Financial Frictions and Productivity Losses: Importance of Default-Led Heterogeneity in Collateral and Loan Rates

Saeed Shaker-Akhtekhane[⋆]

Job Market Paper

Please click here for the latest version

November 27, 2020

Abstract

I develop a model of entrepreneurship with default to quantitatively analyze the impact of financial frictions on total factor productivity (TFP). Default risk justifies the need for collateral. Entrepreneurs are charged higher loan rates if the value of their collateral is low, which favors the wealthy over the poor, regardless of their talent, and discourages poor individuals from self-financing to start or expand their businesses. The close link between deposit rates and loan rates, in most models, is broken. Consistent with empirical evidence, my model can generate a weak self-financing motive while allowing for a highly persistent individual productivity, a challenge for existing models of financial frictions.

Financial frictions in my model stem from three different sources: limited enforceability related to the recovery rate of collateral by financial intermediaries; informational frictions related to inefficiencies in financial intermediaries' evaluation of entrepreneurs' default risks; and frictions related to entrepreneurs' expectations of future loan terms. I use machine learning classification techniques to solve the problem financial intermediaries face evaluating entrepreneurs' default risks. My analysis shows sizeable losses from financial frictions, more than 40% in TFP losses for the U.S. if we were to replace its financial markets with a poorly functioning one. Large TFP losses arise as there is amplification between the three sources of financial friction. Without default and heterogeneity in collateral and loan rates, my model would function similarly to a neo-classical model, and there would be a small impact of financial frictions with only a 7% loss in TFP.

I am indebted to my advisor, Aubhik Khan, for his support and guidance. I am grateful to Julia Thomas, Kyle Dempsey, and Benjamin Moll for their valuable comments and directions. I also thank Abolfazl Setayesh, Soyoung Lee, Benjamin Lidofsky, Rohan Shah, and Michael Carter for their generous help.

[⋆]Department of Economics, Ohio State University. Email: shaker.34@osu.edu

1 Introduction

The relationship between financial development and economic development is a well-established fact in the macroeconomics and finance literature.[1] The main related question is: how large are the TFP losses from financial frictions? Quantitative macroeconomic models find both large and small losses, with the latter prevailing recently.[2] Self-financing, the process of wealth-accumulation to start or expand a business, has a pivotal role in these studies. Individuals save and may eventually overcome financial constraints in order to start or expand their businesses. However, this happens only when current productivity levels persist over a relatively long time. This high persistence of productivity, backed by empirical evidence,[3] leads to strong self-financing motives.[4] This dampens the impact of financial frictions.[5]

However, there is ample empirical evidence against self-financing in countries with less-developed financial markets.[6] Nonetheless, quantitative models have not succeeded in reconciling highly persistent productivity with a small role for self-financing, both of which are supported by empirical observations. In quantitative studies that produce large TFP losses from financial frictions, the self-financing motive is weak, and this is consistent with empirical evidence. However, the persistence of individual productivity is low, which is in contrast to empirical evidence. On the other hand, the studies that produce small TFP losses emphasize the high persistence of productivity but predict a strong self-financing motive. The latter is not consistent with the data.[7]

I develop a model of entrepreneurship with default, consistent with empirical evidence on the persistence of individual productivity and self-financing. That is, my model can account for a high persistence of productivity while generating a weak self-financing motive. This can explain large TFP losses from misallocation caused by financial frictions. My results indicate that the U.S. economy, if it had a highly distorted financial market, would lose 43% of its TFP compared to its relatively undistorted situation. Without default, my model would be isomor-

[1]See Levine (2005), Matsuyama, Gertler and Kiyotaki (2007), Townsend (2010) and Buera, Kaboski and Shin (2015) for comprehensive surveys on this literature.

[2]Buera, Kaboski and Shin (2011) and Buera and Shin (2013) documented large losses while Midrigan and Xu (2014) and Gopinath et al. (2017) documented relatively small losses.

[3]See Pawasutipaisit and Townsend (2011), Midrigan and Xu (2014), and Moll (2014) for evidence on the high persistence of individual productivity.

[4]Throughout the paper, by self-financing motive, I mean both ability and motive to accumulate wealth for self-financing.

[5]The role of the persistence of productivity shocks is discussed in length by Moll (2014).

[6]The evidence is discussed extensively in section 2.3.

[7]See Buera and Shin (2013), Buera, Kaboski and Shin (2011), Midrigan and Xu (2014), and Moll (2014)

phic to a Buera and Shin (2013) model, which, in the absence of distortionary taxes, functions similarly to a neo-classical model. This sharply reduces the impact of financial frictions, and the comparable exercise leads to only 7% TFP loss.

Default risks generate a role for collateral[8] as a means of securing loans. Entrepreneurs differ in their default probabilities.[9] Heterogeneity in entrepreneurs' default probabilities leads to heterogeneity in business loan collateral. Both lead to heterogeneity in loan interest rates across entrepreneurs. As a result, the traditional link, in models without default, between loan rates and deposit rates, weakens. This disproportionately affects those poor individuals with ideas worth implementing but low collateral. In my model, individuals can employ collateral to lower the default risk and, therefore, their cost of funds. This implies that poor entrepreneurs face higher loan rates and therefore are more rationed in terms of investment.[10] Therefore, a highly distorted financial market makes it nearly impossible for talented but poor individuals to start businesses. Also, existing businesses with a large need for external finance would have difficulty expanding. These lead to TFP losses.

A contribution of this paper is that my setup allows me to disentangle the effects of financial frictions due to three different sources: First, there is a limited enforceability problem in the model. This is related to the ownership transfer cost, or recovery rate, of collateral for the financial intermediary. A low collateral recovery rate means a higher cost for financial intermediaries in the case of default, which would drive up loan rates and collateral requirements. Second, there are informational frictions related to the ability of financial intermediaries to accurately assess the default probabilities of the loan applicants. A less efficient evaluation of default risks by financial intermediaries causes large losses for them. This leads to higher loan rates and collateral requirements.[11] I have used a simple, innovative approach borrowed from the machine learning field to handle the assessment problem of financial intermediaries.[12] Third, there are

[8]Collateral in my model is defined as part of the working capital that will be transferred to the financial intermediary if an entrepreneur defaults. This capital is purchased using loans or entrepreneurs' own funds, or a combination of both.

[9]Default probability is related to the current state of the entrepreneur as well as the uncertainty about the future. However, it is evaluated by the financial intermediary with partial access to information about the entrepreneurs' state.

[10]Also, there might be little (or no loans) offered by the financial intermediary to individuals with very low collateral. That is, there is a possibility of a borrowing cap for low levels of collateral.

[11]This is a case where financial intermediary does not know the true default risk and might, for example, end up assigning low risk and offering low rate to an actually high-risk borrower, and vice versa.

[12]In the model, financial intermediaries use decision trees classifier to evaluate the default probability for any given loan contract, based on loan and borrower information available to them. The depth of the decision tree will control the quality of their assessment. I will explain the reasons for choosing decision trees over alternative methods in the modeling section.

informational frictions affecting entrepreneurs' assessment of future loan terms. Individuals in my model are heterogeneous with respect to their creditworthiness. Entrepreneurs with high creditworthiness have a higher aversion to default than those with low creditworthiness.[13] If individuals' creditworthiness does not vary much over time, they can make reliable predictions of their future loan terms and, as a result, make self-financing plans for either starting or expanding their businesses.[14] That is, a high persistence of creditworthiness implies less uncertainty regarding future financing.[15] A low persistence of creditworthiness, on the other hand, means higher uncertainty for entrepreneurs and potential entrants regarding their financing in the future. This uncertainty discourages self-financing because individuals do not know if their savings will be adequate to secure a loan at a reasonably low rate in the near future.

My modeling choice for the severity of financial frictions is motivated by my empirical findings of a rich heterogeneity in collateral rates[16] within countries, and differences in the distribution of collateral rates across countries. Unlike existing models in the literature, where financial development is measured by external dependence, financial development in my model is represented by the distribution of collateral rates across firms.[17] More specifically, I use the first three moments of the collateral rates distribution related to the three mentioned sources of financial frictions to specify the level of financial development in the economy.

To quantitatively discipline my model, I initially calibrate the parameters to reflect U.S. data on wealth and entrepreneurship. Then, I change the U.S. level of financial development by adjusting three financial frictions parameters. Using these parameters, I match the first three moments of the collateral rate distribution for countries in different per-capita income groups. That is, as if the U.S., with all its underlying characteristics, had a less developed financial market. I use this exercise to analyze the effects of financial frictions on TFP, entrepreneurship, and wealth concentration.

In another exercise, I change the three financial market parameters, one at a time, to distorted levels of the previously mentioned exercise while keeping all other parameters at

[13]The creditworthiness can be interpreted as different forms of customer-bank relationships, which might make it easier to get better loan deals for some agents than others. It could also be seen as some limited type of credit score which informs financial intermediaries of the borrowers' default risk. See Chatterjee et al. (2020) for a quantitative theory of credit scores.

[14]This is assuming the fact that productivity is also highly persistent, consistent with empirical evidence.

[15]A more persistent creditworthiness can also be interpreted as a better credit registry in the economy and vice versa.

[16]Throughout the paper, by collateral rates, I mean the ratio of collateral value to loan amount or Value-to-Loan Ratio.

[17]In section A.4, I show the empirical relevance of the collateral rate distribution to economic development.

their U.S. levels. Ownership transfer costs equivalent to that of the countries in the lowest income decile reduces U.S. TFP by 9%. The quality of financial intermediaries' assessment of default risks, together with the persistence of creditworthiness, account for a 22% drop in the U.S. TFP, with the latter being more important; 5% vs. 12%, respectively, when they are considered independently. This implies a relatively significant role for enforceability but an even larger role for informational frictions in explaining TFP losses from financial frictions. Note that for all these exercises, productivity shocks are highly persistent, consistent with empirical evidence.

In my model, unlike existing models in the literature, entrepreneurs face different loan interest rates depending on how much collateral they want to or are able to pledge. Individual loan rates are not the same as the risk-free rate or the deposit rate. Therefore, higher financial frictions, due to limited enforceability or informational frictions, would result in higher collateral requirements and loan rates,[18] but not proportionally higher deposit rates. This would have implications on the wealth accumulation across agents. To clarify this, below, I discuss the wealth accumulation dynamics of potential entrants and existing entrepreneurs, both of which are important for economic development.

Extensive-margin effect (potential entrants): In a standard model[19] with high financial frictions, talented but poor individuals would still be highly motivated to save in order to accumulate the wealth required to start their own businesses. When financial constraints tighten in these models, deposit rates decrease, which discourages savings. Nevertheless, since deposit rates are the same as the loan rates, a lower loan rate encourages savings for high productivity individuals as they want to accumulate wealth to start businesses. Wage rates also decrease in these models implying even larger returns for entrepreneurship. However, in my environment with high financial frictions, deposit rates decrease, discouraging savings. Since the link between deposit and loan rates is broken, loan rates increase on average, decreasing the returns to entrepreneurship and further discouraging self-financing. Besides, the wage drop in my environment is small, altogether implying low returns to entrepreneurship. With higher financial frictions, individuals in my model need to provide higher collateral to avoid paying extremely high interests on their loans when they start a business. This would mean lower expected earnings for prospective entrepreneurs and, given a large gap between the deposit rates and

[18]The increase in collateral and loan rates would vary across the agents, with a smaller increase for some and a larger increase for other agents.

