\partial V^W}{\partial b}(w + rb - p_S c_S - p_M c_M)\\ &+ \frac{\partial V^W}{\partial z_S}\mu(z_S) - \frac{1}{2}\frac{\partial^2 V^W}{\partial z_S^2}\sigma^2(z_S)\\ &+ \frac{\partial V^W}{\partial z_M}\mu(z_M) - \frac{1}{2}\frac{\partial^2 V^W}{\partial z_M^2}\sigma^2(z_M)\end{aligned}$$

HJBVI will be derived from this using the best entry values to either sector, $\max\{V^{*S}, V^{*M}\}$.

The entrepreneurs HJB takes the following form, for $j \in \{S, M\}$:

$$\begin{aligned}\rho V^j \quad (a, k \quad, z_S, z_M, t) &= \max_{c_S, c_M} u(c_S, c_M)\\ &+ \frac{\partial V^j}{\partial a}\left(\frac{(1-a)^2}{(1-C^P)k}(\pi^j - C^{adj} - \mathbf{p}.\mathbf{c}) + a(1-a)(r + \delta - \frac{i}{k})\right)\\ &+ \frac{\partial V^j}{\partial k}(i - \delta k)\\ &+ \frac{\partial V^j}{\partial z_S}\mu(z_S) - \frac{1}{2}\frac{\partial^2 V^j}{\partial z_S^2}\sigma^2(z_S)\\ &+ \frac{\partial V^j}{\partial z_M}\mu(z_M) - \frac{1}{2}\frac{\partial^2 V^j}{\partial z_M^2}\sigma^2(z_M)\end{aligned}$$

Similarly, HJBVI will be derived from this using the maximum of switch and exit values, $\max\{V^{*W}, V^{*-j}\}$.

4.5 Optimal Decision Rules

Given the idiosyncratic shocks and price vector, and using the HJB equations, individuals will choose consumption and investment (if operating), to maximize their value function.

4.5.1 Consumption

A wage worker's optimal consumption of service goods will be given by:

$$c_S^{(W)} = \left[\frac{p_S}{\psi}(\psi + (1-\psi)B^{\epsilon})^{\frac{\epsilon-\sigma}{\epsilon}} \partial_a V^W\right]^{\frac{1}{\sigma-1}}. \tag{4.2}$$

And and entrepreneur's optimal consumption of service goods will be, for $j \in \{S, M\}$:

$$c_S^{(j)} = \left[\frac{p_S}{\psi} . \frac{(1-a)^2}{(1-C^P)k}(\psi + (1-\psi)B^{\epsilon})^{\frac{\epsilon-\sigma}{\epsilon}} \partial_a V^j\right]^{\frac{1}{\sigma-1}}. \tag{4.3}$$

Given the consumption of service goods, it is easy to obtain the consumption of manufacturing goods for any individual in any occupation $\mathcal{O} \in \{W, S, M\}$:

$$c_M^{(\mathcal{O})} = B c_S^{(\mathcal{O})}, \tag{4.4}$$

where

$$B = \left(\frac{\psi}{(1-\psi)} . \frac{p_M}{p_S}\right)^{\frac{1}{\epsilon-1}}.$$

4.5.2 Investment

An entrepreneur in sector j who wants to continue producing in the same sector, faces an investment decision. First we calculate the unconstrained investment as if there were no borrowing constraints, and later we will include the borrowing limit. If the entrepreneur decides to invest, her unconstrained investment will be given by:

$$i_u^+ = \max\left\{0, \left[\left(\frac{V_k}{V_a} - \frac{a(1-a)}{k}\right)\frac{(1-C^P)k}{(1-a)^2} - 1\right]\frac{k}{2C^Q}\right\}. \tag{4.5}$$

where V_x denotes the partial derivative of the value function with respect to x. If she decides to dis-invest, the amount of unconstrained disinvestment will be given by:

$$i_u^- = \in \left\{ 0, \left[\left(\frac{V_k}{V_a} - \frac{a(1-a)}{k} \right) \frac{(1-C^P)k}{(1-a)^2} - 1 + C^P \right] \frac{k}{2C^Q} \right\}. \tag{4.6}$$

The amount of unconstrained investment will be sum of positive and negative components:

$$i_u = i_u^+ + i_u^-$$

4.5.3 Borrowing Limits

The financial constraints matter for every individual in the economy, whether they are wage workers and may want to start business in the future or they are operating a firm in one of the two sectors.