[19]For example, Midrigan and Xu (2014), or Buera and Shin (2013) with relatively high persistence of productivity.

loan rates, would discourage savings by potential entrants.[20] This would further distort entry away from the talented individuals and towards the wealthy and would cause misallocation at the extensive-margin.

Intensive-margin effects (existing entrepreneurs): There is a similar situation evolving around incumbent entrepreneurs. High productivity entrepreneurs would want to accumulate wealth in order to expand their businesses. In standard models with tighter financial constraints, entrepreneurs enjoy lower loan rates and wages which means high returns for investment. This encourages them to save and overcome financial constraints. However, in my environment with higher financial frictions, some entrepreneurs can take advantage of low loan rates, but the rates are high for the remaining, perhaps poor, ones. The resulting lower return on their businesses makes wealth accumulation more challenging for poor entrepreneurs. As a result, poor entrepreneurs with low levels of collateral are disproportionately affected by financial frictions.

The above-mentioned disproportionate effects on poor entrepreneurs and potential entrants are reminiscent of the phrase *"the tyranny of collateral"* by Rajan and Zingales (2004). A combination of these extensive- and intensive-margin effects implies that financial frictions can have large and amplifying effects on aggregate output and TFP. Related, financial frictions can create massive wealth inequality. I will focus on the right tail of the wealth distribution and, consistent with empirical evidence, argue that it matters a great deal for economic development. The right tail of the wealth distribution is shaped by both intensive- and extensive-margin effects discussed above. I show that for high levels of financial frictions, wealth becomes highly concentrated at the top.[21] I will also focus on the distribution of wealth amongst entrepreneurs. In under-developed financial markets, wealth becomes highly concentrated among fewer entrepreneurs. This is because, in an under-developed financial market, the wealthiest entrepreneurs earn far higher returns on their assets. The models introduced in the literature are not successful in producing the inequality amongst the entrepreneurs or the fraction of aggregate resources owned by the very wealthiest.[22]

This paper is organized as follows. After reviewing the related literature in the following, I will extensively discuss the empirical evidence in section 2. In section 3, I introduce the

[20] The severity of this discouragement effect would also depend on preference-related factors.

[21] For the concentration of wealth amongst wealthy we can think of different measures: for example, the wealth share of top 1% over the share of top 10%; the wealth share of top 1% over top 5%; or the wealth share of top 5% over top 10%, etc.

[22] In section 2.3, I will provide some empirical evidence on the inverse relationship between financial development and concentration of wealth at the top, consistent with the argument I have laid out.

quantitative model and its solution. The model results and how they relate to the observations from the data are explained in section 4. Concluding remarks and directions for future research are provided in section 5.

Related Literature

Motivated by empirical observations, this paper contributes to our understanding of finance-development links using a quantitative model of financial frictions where aggregate outcomes are driven by individual decisions on occupation, financing, default, and savings.[23]

The effects of financial frictions are heavily dependent on individuals' ability to accumulate wealth and overcome financial constraints. To a great extent, this is governed by the persistence of individual productivity in the quantitative models in the literature. There is a long tradition emphasizing the persistence of productivity in models of firm dynamics, starting with the seminal work of Hopenhayn (1992).[24] The emphasis on the role of the persistence of productivity in the context of financial frictions is also not recent, [Cooley and Quadrini (2001)]. The fact that a higher persistence of productivity shock dampens the effect of financial frictions on TFP was first elaborated by Caselli and Gennaioli (2013). The dependence of self-financing on the persistence of productivity in the literature explains much of the variations in existing results. See Restuccia and Rogerson (2017) for a discussion. Also, see Moll (2014) for an extensive analytical assessment of the persistence of productivity and self-financing. My paper is the first to resolve an empirical tension between the persistence of productivity and self-financing. I achieve this by introducing default and heterogeneity in loan rates and collateral. In my model, a weak self-financing motive is consistent with highly persistent productivity.

Buera, Kaboski and Shin (2011) and Buera and Shin (2013) attribute large effects to financial frictions.[25] The former develops a two-sector economy and analyzes sectoral dynamics while the latter focuses on transition dynamics in a one-sector economy. Both works characterize financial frictions as a form of collateral constraint. In contrast to my work, neither has default or heterogeneity in loan rates and collateral.[26] The main driver of the large effects of financial

[23]There is a large empirical, theoretical and quantitative literature trying to explore the links between financial development and economic development. Extensive surveys are conducted by Levine (2005), Matsuyama, Gertler and Kiyotaki (2007), Townsend (2010) and Buera, Kaboski and Shin (2015).

[24]See Shaker Akhtekhane (2017) for an analysis of Hopenhayn's model in a continuous-time setting.

[25]Many other works in the literature have also documented large effects from financial frictions, [Jeong and Townsend (2007), Amaral and Quintin (2010)].

[26]Buera, Kaboski and Shin (2011) use an endogenous form of collateral constraint related to contract enforceability, but it has no implications on loan rate.

frictions in their models is the weakness of self-financing due to a relatively low persistence of productivity shocks. In my model, consistent with empirical evidence, the persistence of productivity is high, but the self-financing motive is relatively strong in developed financial markets while weak for under-developed financial markets.[27]

Midrigan and Xu (2014) argue that the effects of financial frictions on TFP are small.[28] They develop a model with technological choice in an economy with formal and informal sectors. They provide evidence for a high persistence of individual productivity and reflect it in their quantitative analysis. The high persistence of productivity leads to a strong self-financing motive, making wealth accumulation easy, especially when agents enter the productive sector. This dampens the impact of financial frictions on TFP. In my model, default and heterogeneity in loans make wealth accumulation for self-financing very difficult when financial frictions are high.

More recently, the adoption of productivity processes that are different from the standard AR(1) has gotten attention in attempts to be consistent with the high persistence of individual productivity while having a weaker self-financing motive. Ruiz-Garcıa (2020) has used non-linear and non-Gaussian productivity,[29] and produced relatively large TFP losses from financial frictions. Different from his work where the productivity process drives the results, my results are driven by default and a rich heterogeneity in collateral and loan rates while using a standard AR(1)-type process.[30] Apart from the mentioned underlying differences in the mechanism, another reason that my model generates larger losses than Ruiz-Garcıa' is that in my model, financial frictions arise from multiple sources, enforceability and informational frictions, each of which has different implications on certain margins as well as the amplifying effects. In contrast, his financial frictions stem from a size-dependent collateral constraint similar to the one in Gopinath et al. (2017).[31]

My paper also contributes to another strand of literature relating to financial development and wealth inequality. In a closely related work, Cagetti and De Nardi (2006) show that more

[27] See the discussion in section 2.3.1.

[28] See also Gopinath et al. (2017) who produce small losses from financial frictions.

[29] Jo and Senga (2019) also uses non-Gaussian productivity process to assess the policies that alleviate the financial burden of small and young businesses. De Nardi, Fella and Paz-Pardo (2020) use similar processes in the context of household earnings dynamics and welfare analysis.

[30] I use Ornstein-Uhlenbeck process which is the equivalent of AR(1) in continuous time.

[31] The use of different forms of collateral constraint has a long tradition in the literature. The earlier contributions on models of financial constraints that use collateralized assets as the basic cost of financing are Bernanke and Gertler (1989), Banerjee and Newman (1993), Kiyotaki and Moore (1997) and Berger and Udell (1990) to name a few.

restrictive financial constraint would result in less wealth concentration. I focus on wealth concentration at the top and amongst entrepreneurs, arguing that they are the most relevant for economic development. However, in my mechanism with heterogeneity in loan rates, financial frictions increase wealth concentration at the top, with the effects being more severe at low levels of financial development.[32]

In a related paper, Chatterjee and Eyigungor (2020) rationalize the increase in firm concentration through low levels of the risk-free rate in an environment with financial frictions. In my paper, financial frictions generate high firm concentration that I discuss in the context of wealth inequality amongst entrepreneurs. In their mechanism, low interest rates benefit large firms, and in my environment, larger firms (i.e., wealthy entrepreneurs) can enjoy lower interest because they can pledge high collateral, which generates higher firm concentration, i.e., higher wealth inequality amongst entrepreneurs.

Regarding the underlying sources of financial frictions, apart from enforceability,[33] my paper contributes to the literature that relates economic development to informational frictions. David, Hopenhayn and Venkateswaran (2016) develop a model where firms make production decisions under imperfect information,[34] and produce relatively sizeable TFP losses from informational frictions. My source of informational frictions affects both financial intermediaries and entrepreneurs. Although the target of informational frictions is different in my model, the magnitude of productivity losses from these frictions is comparable to their results.

My paper also contributes to the literature that provides different measures and indicators for financial development to explain economic growth and development. In an influential industry-level study Rajan and Zingales (1998) use the ratio of private credit to GDP and stock market capitalization, also as a ratio to GDP, to show that higher financial development facilitates economic growth.[35] Influenced by Rajan and Zingales' work, the external dependence as an indicator of financial development has become extremely popular in the quantitative

[32]See Jalilian and Kirkpatrick (2005) for an empirical analysis on this. Also, see Madsen, Islam and Doucouliagos (2018) who discuss the role of inequality on economic development conditional on financial development. In a sample of OECD data, they show that inequality limits economic development when financial markets are under-developed, but it has little to no effect when financial markets are developed. This is consistent with my results and the evidence on declining self-financing ability in under-developed financial markets.

[33]See, for example, Amaral and Quintin (2010) for a study on the importance of limited enforcement for economic development.

[34]See Bloom et al. (2013) for another related work.

[35]Goldsmith (1969) was the first to use total assets over GDP as an indicator of the size of the financial sector to show its positive correlation with economic growth. Many other works have also used credit to GDP as measures of financial development in a similar context, e.g., Arcand, Berkes and Panizza (2015) and Dabla-Norris and Srivisal (2013)

literature where most models use this measure to govern the level of financial frictions in the economy.[36] The state of financial development in my economy is measured by the distribution of collateral rates. I argue that this distribution is very relevant in the context of economic development since it contains information from both financial intermediaries' and firms' side, e.g., information related to default risks and how they are evaluated. Across countries, the distribution of collateral can inform us about the underlying legal and institutional differences related to enforceability and informational frictions.[37]

Finally, in a broader sense, my paper is related to the literature that studies the effects of factor misallocation on aggregate outcomes. The leading works in this literature are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Reasonably large TFP losses from resource misallocation are documented by Hsieh and Klenow.[38] My paper also reports large losses from misallocation caused by financial frictions, particularly limited enforcement and informational frictions.

2 Empirical Considerations

In this section, I will use firm-level data from World Bank's Enterprise Survey[39] in conjunction with other standard cross-country data sets[40] to provide evidence on: the importance of collateral and its implications for misallocation, the relevance of the distribution of collateral rates for financial and economic development, and the moments of the distribution of collateral and the significance of the first three moments. I close this section with a discussion on a weakening self-financing motive in financially less-developed countries as well as wealth concentration and how it relates to economic and financial development.

The firm-level data from the World Bank's Enterprise Survey mainly consists of low-income and developing countries and some developed countries. Many businesses are surveyed about their financing as well as the limitations they face regarding their operations. The most relevant

[36] As argued by Čihák et al. (2012) and Aizenman, Jinjarak and Park (2015), there are other important features of financial systems that should be considered as indicators of financial development. They consider some forms of access and efficiency in addition to the depth of financial markets and institutions.

[37] I extract various indicators from the collateral rate distribution and inspect their relevance to institutional indicators as well as economic development indicators. I find that the first three moments of the collateral rate distribution are the most relevant ones, and they remain significant after controlling for several existing indicators of financial development.

[38] They calculated about 40 percent loss in the manufacturing sector in India and China.

[39] See Appendix A.1 for more information about this data set and my preparation steps.

[40] e.g., World Bank's World Development Indicators, Penn World Tables, International Monetary Funds' Financial Development Index, etc.

piece of information in the survey to the purpose of this study is the loan and collateral values reported by firms as well as the main obstacles they face regarding financing.

In the entire sample, one-third of all firms do not apply for loans or lines of credit despite their needs. This is substantial and significant in terms of access and allocation of capital. It would cause severe misallocation in the intensive margin as, on average, one-third of the businesses are under-financed and are operating under their desired capacity. The number of financially constrained firms in the sample varies across countries.

The survey also asks firms about the reasons that kept them from getting the needed financing. The reasons are: unfavorable interest, complex application procedures, too high collateral requirement, no hope for loan approval, insufficient loan size and maturity, and other reasons. Among the various reasons, loan rates and collateral requirements are the dominant ones. They are the main reasons for more than half of the firms in the sample that did not apply for the needed loans. In terms of these deterring reasons' explanatory power regarding economic development, machine learning classification techniques show that collateral is the most relevant to TFP and GDP per capita. See Appendix A.2 for an extensive discussion on collateral and its relevance to misallocation.

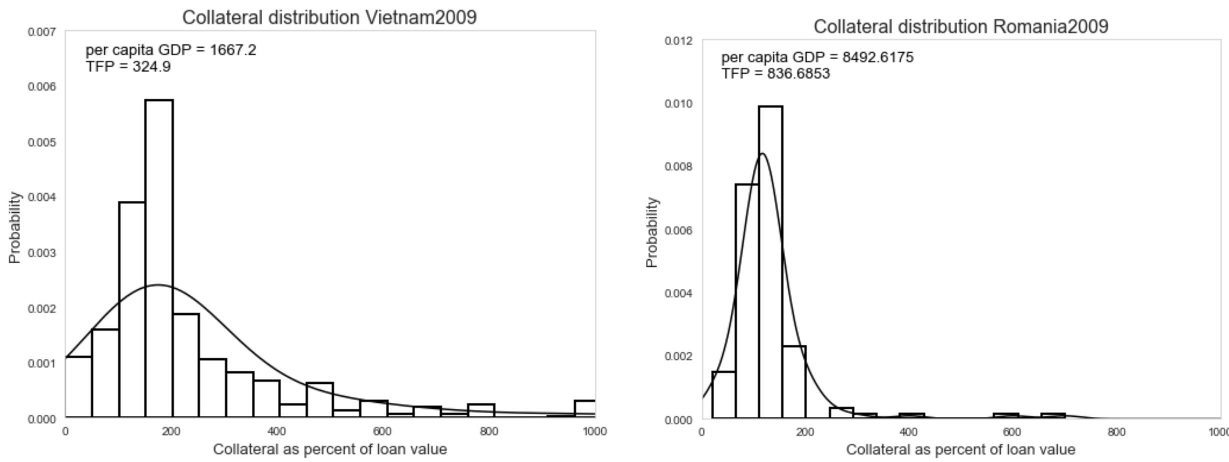
2.1 Collateral Rates Heterogeneity

The discussion above[41] provides us with the evidence that loan rates and collateral are the main reasons deterring many businesses from obtaining the financing they need. Compared to other factors, collateral is the strongest factor in explaining TFP and GDP per capita differences across countries. As a result of this, and the fact that the survey does not cover loan interest rates paid by the businesses, I now explore the role of collateral and its variation within and across countries.

Within any given country, I observe a rich heterogeneity in collateral rates.[42] Figure 2.1 shows this heterogeneity through the distribution of collateral rates for Vietnam (2.1a) and Romania (2.1b). This rich heterogeneity is an indicator of default risks in the economy. If there were no default risks, collateral as a tool for securing loans would be irrelevant. In an environment without default, agents could easily get the funds they need at the same risk-free

[41]See a more detailed discussion in Appendix A.2

[42]Collateral rate is defined as the value of collateral as a ratio of the loan value.



(a) Collateral rate distribution: Vietnam 2009 (b) Collateral rate distribution: Romania 2009

Figure 2.1: Distribution of collateral rates

rate[43] which is the case for most models in the literature.[44] However, the fact that agents in an economy vary in their likelihood of default creates the need for collateral. As a result, financial intermediaries specialize in loans for customers with different underlying default risks. Since financial intermediaries try to avoid losing money in the long run, a relationship will be established between default risks, collateral rate, and the interest charged on loans. This will generate a rich heterogeneity in collateral rates observed in the data.

Across countries, we observe fundamental differences between distributions of collateral rates. Figure 2.1 illustrates the differences between the distribution of collateral rates in Vietnam and Romania. As we can see in figure 2.1, margins of difference could be relevant for financial and economic development. It is unclear whether the mean, median, standard deviation, skewness or kurtosis best capture the quantitative importance of cross-country collateral heterogeneity. Nevertheless, before any deeper investigation, we can see some stark differences between Romanian and Vietnamese collateral rate distributions that can inform us about the state of financial development in these countries. Next, I turn to analyzing and extracting some relevant indicators from the collateral rate distribution that can help explain cross-country differences in income and TFP.

[43] There might be other costs such as depreciation, but the idea is that the rates would be identical for different agents.

[44] Buera, Kaboski and Shin (2011), Buera and Shin (2013), Midrigan and Xu (2014) to name a few.

2.2 Relevant Moments of Collateral Distribution

Here I explore whether certain features of the collateral distribution can explain economic development beyond what the existing indicators of financial development do. Despite the strong relationship between finance and development, there remains a great deal of variation unexplained through the conventional indicators of financial development,[45] say external dependence.[46] I will show that differences in collateral distribution can help explain the variation in TFP and GDP per capita. Also, my choice of modeling is related to this, where the collateral distribution determines the level of financial development.[47] Cross-country data exhibits a relatively strong association between external dependency and economic development indicators such as GDP per capita and total factor productivity (TFP). This is depicted in figure 2.2 along with an example of the same two countries, Romania and Vietnam, displaying a completely different picture.[48]

An important takeaway from figure 2.2 is that: Romania lags Vietnam in financial development[49] but leads in economic development, and the differences are somewhat stark. Combining this piece of information with that provided in figure 2.1, we can see the possibility that the information in collateral rate distribution can help explain the differences in income and productivity between Romania and Vietnam, as well as across other countries in the sample. After exploring such a possibility, I observe that there actually is useful information related to economic development in the collateral rates distribution. To analyze this, I use different measures extracted from the collateral distribution, including simple mean, variance, and higher moments of the distribution as well as some other more subtle features such as the moments

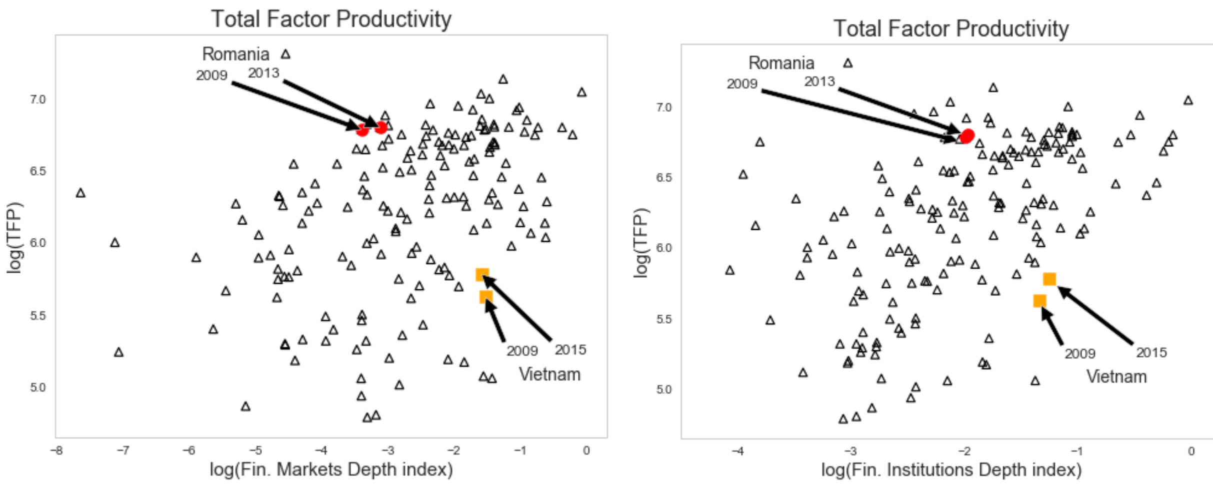
[45]Some indicators do a better job than others, but there remains unexplained variation in TFP and GDP per capita. Also, some of the existing indicators include information related to household financing rather than business financing. As a result, even though some existing measures provide a relatively good fit for TFP or GDP per capita, my measures related to collateral distribution would still be valuable as they are directly and only related to firms' financing.

[46]I use the financial markets' and financial institutions' depth indexes from the IMF as measures of external dependence. I do so because the richness of IMF's cross-country data allows me to match most of the country-year observations in the sample of collateral distributions from World Bank's Enterprise Survey. See Appendix A.3 for an explanation on these indicators, and to see how these measures are related to external dependence measure.

[47]This is different from the standard practice in the literature, where mainly the external dependence indicator determines the level of financial development or financial frictions.

[48]Measure of GDP per capita is taken from World Bank's Development Indicators, TFP measure constructed using the same method of Klenow and Rodriguez-Clare (1997) using Penn World Tables 9.1, and external dependency is taken to be financial market's depth index from International Monetary Fund. Also, note that in order to be consistent with my analysis throughout the paper, in figure 2.2 I have used the same countries across the same years for which data is available in the firm-level data set of World Bank's Enterprise Survey.

[49]Financial development measured by conventional indicators.