Entrants:

After observing a reasonably high productivity shock related to either sector, a wage worker may decide to start business, say in sector j. However, the amount of capital would depend on her assets, b, as well as the financial development of the economy depicted by parameter ϕ. She can finance the capital she buys as well as the entry cost. The financial intermediary will set up a contract that takes into account the depreciation of the capital as well as the irreversibility component of the adjustment cost, to ensure that the entrant can repay her debt and end up with a non-negative wealth. The amount of capital that an entrant can start business with is given in the following proposition.

Proposition 3: Regardless of the sector that the entrant wants to start production in, the amount of capital she can borrow is constrained by the following:

$$k \leq \frac{(1-\underline{a})b}{1 + (1-\underline{a})C^{ENT} - \underline{a}C^P}, \tag{4.7}$$

where $\underline{a}$ is the lowest value for deposits-to-wealth ratio, a:

$$\underline{a} = \frac{\phi}{\phi - (1 - C^P)}.$$

See Appendix for a proof. □

Switching Entrepreneurs:

An entrepreneur who operates in sector j, may figure that she can get higher value from operating in the other sector, $-j$, and may consider to switch. Since capital is not transferable from one sector to another, the entrepreneur needs to sell her capital in sector j which is subject to irreversibility costs, and then buy new capital she needs and pay the entry cost to start operating in sector $-j$.

Now, the amount of capital would depend on her states: k and a, as well as the parameter ϕ. Similar to the entrant's case, the financial intermediary sets up a contract that takes into account the capital depreciation and the elements of adjustment cost to ensure that the entrepreneur doesn't fail to repay her debt and ends up with a non-negative wealth. The amount of capital that a switching entrepreneur can borrow is given in the following proposition.

Proposition 4: The investment of a switching entrepreneur is constrained by the following:

$$i \leq \frac{(1-\underline{a})\left(\frac{a}{1-a}(1-C^P)k-(C^P+C^{ENT})k\right)-(1-C^P)(1-\delta)\underline{a}k}{(1-\underline{a})(1+C^{ENT})+\underline{a}(1-C^P)},$$

where $\underline{a}$ is the same as in Proposition 3. See Appendix for a proof. □

Continuing Entrepreneurs:

An entrepreneur who operates in sector j, and wants to stay in the same sector for the next period, may want to adjust the capital to optimize her value. If she decides to invest/disinvest, she will face the adjustment cost consisting of quadratic and partial irreversability elements.

Again, the capital depends on her states: k and a, as well as the parameter ϕ. Also, the financial intermediary sets up a contract to ensure that the entrepreneur always ends up having a non-negative wealth. The following proposition gives the investment limit for the continuing entrepreneur.

Proposition 5: The investment of a continuing entrepreneur is constrained by $i \leq \eta$, where η is the real positive root of the following quadratic equation with respect to i:

$$\tilde{A}i^2 + \tilde{B}i + \tilde{C} = 0, \tag{4.8}$$

where

$$\begin{aligned}
\tilde{A} &= (\underline{a} - 1)\frac{C^Q}{k}, \\
\tilde{B} &= (\underline{a}C^P - 1), \\
\tilde{C} &= (1 - \underline{a})(1 - C^P)\frac{a}{1-a}k - (1 - C^P)(1 - \delta)k\underline{a},
\end{aligned}$$

See Appendix for a proof. □

4.6 Entry, Exit and Switching

It is important to understand the mechanism through which the individuals switch their occupations. There are corresponding areas in the state space that an individual who switches her occupation will move through. For instance, consider an entrepreneur with capital k and deposits-to-wealth ratio a, who is operating in sector j. We are interested in finding values of her new state variables if she wants to either exit or switch to sector $-j$. Obviously, the exogenous states remain the same as before, (z_S, z_M).

Exiting Entrepreneur:

This is the easiest case because if an entrepreneur finds it optimal to shut down and exit, she will simply sell the capital and leave to start working for a wage. In this case we need a correspondence from k and a to b, that is from the states of an entrepreneur to the state of a wage worker. This correspondence is as follows:

$$b = (1 - C^P)\frac{a}{1-a}k + (1 - C^P)k, \tag{4.9}$$

where the first term in the right hand side is equal to deposits of the entrepreneur, and the second term is equal to the sale value of the capital she owns.