(a) Financial markets depth vs. TFP

(b) Financial Institutions depth vs. TFP

Figure 2.2: External dependence vs. TFP

within certain inter-quartiles of the distribution and other more complex measures such as entropy, divergence, etc. In order to visually inspect the significance of these features, I use Linear Discriminant Analysis (LDA), a tool from Machine Learning, to create an index from the information extracted from collateral distribution.[50] I call it the collateral distribution index. Note that I use this technique to reduce a large number of relevant variables into a few (one in this case), and as a result, there will be some useful information lost in the process. However, this helps us visualize the relationship between the information extracted from collateral distribution and economic development indicators. This also provides evidence on the relevance and significance of collateral distribution in explaining economic development. Figure 2.3 shows the relationship between the collateral distribution index and TFP as well as GDP per capita.

What we see in figure 2.3 is a clear association between the collateral distribution index and TFP and GDP per capita. Again, note that we lose some information related to the rich distribution in the process of reducing it to a single variable. Despite this, the evidence on such a relationship is clear. This strengthens the idea that the collateral rates distribution as a firm-related indicator of financial development can well-explain economic development.

Since the collateral distribution index is more of an abstract measure and does not have a model counterpart, I will try to extract very few useful features of the distribution related to my quantitative model. To do so, I will use the Random Forest feature importance technique to identify the features of the collateral distribution that have the highest explanatory power

[50] The method is explained in the Appendix A.4.

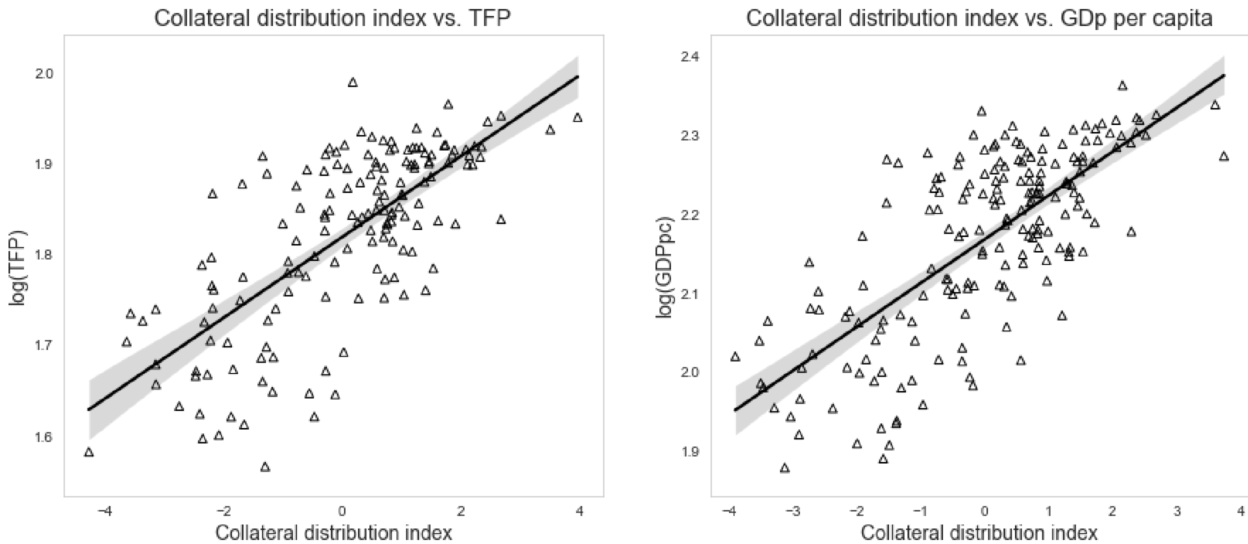


Figure 2.3: Collateral distribution index explaining development

regarding GDP per capita or TFP. I find that the standard deviation of the bottom half of the distribution is the most relevant, followed by the distribution's skewness. I will also use the mean of the distribution because of its simplicity. There are many other features of the distribution that are worth considering. However, given the simplicity of these three moments and their model counterparts, I will use these three features to summarize the information in the collateral rate distribution. Regression results are provided in Appendix A.4.

2.3 Self-financing and Wealth Concentration

In this section, I discuss the existing evidence on self-financing and its relationship with financial development, followed by my observations on wealth concentration at the top and amongst entrepreneurs in the data. Self-financing is the most crucial determinant of the impact of financial frictions on economic aggregates in the models with financial frictions. Its relationship with the persistence of individual productivity causes inconsistency in these models. In existing models, self-financing is strong (weak) if the persistence of individual productivity is high (low). However, empirical evidence suggests that self-financing is weak, especially in financially less-developed countries, while individual productivity is highly persistent.[51] As a result of this inconsistency and other model features, these models predict that wealth concentration at the top and amongst entrepreneurs decreases with the tightening of financial constraints, which

[51] Evidence on high persistence of productivity is provided in Pawasutipaisit and Townsend (2011), Midrigan and Xu (2014), and Moll (2014).

contrasts with empirical observations.

2.3.1 Self-financing

Here, I survey the evidence showing that self-financing becomes more difficult if the financial markets are less-developed. This diminishing effect is worth considering in its implications for both the intensive margin related to incumbent firms and the extensive margin related to potential entrants.

In the intensive margin, self-financing determines the ability of entrepreneurs to accumulate wealth and expand their businesses. Feasibility of self-financing in the intensive margin would imply: 1. much higher saving rates for entrepreneurs than non-entrepreneurs, and 2. no significant increase in return to investment resulted from a small, exogenous increase in entrepreneurs' wealth. We have evidence for the first criteria in financially more developed countries and against the second criteria in financially less developed countries. Using U.S. data, Gentry and Hubbard (2004) and Quadrini (1999) document that there is an acceleration of wealth accumulation among entrepreneurs resulting in much higher wealth-to-income ratios compared to that of non-entrepreneurs. Similarly, using Thai data Pawasutipaisit and Townsend (2011) find higher saving rates for high productivity households and those with higher returns on business assets. These findings indicate that after entry, self-financing might be feasible. However, this is more difficult in less developed countries. Multiple studies have shown large rates of returns for capital, far exceeding market interest rates, from small grants to small businesses: De Mel, McKenzie and Woodruff (2008) in Sri Lanka; McKenzie and Woodruff (2008) in Mexico; Fafchamps et al. (2011) in Ghana, all find that small grants significantly increase the rate of return on capital for small entrepreneurs.[52] This implies large barriers to wealth accumulation for poor entrepreneurs in less-developed economies. Such high returns on capital should otherwise attract many entrepreneurs to save and accumulate wealth. The fact that this is not evident absent the small grants suggests a difficulty, and in some cases impossibility, of self-financing in the intensive margin in countries with less-developed financial markets.

We can ask two related questions in the extensive margin: 1. is wealth a determinant of entry? and 2. does a sudden, exogenous increase in wealth increase the probability of starting a business? The answers to these questions will clarify the extent to which self-financing is feasible. There are many studies addressing the first question. The most notable work using

[52]Also, McKenzie (2015) finds similar results using much larger amounts of randomized grants. He also finds evidence for employment growth.

the U.S. data is Hurst and Lusardi (2004). They find that wealth is not a determinant of entrepreneurship except in the top 5% of the wealth distribution. They show that exogenous shocks to wealth are more relevant to entrepreneurship than wealth itself. Nykvist (2008) runs a similar exercise to Hurst and Lusardi' using Swedish data and finds that liquidity constraints are somewhat more extensive than they are in the U.S. In Thailand, Paulson and Townsend (2004) find that financial constraints play a crucial role in entrepreneurial activity. The differences are also stark between the wealthy region and the poor region, with the entrants in the latter being affected more severely by financial constraints. The answer to the first question is that wealth is an important determinant of entry to entrepreneurship. Regarding the second question, Hurst and Lusardi (2004) find that inheritance has a positive relationship with the probability of starting a business. Using British data, Taylor (2001) shows that the probability of entering entrepreneurship is an increasing function of the size of windfall payments received. Similar results have been found using Swedish data, Lindh and Ohlsson (1996), and using German data, Schäfer, Talavera and Weir (2011). Using Spanish lottery data, Bermejo et al. (2018) show that entrepreneurial activity increases in regions with a higher concentration of lottery winners. These results clarify the answer to the second question. Exogenous increases in wealth significantly increase the probability of entry.

The evidence provided for both questions shows that wealth by itself is not a strong determinant of entry in the U.S. while it is in other countries. Also, the probability of entering into entrepreneurship increases for the receivers of a windfall payment. Since it is unlikely that the windfall gains increase the probability of entry for the wealthy (they already would have entered if they wanted to), the increase in the entrepreneurial entry due to windfall gains can be attributed to the participation by the poor into entrepreneurship.[53] From another perspective, this might explain the results of Hurst and Lusardi (2004) who only find a relationship between wealth and entry at the top of the distribution. Exogenous (inheritance) shocks, as an instrument, do not have as much effect amongst the wealthy as they do amongst the poor. Therefore, controlling for exogenous increases in wealth may explain an increase in entrepreneurship amongst the less wealthy, but it cannot explain an increase in entrepreneurship amongst the wealthiest.

Combining the evidence on both extensive- and intensive-margin effects, these findings indicate that self-financing is not as strong as one might think, and it is particularly weaker in

[53]That is, these are the poor who were financially constrained, and the windfall gains make them overcome the constraint and start their businesses. The wealthy would not be as sensitive to the windfalls and start a business.

less wealthy countries and those with under-developed financial markets. This, together with the evidence on the high persistence of individual productivity, relates to the above-mentioned inconsistency in existing models.

2.3.2 Wealth Concentration

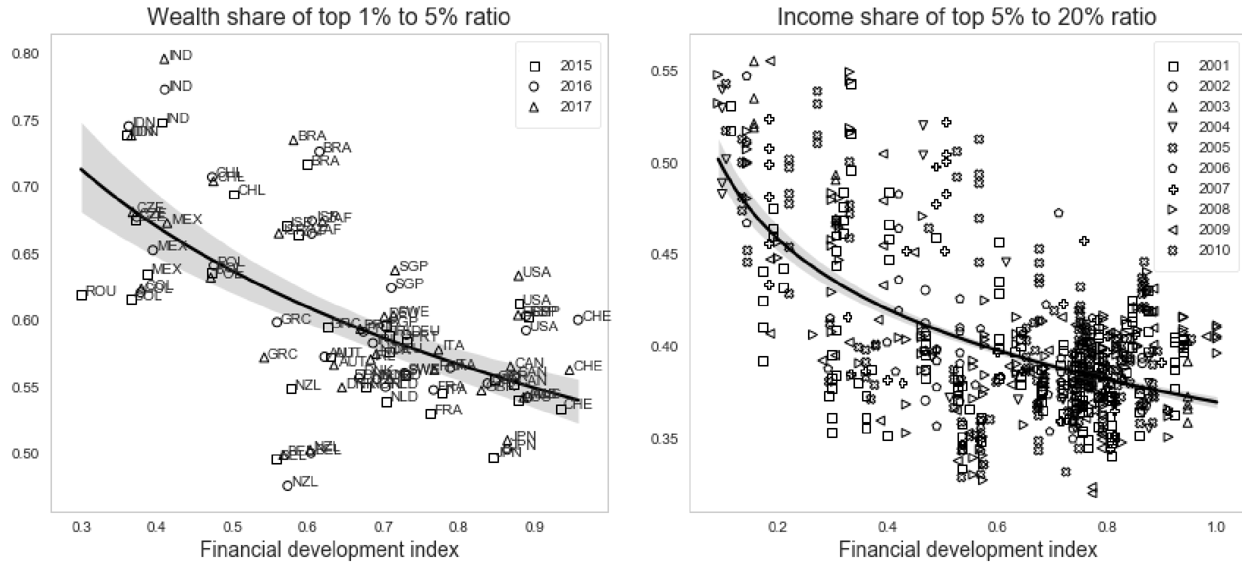
Related to the strength of self-financing, an important point is that the tightening of financial constraints can create a high concentration of wealth at the top and amongst entrepreneurs. This is mostly neglected in the literature, and most existing quantitative models produce results inconsistent with empirical observations.

In order to explore the relationship between financial development and wealth concentration, I use cross country wealth data from the Credit Suisse Research Institute, in conjunction with the financial development index from the International Monetary Fund.[54] The wealth data used is for the years 2015 to 2017. The correlation of the financial development index with the wealth share of the top 1% over the wealth share of the top 5% is -0.56, which is large. This relationship is consistent with other wealth groups, say top 1% over top 10%, or top 5% over top 10%, all of which exhibit strong negative correlations with financial development.

Because of the fact that the wealth data from the Credit Suisse Research Institute mostly contains developed countries, one might want to know the same relationship, including less-developed economies. For this reason, I use income data as a proxy for wealth data across countries for which there are richer data sets available. For income data, I use the World Income Inequality Database (WIID) of UNU-WIDER, from the year 2000 to 2017. The correlation of financial development with the income share of top 5% over the income share of top 10% is also large, about -0.52. This is also consistent when we use other income groups instead, say top 5% over top 20%. Figure 2.4 shows the relationship of financial development with both wealth and income concentration at the top.

Regarding the wealth inequality amongst entrepreneurs, a cross country data that focuses only on entrepreneurs or business owners' wealth shares would be helpful, but such data is not available. Instead, as a proxy, I look at the Herfindahl-Hirschman Index (HHI) across countries from the World Bank's WITS database. This is a firm concentration measure, and to a limited extent, would inform us about the inequality amongst the entrepreneurs if we assume that there is a reasonably high correlation between the size of the businesses and the wealth of the business

[54]Note, that throughout the paper I demonstrate the results related to financial development using the financial development index of International Monetary Fund, but I have also used other measures such as private capital to GDP, financial markets/institutions depth index, all of which produce similar results.



(a) Wealth concentration

(b) Income inequality

Figure 2.4: Wealth/Income concentration vs. financial development

owners. The correlation of financial development with HHI is not very high, -0.22, but it still is an indicator of a negative relationship. The coefficients of the regression of HHI against financial development index are significant.[55] This implies that wealth concentration amongst the top wealthy entrepreneurs is much larger in economies with under-developed financial markets than it is in those with developed financial markets. The evidence from the previous subsection on self-financing in the intensive margin would strengthen this idea. Further discussion on wealth concentration is provided in Appendix A.5.

3 Model

In this section, I discuss a model of entrepreneurship with financial frictions and default. The model follows the setup of Buera and Shin (2013), and for the within period loan structure, the model is similar to Cooley and Quadrini (2001). The agents in my model make choices regarding their occupations, consumption-saving, financing, and default.

[55] I also control for multiple related factors. See Appendix A.5 for regression results.

3.1 Outline of the Model

Time is continuous. There are two main types of agents in my model, where each of them makes certain decisions. I will have individuals as well as a financial intermediary.

Individuals: There are measure 1 of infinitely lived individuals who can choose to work for a wage or be entrepreneurs. Individuals are trying to maximize their lifetime utility from consuming a homogeneous good produced in the economy. Preferences are characterized by a CES utility form given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

The individuals are heterogeneous in their wealth, a, productivity ξ, and creditworthiness, κ. Wage workers receive wages and make consumption-savings decisions only. Entrepreneurs will use capital and labor inputs to produce using the following technology:

$$y = \xi f(k, l) = \xi k^{\alpha} l^{\theta},$$

where $\alpha + \theta < 1$. The entrepreneurial productivity, ξ, has two components: a persistent part that is known at the beginning of the period before making production decisions, z; and an unknown part, ϵ, that they find out about in the middle of the period after the production decisions are made.

$$\xi = z + \epsilon$$

The shocks take the form of diffusion processes.

$$dz_t = \mu_z(z_t)dt + \sigma_z(z_t)dW_t$$
$$d\epsilon_t = \sigma_\epsilon(\epsilon_t)dW_t,$$

where W is a Brownian motion, μ_z and σ_z are drift and diffusion of the known part of the productivity process, and σ_ϵ is the diffusion of the unknown part of the productivity process. For z I will use an Ornstein-Uhlenbeck process, which is equivalent of AR(1) in continuous time. I denote the persistence of z process by ρ_z.

Financing and financial intermediary: Entrepreneurs can borrow to finance their capital. There is a competitive financial intermediary that collects deposits from savers and offers loans at different rates to entrepreneurs. Financial intermediary offers loans based on four criteria: loan amount, b, own investment, d, collateral amount, x and borrowers' creditworthiness, κ.

The information set of the financial intermediary is also limited to these four criteria. Working capital will be the sum of loan amount and own investment: $k = b + d$. Also, only working capital can be collateralized because the financial intermediary does not have information on entrepreneurs' assets: $x \leq k$.

Based on the available offers, entrepreneurs decide whether to get loans or not, and if they get loans, they choose the contract. The entrepreneurs make the loan decisions knowing their wealth, a, creditworthiness, κ, and the known part of their productivity, z, while they do not know the uncertain part of the productivity, ϵ. This creates the risk of default. At the end of the period, after the realization of ϵ, entrepreneurs can decide whether to repay their loan plus interest or to default. Creditworthiness also follows a similar diffusion process with drift and diffusion components as following:

$$d\kappa_t = \mu_\kappa(\kappa_t)dt + \sigma_\kappa(\kappa_t)dW_t.$$

Similarly, for creditworthiness, I use an Ornstein-Uhlenbeck process with persistence denoted by ρ_κ.

Default: In the case of default, an entrepreneur gives up all of the pledged collateral, x, but can keep any assets that were not part of the collateral. There are no additional financial consequences other than losing the collateral. Nevertheless, there is a stigma for entrepreneurs if they default. The stigma is a utility cost as a function of entrepreneurs' creditworthiness. That means entrepreneurs with higher creditworthiness will try harder to avoid default.

Also, the financial intermediary incurs an ownership transfer cost γ. In case of default, only $(1 - \gamma)$ portion of the collateral is recoverable by the financial intermediary. This loss is related to the limited enforceability of contracts. A higher transfer cost implies higher loan rates and collateral requirements because the financial intermediary is competitive and does not lose money in equilibrium.

Timing: The model's timing is as the following: At the beginning of the period, knowing their wealth, creditworthiness, and the known part of productivity, individuals make occupation decisions. Wage workers' problem is easy as they earn wages and then make consumption and savings decisions. On the other hand, entrepreneurs calculate how much capital they need, given the risk-free rate and wages and considering their productivity. They decide whether they want a loan or not, and if they choose to get loans, they decide on the loan contract. Then the unknown part of the productivity is realized, and entrepreneurs produce given their

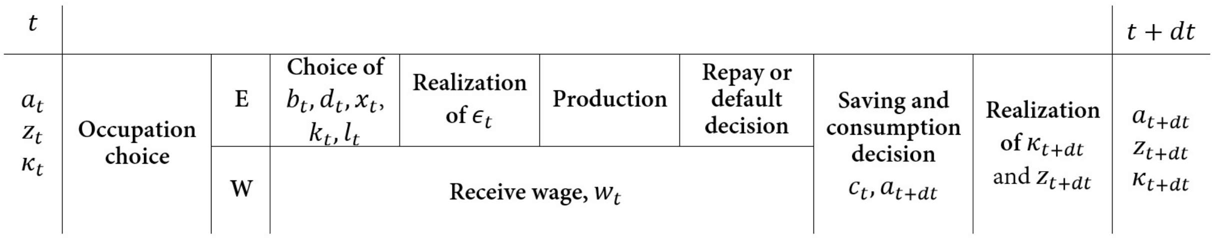


Figure 3.1: Timing of the model

productivity. After production, they decide whether to repay the loan plus interest or to default. See figure 3.1 for timing of the model.

3.2 Loan Pricing

As discussed earlier, financial intermediary's information set is (b, d, x, κ). Also, the financial intermediary is competitive. Zero profit condition on each loan gives the following:

$$\mathbb{E}_{\epsilon_t}[R_b(b_t, d_t, x_t, \kappa_t; \epsilon_t; w_t, r_t; r_{b,t})] = (1 + r_t)b_t \tag{3.1}$$

where

$$R_b(.) = \begin{cases} (1 + r_b)b_t, & \text{no default} \\ (1 - \gamma)x_t, & \text{otherwise} \end{cases}$$

Loan rate, r_b, is the unique solution (if it exists) for the above problem. This means that the expected earnings from loans should be equal to deposit payments and interests paid on deposits. The expected loan earnings will be a combination of earnings from defaulters as well as non-defaulters. The non-defaulting entrepreneurs pay back the loan plus the loan interest rate. For the defaulting entrepreneurs, the financial intermediary recovers $(1 - \gamma)$ fraction of their collateral.

Now, the financial intermediary solves an inference problem to assign default probabilities to the loan space. In order to control the accuracy of financial intermediary's default evaluations, I use the Decision Tree Classification method. This works in the following way: Knowing the past outcome of default in their information space (b, d, x, κ), financial intermediary uses Decision Tree Classifier to classify the space into default and no-default zones and assigns default probability for any given point in the information space. I have chosen the decision trees classifier because they are simple, and it is easy to adjust the accuracy of the classification using the depth parameter, ζ. Therefore, a low value of ζ means a poor and inefficient assessment of the probability space, and as depth increases, the classification becomes more accurate.[56] We can also think of the depth parameter, ζ, as financial intermediaries' quality or access to borrowers' credit information. See Appendix B.1 for a simple example of how a decision tree classifier works in the environment of my model.

Proposition 1. *Let $P^D(b, d, x, \kappa; \epsilon; \zeta)$ be the default probability of contract (b, d, x) for an entrepreneur with creditworthiness of κ that receives the unknown shock ϵ. We have the following loan pricing:*

$$r_b(b, d, x, \kappa) = \frac{rb + (b - (1 - \gamma)x)\mathbb{E}_\epsilon[P^D(b, d, x, \kappa; \epsilon; \zeta)]}{b(1 - \mathbb{E}_\epsilon[P^D(b, d, x, \kappa; \epsilon; \zeta)])} \tag{3.2}$$

The proof is straightforward and follows from equation (3.1). The loan pricing algorithm is explained in section 3.6.