Entrant:

This case is a little bit involved. The entrant's state is only given by her assets in the form of deposits. As a result, she faces a borrowing constraint given by Proposition 3. Let's denote the maximum amount of capital provided in equation (4.7) by k^*. Also, let's assume that the entrant wants to start production in sector j. We will need to look at all the combinations of k and a that are consistent with b, where k is within the borrowing limit. From these combinations, we need to find the one that maximizes the value function at sector j. Given the deposits of the entrant, b, we have:

$$\begin{aligned} (k, a) &= \arg\max_{0 \leq k \leq k^*} V^j(a, k, z_S, z_M) \qquad (4.10) \\ \text{subject to} & \\ a &= \frac{b}{b + (1 - C^P)k}. \end{aligned}$$

Switching:

This case is closely related to both the exit and entry cases discussed above. The reason is that the switching entrepreneur is acting as if she exits the market and then re-enters which is because the capital is not transferable between sectors. After exit her assets are given by equation (4.9). Then given the assets, b, we obtain the optimal amount of k and a in the new sector using equations (4.11).

At any state, the individuals use the occupation switch states explained above to obtain the corresponding values functions. Eventually, they will decide on continuation or switching after evaluating different outcomes using the relevant value functions.

4.7 Distribution

Now we want to solve for the stationary distribution of the economy. The density function will be obtained using Kolmogorov Forward Equation (KFE), which is a partial differential equation similar to HJB, and will be solved numerically using the finite difference method. However, the complexity that arises from the occupational choices discussed in the previous sub-section, will also make solving KFEs complicated.

After solving the value function for different individuals using HJBVIs,[6] the relevant value functions will provide the areas in the state space where

[6] The HJBVIs are solved as Linear Complementarity Problem (LCP) which is a technique based on finite difference method.

the individuals switch occupation. This gives us six cases to deal with while solving for Kolomogorov Forward Equation (KFE). For any two distinct $\mathcal{O}_1, \mathcal{O}_2 \in \{W, S, M\}$, we consider switching from $\mathcal{O}_1$ to $\mathcal{O}_2$.

Let $m(\mathcal{O}_1, \mathcal{O}_2)$ denote the distribution of the individuals switching from $\mathcal{O}_1$ to $\mathcal{O}_2$. The KFEs will be defined using m.

Wage workers' KFE:

$$\begin{aligned}\frac{\partial g^W(b, z_S, z_M, t)}{\partial t} = & - \frac{\partial}{\partial b}\left[(w + r(t)b - p_S c_S - p_M c_M)g^W\right] \\ & - m(W, S) - m(W, M) \\ & + m(S, W) + m(M, W)\end{aligned} \tag{4.11}$$

Entrepreneurs' KFE in sector j:

$$\begin{aligned}\frac{\partial g^j(a, k, z_S, z_M, t)}{\partial t} = & \frac{1}{2}\frac{\partial^2}{\partial z_S^2}\left(\sigma^2(z_S)g^j\right) - \frac{\partial}{\partial z_S}\left(\mu(z_S)g^j\right) \\ & + \frac{1}{2}\frac{\partial^2}{\partial z_M^2}\left(\sigma^2(z_M)g^j\right) - \frac{\partial}{\partial z_M}\left(\mu(z_M)g^j\right) \\ & - \frac{\partial}{\partial a}\left[\dot{a}g^j\right] - \frac{\partial}{\partial k}\left[(i + \delta k)g^j\right] \\ & - m(j, -j) - m(j, W) \\ & + m(-j, j) + m(W, j)\end{aligned} \tag{4.12}$$

where $-j$ means the sector other than j, and $\dot{a}$ is the evolution of a given by equation (4.1). I have also suppressed the arguments of $g(a, k, z_S, z_M, t)$ in the right hand side of the equation.

4.8 Equilibrium

The stationary equilibrium of the model is obtained by joint solution of HJBVIs and KFEs given the occupational choice, optimal decision rules for consumption, saving and investment, and the borrowing constraint. The number of individuals in the economy is normalized to 1. I solve the HJBVIs as a Linear Complementarity Problem (LCP) which is based on the finite difference method. KFEs are also solved using the finite difference method. See Achdou et al. (2014) for a detailed explanation on the application of finite difference method on a heterogeneous agent problem.