3.3 Occupation Decisions

Workers receive wages, w, and earn interest, r on their deposits. That is, every period, they earn $w + ra$. I now consider the entrepreneurs' options and profits, which, compared to the wages and interests earned by workers, will help determine the occupational choice.

Given their knowledge about their wealth, known part of productivity and creditworthiness, and considering the loan menu as well as the uncertainty they will face, entrepreneurs will choose the optimal amount of production factors, as well as loan contract consisting of the loan amount, own investment, and collateral.

To make things simple in the model, I assume that the only inter-temporal feedback regarding the loan and production choice is related to the default indicator, i.e., related to the values of unknown shocks ϵ that will lead to default. The entrepreneurs will solve the following

[56] Note that too high values of depth might cause over-fitting issues which I avoid by choosing a smaller range for depth and not choosing too high values.

problem to determine the loan and production factors.

$$\begin{aligned}
\pi^E(a,z,\kappa) &= \max_{b,d,x,l} \mathbb{E}_\epsilon\{f(z+\epsilon,k,l) - wl - \delta k + r(a-d) \\
&\qquad -(1-I_{\mathcal{D}}(a,z,\kappa,\epsilon))r_b b - I_{\mathcal{D}}(a,z,\kappa,\epsilon)(x-b)\}, \qquad (3.3)
\end{aligned}$$

subject to

$$\begin{aligned}
0 \leq \quad d & \leq a \\
0 \leq \quad x & \leq k \\
r_b &= r_b(b,d,x,\kappa) \text{ , solution to (3.2)} \\
k &= b+d \\
I_{\mathcal{D}}(a,z,\kappa,\epsilon) & \quad \text{is default indicator taking 1 and 0}
\end{aligned}$$

Knowing the expected profits given by (3.3), and knowing the earnings for a wage worker, individuals will choose their occupation as the following:

$$\Pi(a,z,\kappa) = \max\{\pi^E(a,z,\kappa), w+ra\}, \qquad \text{where } \pi^E \text{ is given by (3.3).} \qquad (3.4)$$

After the realization of the unknown part of productivity, ϵ, we will have the following earnings for entrepreneurs and wage workers:

$$\begin{aligned}
\tilde{\Pi}^E(a,z,\kappa;\epsilon) &= (z+\epsilon)f(k,l) - wl - \delta k + r(a-d) - r_b b, \qquad (3.5) \\
\tilde{\Pi}^W(a,z,\kappa;\epsilon) &= w + ra
\end{aligned}$$

where all the loan and production decisions are given by (3.3). Savings for continuing individuals is given by:

$$\dot{a} = \tilde{\Pi}^j(a,z,\kappa;\epsilon) - c, \qquad \text{for} \quad j \in \{E,W\} \qquad (3.6)$$

3.4 Value Function

Because of the sudden change in the state and entrepreneurs incurring a utility cost (stigma) in case of default, I will model it as a stopping time problem. Individuals will solve the following problem, which is the expected lifetime value:

$$\begin{aligned}
&V^j(a,z,\kappa,\epsilon) = \max_{c_t,\tau}\left\{E_0\int_0^\tau e^{-\rho t}u(c_t)dt + e^{\rho\tau}V^{*j}(a,z,\kappa,\epsilon)\right\}\\
&\text{subject to}\\
&\dot{a}_t = \tilde{\Pi}^j_t(a_t,z_t,\kappa_t;\epsilon) - c_t\\
&dz_t = \mu_z(z_t)dt + \sigma_z^2(z_t)dW_t\\
&d\epsilon_t = \sigma_\epsilon^2 dW_t\\
&d\kappa_t = \mu_\kappa(\kappa_t)dt + \sigma_\kappa^2(\kappa_t)dW_t
\end{aligned} \tag{3.7}$$

where $\tilde{\Pi}^j(a,z,\kappa;\epsilon)$ is given by (3.5); and V^{*j} is the default value only available for entrepreneurs with $b > 0$.

In case of default, the collateral is transferred to the lender, and the defaulting borrower can keep whatever savings she has extra to the collateral value. The defaulting entrepreneurs will incur a utility cost, which is a function of their creditworthiness. The switching value at default is given by the following:

$$V^*(a,z,\kappa,\epsilon) = V(a^D,z,\kappa,\epsilon) - h(\kappa) \tag{3.8}$$

where

$$a^D = a + b - x,. \tag{3.9}$$

The stigma cost is a function of entrepreneurs' creditworthiness. To get the scale right, I will relate the stigma to the average value in the state space. I also assume $h()$ is an increasing function of creditworthiness, and it is weakly convex. That is, the individuals will differ more at the highest levels of creditworthiness. I will use the following quadratic form for stigma function:

$$h(\kappa) = (h_0 + h_1\kappa + h_2\kappa^2)\bar{V}$$

where h_0, h_1 and h_2 are the parameters to be determined in calibration and $\bar{V}$ is the average value across the state space.

I solve for value functions using the Hamilton-Jacobi-Bellman variational inequality (HJBVI). Since the individuals can make default decisions, which leads to a sudden change in the state and the value function, I formulate the value function as a stopping time problem. To obtain the problem's solution, I modify the main problem to derive a Hamilton-Jacobi-Bellman variational inequality (HJBVI). Since ϵ shocks are independent Brownian incidents, we can solve for the value functions independently for different values of ϵ. Individual's HJB has the following form. For occupations $j \in \{W, E\}$, where W stands for wage workers and E stands for entrepreneurs.

$$\begin{aligned}
\rho V^j(a,z,\kappa;\epsilon,t) &= \max_c u(c) + \frac{\partial V^j}{\partial a}\left(\tilde{\Pi}^j(a,z,\kappa;\epsilon) - c\right) \\
&+ \frac{\partial V^j}{\partial z}\mu_z + \frac{1}{2}\frac{\partial^2 V^j}{\partial z^2}\sigma_z^2 + \frac{\partial V^j}{\partial \kappa}\mu_\kappa + \frac{1}{2}\frac{\partial^2 V^j}{\partial \kappa^2}\sigma_\kappa^2.
\end{aligned}$$

The HJBVI will be derived from this using the default value, given by $V^*(a,z,\kappa;\epsilon)$.

3.5 Distribution

Now we want to solve for the stationary distribution of the economy. The density function will be obtained from Kolmogorov Forward Equation (KFE), a partial differential equation similar to the HJB. Similar to the value function, KFE will be solved numerically using the finite difference method.

After solving the value function for different individuals using HJBVIs,[57] the relevant value functions will provide the areas in the state space where the individuals default. The KFE will be as the following for $j \in \{W, E\}$:

$$\begin{aligned}
\frac{\partial g^j(a,z,\kappa;\epsilon,t)}{\partial t} &= \frac{1}{2}\frac{\partial^2}{\partial z^2}\left(\sigma_z^2 g^j(a,z,\kappa;\epsilon,t)\right) - \frac{\partial}{\partial z}\left(\mu_z g^j(a,z,\kappa;\epsilon,t)\right) \qquad (3.10)\\
&+ \frac{1}{2}\frac{\partial^2}{\partial \kappa^2}\left(\sigma_\kappa^2 g^j(a,z,\kappa;\epsilon,t)\right) - \frac{\partial}{\partial \kappa}\left(\mu_\kappa g^j(a,z,\kappa;\epsilon,t)\right) \\
&- \frac{\partial}{\partial a}\left[\dot{a} g^j(a,z,\kappa;\epsilon,t)\right] \\
&- g^D(a,z,\kappa;\epsilon,t) + g^D(a^D,z,\kappa;\epsilon,t) \qquad (3.11)
\end{aligned}$$

where $\dot{a}$ is the evolution of a given by equation (3.6). Also, g^D is the distribution of entrepreneurs that default, and a^D is given by (3.9). Let's denote the cumulative distribution by $G(\cdot)$.

[57] The HJBVIs are solved as Linear Complementarity Problem (LCP) which is a technique based on finite difference method.

3.6 Equilibrium and Model Solution

The model's stationary equilibrium is obtained by a joint solution of HJBVIs and KFEs, given the occupational choice and optimal decision rules for consumption, saving, production, and financing. I solve the HJBVIs as a Linear Complementarity Problem (LCP). KFEs are also solved using the finite difference method. See Achdou et al. (2017) for a detailed explanation of the application of the finite difference method on a heterogeneous agent problem.

3.6.1 Market Clearing Conditions

After solving for the distribution and the value function, we can use them along with the decision rules to calculate the aggregates and update loan rates. After the loan rates converge, we update wages and risk-free rates using the market clearing conditions in an outer loop. For simplicity in notation, let's denote a general state vector as $S = (a, z, \kappa)$. Also let's define $S' = (a, z, \kappa, \epsilon)$.

i. Loans Market: The zero-profit condition for financial intermediary means that payments for deposits plus interest should be equal to loans plus interest received from borrowers who do not default and the recovered collateral from defaulting entrepreneurs. This gives the following loan market-clearing condition:

$$\begin{aligned}&\int_{S'} (1+r)\, b(S)\mathrm{d}G(S') = \\ &\int_{S'^{ND}} \left[1 + r_b(b(S), d(S), x(S), \kappa)\right] b(S)\mathrm{d}G(S') + \int_{S'^{D}} (1-\gamma)x(S)\mathrm{d}G(S')\end{aligned} \tag{3.12}$$

where S'^{ND} is the part of the state space that default does not occur, and S'^{D} is the area that default occurs. Note that this market clears as a result of the financial intermediary's loan pricing given by (3.1).

ii. Capital Market: Here, the amount of capital used in production by entrepreneurs equals the amount deposited by all individuals. That is:

$$\int_{S'^{E}} (b(S) + d(S))\mathrm{d}G(S') = \int_{S'} a\mathrm{d}G(S') \tag{3.13}$$

where S'^{E} is the space of entrepreneurs.

iii. Labor Market: The demand for labor by entrepreneurs equals the supply of labor by wage workers:

$$\int_{S'^E} l(S)\mathrm{d}G(S') = \int_{S'^W} \mathrm{d}G(S') \tag{3.14}$$

where similarly S'^W is the space of wage workers.

3.6.2 Computation Algorithms

Loan Pricing Algorithm:

Given wages, w, and risk-free rates, r, begin with an initial guess for loan rates $r_b^0(b, d, x, \kappa)$,

1 Solve for the loan decisions and the value function.

2 Obtain the default regions at the state space (a, z, κ) for any value of ϵ.

2 Solve for the distribution of agents across state space.

4 Knowing the agents' loan choices, identify defaulters in the financial intermediary's information space (b, d, x, κ).

5 Using the defaulters vs. non-defaulters and their corresponding density in the loan space, use the Decision Tree Classifier to assign default probability to each possible loan in the space (b, d, x, κ).

6 Using the default probabilities from step 5, update the loan rates using equation (3.2). Go back to step 1.

- Repeat until loan rates converge.

Equilibrium Algorithm:

A simplified algorithm for solving the equilibrium is described in the following.

Start with an initial guess for wages, w^0, and interest rate, r^0. Then, for $s = 0, 1, 2, ...$ do as follows:

1 Given the prices and loan rates, use the loan pricing algorithm to solve for loan interest rates.

2 Check for capital and labor market clearing conditions. Update the wages and risk-free rates accordingly, and go to step 1.

- Stop iteration if the markets clear.

This provides the stationary equilibrium of the economy.

4 Results

In this section I report the results of the model. I start with a benchmark model calibrated to U.S. moments. Then I will analyze the impact of financial distortions on the U.S. economy. I will also provide the results of a model without default with both high and low persistence of individual productivity shocks.

4.1 Calibration

Motivated by cross-country observations regarding the heterogeneity in collateral rates, I identify financial frictions in the economy using the collateral rates distribution. This is not a conventional way to identify the level of financial development (or financial frictions) in the literature, as most works relate the financial frictions to a single parameter measured by external dependency. The main reason for my choice of collateral distribution is that collateral and loan rates are inter-connected, and in the sample of countries in the World Bank's Enterprise Survey, they account for more than half of the firms avoiding financing despite needing it. Another reason is that, as discussed in section 2, the collateral distribution contains information that provides a good fit for development indicators such as TFP and GDP per capita. Also, as discussed in section 2, I will choose the first three moments of the collateral distribution to proxy for the whole distribution.

In the model, I need at least three financial frictions parameters to match the first three moments of the collateral distribution. Ownership transfer cost, γ, which is related to limited enforceability, is one parameter. The other one is the persistence of creditworthiness, ρ_κ. This affects agents' savings decisions because of the future uncertainty regarding their loans, and it can shape the collateral distribution through savings and self-financing channels. The third parameter is related to the financial intermediary's ability to assess the loan applicants' default probabilities which is governed by the depth parameter of decision tree classifier, ζ.

For the calibration of the U.S. moments, I will set the financial friction parameters freely. Thereafter, to assess the effect of financial frictions, I will adjust these parameters to reproduce the distribution of collateral in other countries. I will match collateral distribution moments of the countries in the lower decile of the per-capita income distribution.

I have chosen $\gamma = 0.2$ to be consistent with a high collateral recovery rate of around 80% for the U.S. I have also set a very high value for the persistence of creditworthiness, $\rho_\kappa = 0.98$ to reflect the high quality of credit registries and, as a result, reliability of credit information from

Table 1: Freely Calibrated parameters

Parameter	Description	value
σ	CRRA	1.5
δ	Capital depreciation	0.05
γ	Collateral recovery rate	0.2
α	Capital share	0.3
θ	Labor share	0.5
ρ_κ	Creditworthiness persistence	0.98
h_0	Stigma function parameter	0
h_1	Stigma function parameter	0
ζ	Decision Tree depth	20

CRRA is the utility parameter, coefficient of relative risk aversion.

the borrowers point of view. For the depth of the decision tree that governs the accessibility of credit information, I have set $\zeta = 20$. Values beyond 20 do not significantly affect my results, and I do not choose too high values for this parameter to avoid issues that may arise from over-fitting decision trees classifiers.

There are six other parameters that I set using the values from the literature. I set the coefficient of relative risk aversion for utility function, $\sigma = 1.5$, the 1-year depreciation rate of capital, $\delta = 0.05$, share of capital, $\alpha = 0.3$ and share of labor, $\theta = 0.5$. The labor and capital shares imply a span of control of 0.8.[58] Finally, to keep things simple, I choose the stigma function parameters, h_0, and h_1, both equal to zero. This means my stigma function will only have the quadratic part calibrated jointly with other remaining parameters. I have listed the free parameters in table 1.

There remain eight parameters to be jointly calibrated to match distributional and aggregate moments of the U.S data. These parameters are the rate of time preference, ρ, the persistence of productivity, ρ_z, volatility of productivity, σ_z, mean productivity, μ_z, volatility of creditworthiness, σ_κ, mean of creditworthiness, μ_κ, volatility of unknown shock, σ_ϵ and the quadratic coefficient of stigma function, h_2.

These parameters are jointly calibrated to match the risk-free rate, the share of entrepreneurs in the population, firms exit rate, default rate of entrepreneurs, average collateral rate,[59] wealth shares of top 1%, top 5%, and top 10%. These moments and their values are

[58]These parameter values follow the standard practice in the literature. Buera and Shin (2013) set $\sigma = 1.5$. The capital depreciation rate is set to 0.06 in Buera and Shin (2013) and to 0.05 in Moll (2014). Span of control and capital income share are 0.79 and 0.33 in Buera and Shin (2013), and 0.85 and 0.3 in Midrigan and Xu (2014).

[59]I did not have data on the average collateral rate for the U.S., and instead I used the average collateral rate

Table 2: Jointly Calibrated parameters

Parameter	Description	Value
ρ	Time preference	.053
ρ_z	Productivity persistence	.97
σ_z	Productivity volatility	.39
μ_z	Productivity mean	2.28
σ_κ	Creditworthiness volatility	.57
μ_κ	Creditworthiness mean	3.06
σ_ϵ	volatility of unknown shock	.14
h_2	Stigma function parameter	.06

Table 3: Targeted moments

Targets	Model	Data
Risk-free rate	0.04	0.04
Entrepreneurs share pop. %	7.5	7.5
Entrepreneurs exit rate	0.1	0.1
Default rate %	2.3	2.85
Average collateral rate	1.34	1.4*
Wealth share top 1%	30	30
Wealth share top 5%	54	54
Wealth share top 10%	66	67

listed in table 3. The jointly calibrated parameters are also reported in table 2.

4.2 Default Probabilities

The most distinct ingredient of my model is the default risk, so it is worthwhile to check the default probabilities produced by the model and see how they are affected by variations in the individual state. Financial intermediary evaluates the probability of default in the space of (b, d, x, κ). I map it to the state space (a, z, κ) using the outcome of entrepreneurs' loan decisions. To graphically illustrate the default probability across the state space (a, z, κ) I will show them across different state-space dimensions for chosen two state variables at a time. Figure 4.1 shows the default probability in the asset-productivity space for individuals with low creditworthiness (left panel) as well as for those with high creditworthiness (right panel). As we can see in figure 4.1, default risk decreases with assets for a given level of productivity. Also, individuals with low creditworthiness are much more prone to default. Note that some of

for the top 5% of the richest countries in the Enterprise Survey sample.

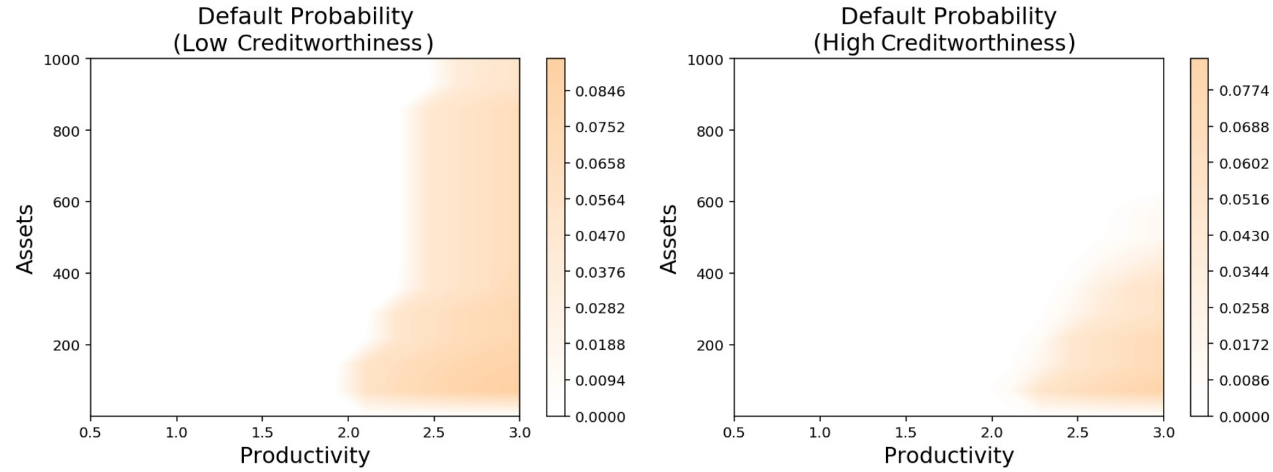


Figure 4.1: Default probabilities in the asset-productivity space

the white space in the figure belongs to wage workers who do not have a default option, and default probability is not evaluated for them.

Figure 4.2 shows the default probability in the asset-creditworthiness space for both low productivity (left panel) and high productivity (right panel) individuals. It can be seen in the right panel of figure 4.2 that default probability decreases with both assets and creditworthiness, and there is almost no default risk at the top-right corner, which belongs to agents with very high assets and creditworthiness. There is no default probability shown in the left panel because these are the lowest productivity individuals, and none of them are entrepreneurs, and as a result, no default risk is evaluated for them.

Finally, figure 4.3 shows the default probabilities in the productivity-creditworthiness space for both low assets (left panel) as well as high assets (right panel). Similarly, it can be seen that default probability decreases with assets, productivity, and creditworthiness conditional on entrepreneurship.

4.3 Effect of Financial Frictions

I will study the effect of financial development by evaluating U.S. economy if it had other countries' financial markets. I use the first three moments of the collateral distribution as my indicators of financial development. Figure 4.4a shows the distribution of collateral rates for the benchmark U.S. model. The main moments of this distribution are as follows. The mean collateral rate is 138%; the standard deviation of the bottom half of the distribution is 14% and; skewness is equal to 2.7. Since I do not have the U.S. data on collateral rates, I cannot directly