Market Clearing Conditions:

After solving for the distribution and the value function, we can use them along with the decision rules to calculate the aggregates and update prices using the market clearing conditions. There are five markets in the economy that need clearing. Let's denote the state vector as $\xi^j = (a, k, z_S, z_M)$ for entrepreneurs in sector j, and $\xi^W = (b, z_S, z_M)$ for wage workers. For simplicity in notation let's denote a general state vector as ξ that applies to everyone in the economy.

i. Credit Market: The total net deposits is equal to zero, and the zero profit condition for financial intermediaries imply that $R = r$.

ii. Market for Service Goods: For this market to clear, the amount consumed by all the individuals in the economy should equal the amount produced in the service sector. That is:

$$\int c_S(\xi)g(d\xi) = \int_{\mathcal{O}=S} \left(z_S f_S(k,l) - \kappa_S\right) g(d\xi) \tag{4.13}$$

iii. Market for Manufacturing Goods: In this market, the amount consumed by all the individuals in the economy plus the depreciated capital used in production should equal the amount produced in the manufacturing sector. That is:

$$\int c_M g(d\xi) + \delta \int_{\mathcal{O}\in\{S,M\}} k g(d\xi) = \int_{\mathcal{O}=M} \left(z_M f_M(k,l) - \kappa_M\right) g(d\xi) \tag{4.14}$$

iv. Capital Market: Here, the amount of capital used in production plus adjustment costs (including entry costs) equals the amount deposited by the individuals. That is:

$$\int_{\mathcal{O}\in\{S,M\}} (k + C^{adj}) g(d\xi) = \int_{b\geq 0} b g(d\xi) \tag{4.15}$$

v. Labor Market: The demand for labor by entrepreneurs equals the supply of labor by wage workers:

$$\int_{\mathcal{O}\in\{S,M\}} lg(d\xi) = \int_{\mathcal{O}=W} g(d\xi) \tag{4.16}$$

Algorithm:

The algorithm for solving the stationary equilibrium is described below: Start with an initial guess on output prices, (p_S^0, p_M^0) as well as wages, w^0, and interest rate, r^0. Then, for $s = 0, 1, 2, ...$ we do as follows:

1. Given the prices, for any occupation $\mathcal{O} \in \{W, S, M\}$ solve for the optimal consumption $(c_S^{\mathcal{O}}, c_M^{\mathcal{O}})^s$ using (4.2, 4.2, 4.4). For the entrepreneurs in sector $j \in \{S, M\}$ solve for the optimal investment, $(i^j)^s$, using (4.5, 4.6) as well as using the borrowing limit, (4.8). Then solve for the corresponding labor input, output and profits $(\pi^j)^s$ for any point in the state space.

2. Solve for the outside option for the individuals at any occupation using the discussion in section 4.6, and use those scrap values to solve for the value function using LCP, and obtain $(V^{\mathcal{O}})^s$.

3. Solve for the stationary distributions, $(g^{\mathcal{O}})^s$, from (4.12 and 4.13) using finite difference method.

4. Solve for the aggregate outcomes using the distribution obtained in step 3. Use the market clearing conditions explained above to update the prices.[7]

5. Given the newly obtained values for prices and aggregate quantities, return to step 1. Stop iterating if the changes in prices (or aggregate quantities) are very small. That is, stop if the following condition holds for some desirably small ϵ:

$$\left|p_S^{s+1} - p_S^s\right| + \left|p_M^{s+1} - p_M^s\right| + \left|w^{s+1} - w^s\right| + \left|r^{s+1} - r^s\right| < \epsilon.$$

This provides the stationary equilibrium of the economy which is given by $\left(p_S^s, p_M^s, w^s, r^s, (V^{\mathcal{O}})^s, (g^{\mathcal{O}})^s\right)$ and the aggregate outcomes.

[7] I have used Broyden's method to update prices using aggregates and market clearing conditions

5 A Quantitative Analysis (Preliminary)

In this section I report the numerical results of the model. The numerical solution of the model and my extensions to the solution method in continuous time is explained in details in the Appendix. Since it's a preliminary version of an ongoing project, I will only provide the initial output of my model. A full quantitative exercise will be carried out in the near future.

5.1 Parameter Values

Note that these are preliminary results and are only for the purpose of a quantitative exercise. A careful calibration of the model will be carried out in the next version of the draft. Most of the parameter values such as preferences and production parameters are taken from Buera et all (2011) whose work is closely related to this paper. The parameter values for adjustment costs of capital are taken from Bloom (2009). Table 2 provides the values I have used in this paper for different parameters. Since I haven't done a model based calibration, I won't be using different factor shares for different sectors. I will activate this feature of the model after doing some data work and a model calibration that justifies the addition of those parameters into my model. For now, consistent with Buera et al (2011) I will only use fixed per period cost to distinguish the sectors.

5.2 An overview of the results

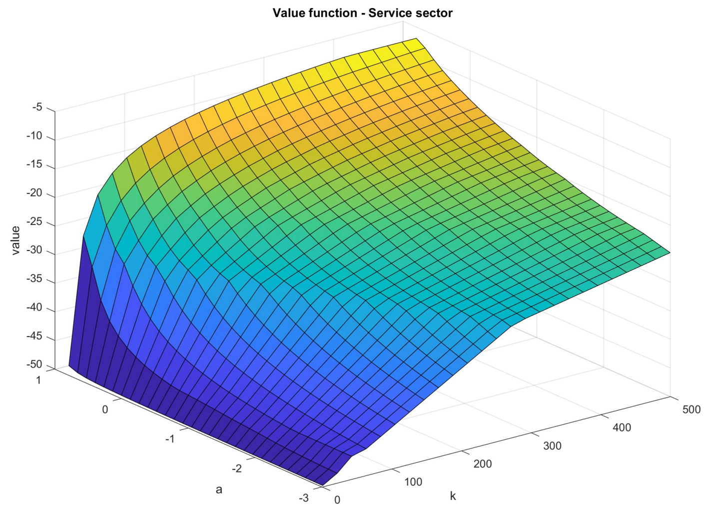
In this part I simply report some initial outputs of the model. All the output reported in this section is for the value of $\phi = 0.5$, which is about an average financial development parameter. A sample value function for service sector is shown in Figure 3a and the investment/disinvestment decisions are shown in Figure 3b. The investment figure is intuitive. It says that, the entrepreneurs with low (negative) deposits-to-wealth ratio, will do nothing if they have low capital (since they cannot invest), and will start disinvesting after some capital threshold. After some point the disinvestment rate will change to account for the amount of capital they own. But, for higher deposit-to-wealth ratios the entrepreneur will invest more as capital increases up to some threshold. This is because the borrowing constraint is tighter for those with less capital. After the threshold, the entrepreneur will start buying less capital as they are approaching their optimal level of

Table 2: Parameter values

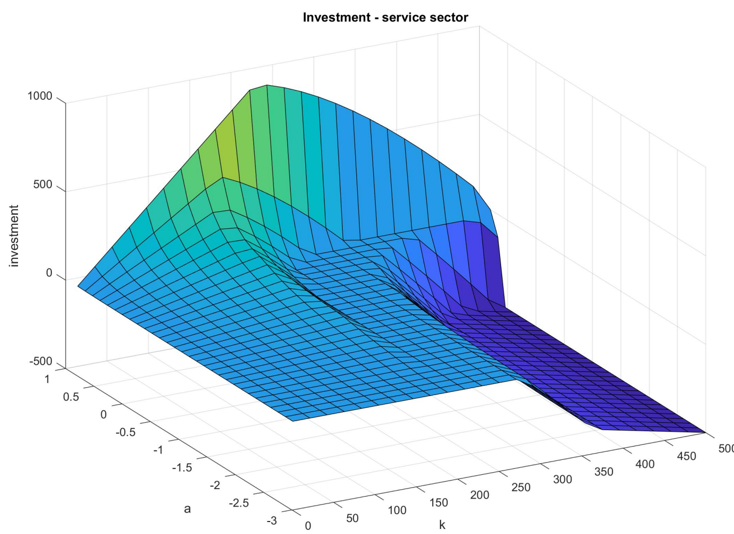
Parameter	Description	Values
α	Capital share in production	0.26
θ	Labor share in production	0.53
κ_S	per period cost - services	0.00
κ_M	per period cost - manufacturing	4.68
δ	Capital depreciation	0.06
ρ	discount factor	0.05
σ	$(1-\sigma)$ is CRRA	-0.5
ϵ	$\frac{1}{1-\epsilon}$ is IES between S and M goods	-0.33
ψ	Share of S goods in consumption expenditure	0.91
C^Q	Quadratic parameter for adjustment cost	1.0
C^P	Partial irreversibility	0.33

capital. They will increase their capital in a decreasing rate until a point where the partial irreversibility kicks in, and they will get into the inaction region where they rather operate with non-optimal capital than selling it very cheaper than buying price. They will tolerate this inaction situation until the region where it is optimal for them to sell some portion of their capital. At this point they are way above the capital that matches their productivity, and more capital means higher opportunity cost since they can earn interest if they sell some of their capital.

Figure 4 shows the relationship between TFP and entry cost with different financial and sectoral structures. The blue curve (square-marked) is the relationship between TFP and entry cost when there's almost no financing available, $\phi = 0.01$. As we can see the curve is downward sloping for all the values I examined in this example, which means that higher entry costs under poor financial markets leads to lower TFP. This is consistent with our observations in cross-country data. The red curve (diamond-marked) in the figure depicts the relationship when the financial markets are near-perfect, $\phi = 0.8$, but the manufacturing sector is highly concentrated. In this case, entry cost initially has almost no impact on TFP, but when it is high it leads to lower TFP. The green curve (circle-marked) is also related to good financial markets but a less concentrated manufacturing sector. As we can see, initially and increase in entry cost leads to increase in TFP through



(a) Value function



(b) Investment

Figure 3: Sample value function and investment - service sector - $\phi = 0.5$

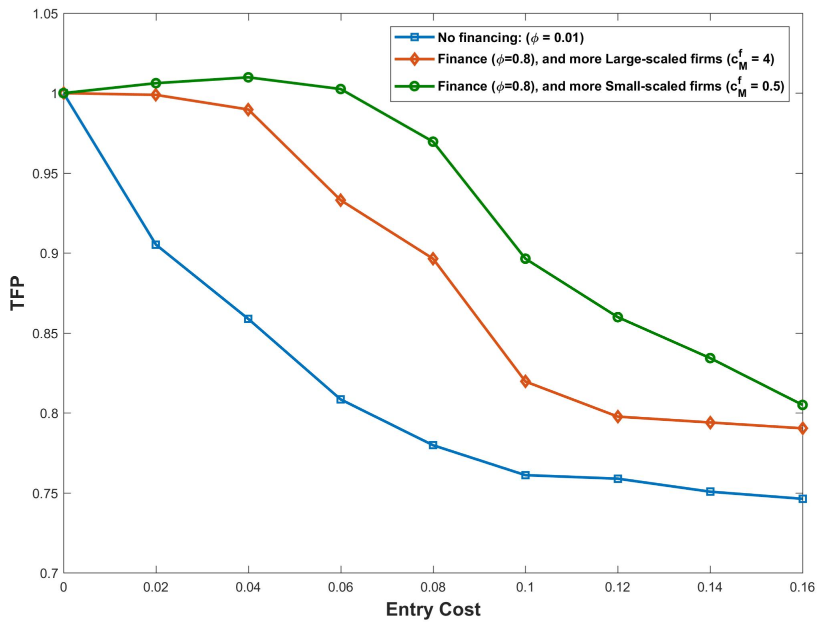


Figure 4: TFP vs Entry cost

the mechanisms explained earlier in the paper. TFP will eventually drop as a result of increase in entry costs beyond some threshold. These are all consistent with the cross country data presented as well as the intuition and mechanisms provided in earlier sections of this paper.

6 Conclusion

In this paper I have developed and solved a two-sector model of entrepreneurship in continuous-time. The model includes entry costs, financial frictions in the form of borrowing constraint and physical capital adjustment costs with quadratic and partial irreversibility elements. The main goal of the paper is to analyze the impact of entry costs on aggregate productivity in economies with perfect and imperfect financial markets.

Because of the richness of my model, it can also be used to answer some other questions: For instance, I can use the model to study the effect of capital adjustment costs on TFP in the presence of financial friction, or to study the impact of financial frictions on TFP, and interactions between physical and financial frictions. Also, a transitional dynamics study might be very interesting.

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