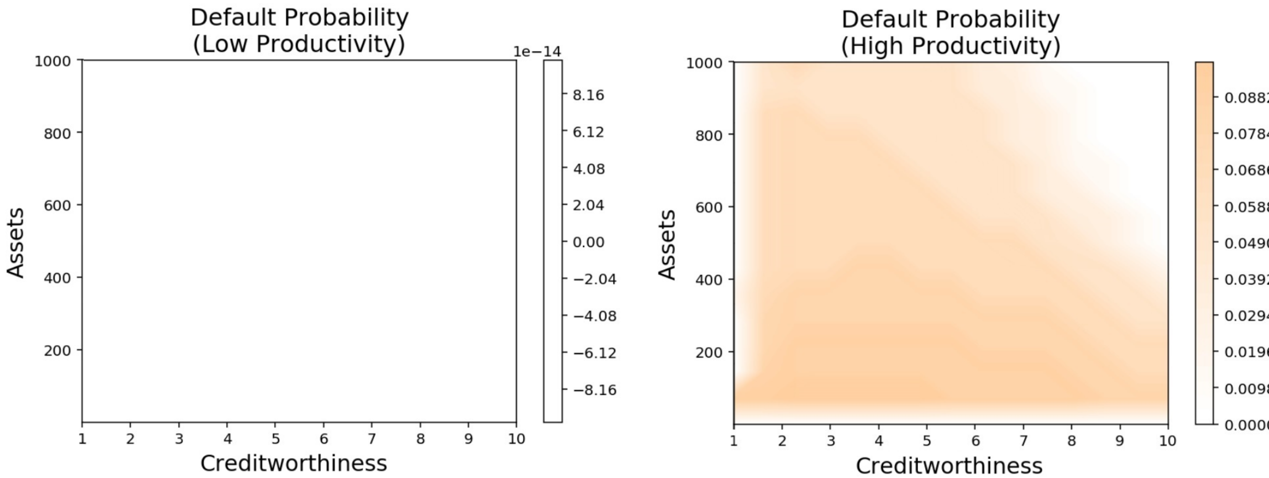


Figure 4.2: Default probabilities in the asset-creditworthiness space

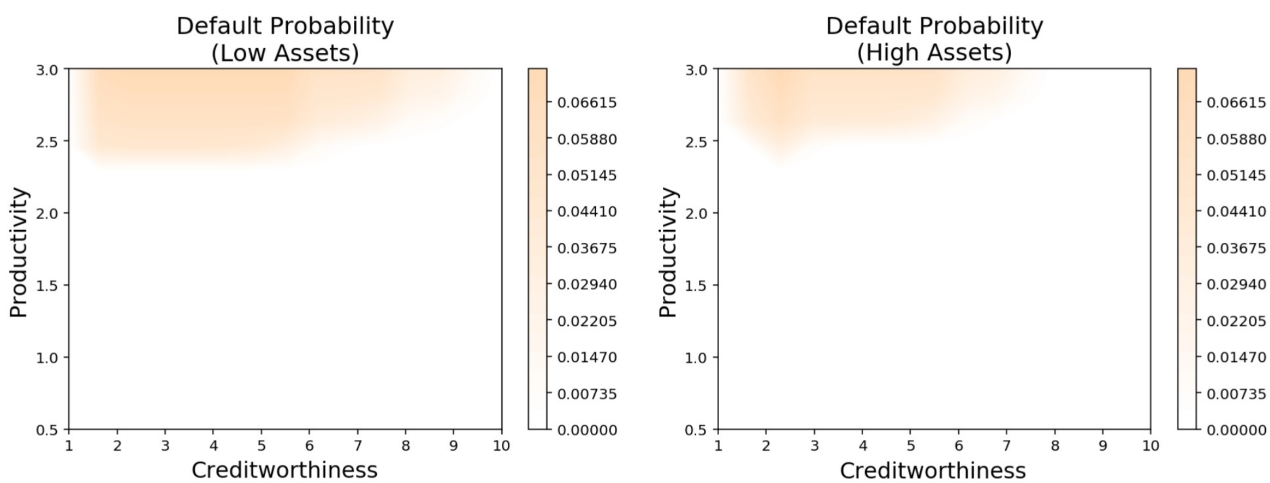
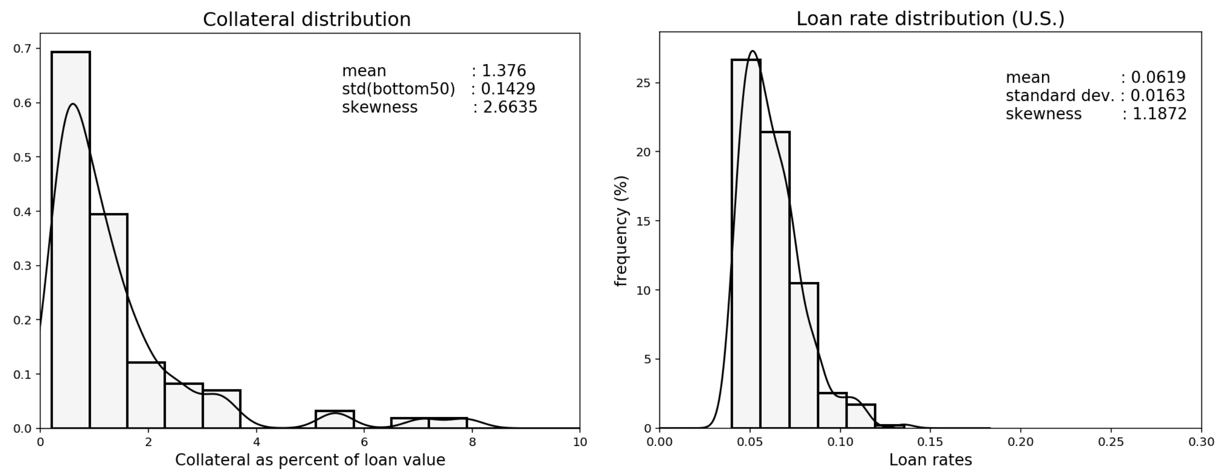


Figure 4.3: Default probabilities in the productivity-creditworthiness space



(a) Collateral rates distribution

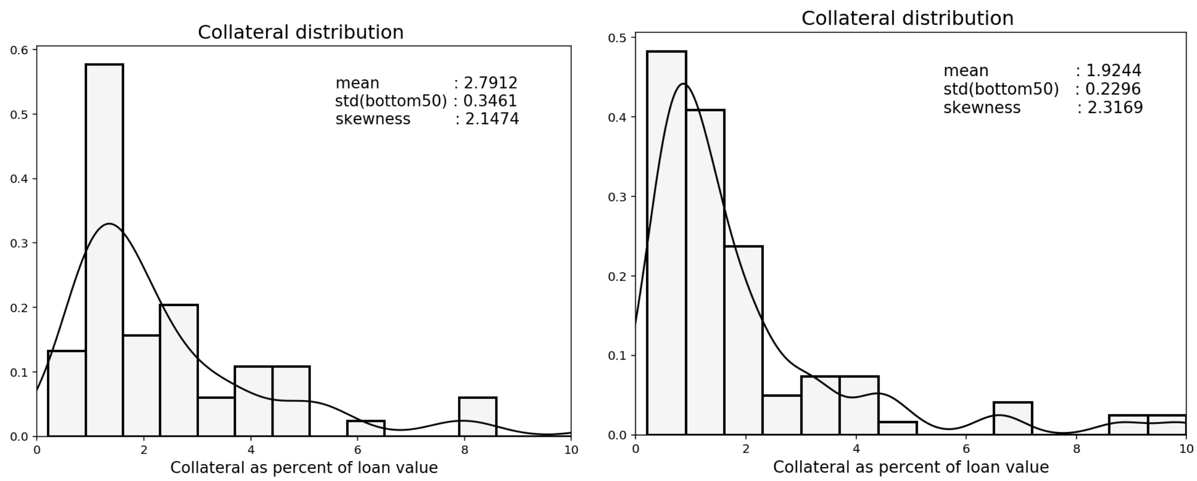
(b) Loan rates distribution

Figure 4.4: Collateral and loan rate distributions - undistorted U.S.

check my model's validity in producing the U.S. collateral rate distribution. However, we can compare these mentioned moments with the distributional moments of the countries at the top 1% of GDP per capita in the World Bank's Enterprise Survey. The collateral rate distribution moments for the countries at the top 1% of GDP per capita distribution are the following. The mean collateral rate is 112%; the standard deviation at the bottom half is 13% and; the skewness is equal to 3.3. As we can see, the benchmark model's collateral rate distribution moments are comparable to those of the rich countries in my sample. Similarly, figure 4.4b shows the distribution of loan rates for the benchmark U.S. economy.

I will adjust the three parameters related to financial frictions to change the U.S.'s collateral distribution so that it matches that of lower-income countries. In the first exercise, I vary all three parameters at the same time to the level of countries in the middle 20% of GDP per capita amongst all countries in the sample. In another exercise, I adjust the parameters to match the collateral distribution of the countries at the bottom 10% of GDP per capita distribution. Given these exercises, I can look at multiple outcomes and analyze the changes. Figure 4.5 shows the collateral rate distribution of the distorted U.S. financial markets to that of the middle- and low-income countries.[60] For the U.S. economy with middle-income countries' financial markets, the model produces the following moments for the collateral rate distribution. The mean is 192% (v.s. 194% in data); the standard deviation at the bottom half is 23% (v.s. 33% in

[60]Middle and low-income countries are the countries in the middle 20% and those at the bottom 10% of the GDP per capita distribution.



(a) Bottom 10%

(b) Middle 20%

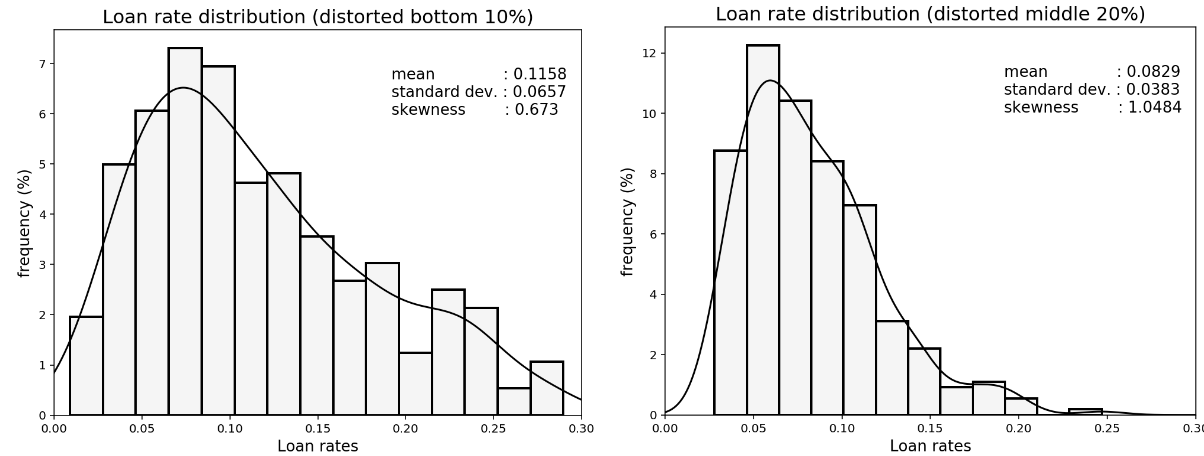
Figure 4.5: Collateral rate distributions - distorted

data) and; the skewness is 2.3 (v.s. 2.6 in data). Similarly, for the U.S. economy with low-income countries' financial markets, the following moments for the collateral rate distribution are produced. The mean is 279% (v.s. 245% in data); the standard deviation at the bottom half is 35% (v.s. 30% in data) and; skewness is 2.1 (v.s. 2.1 in data).

Similarly, figure 4.6 shows the loan rate distribution of the distorted U.S. financial markets to that of the middle- and low-income countries. The average loan rate is 0.083 with a mild distortion, and it is 0.112 with a severe distortion. The standard deviation (skewness) of loan rates also increase (decrease) with financial frictions. This implies a disproportionate impact of financial frictions.

Regarding the effects of financial frictions, we explore the right tail of the wealth distribution, TFP, and the fraction of entrepreneurs. Table 4 shows the values of the distorted economy as well as the benchmark U.S. economy. We observe that a mild distortion[61] reduces the U.S. TFP by 13% whereas a more severe distortion[62] reduces the U.S. TFP by a large amount, 43%. Entrepreneurship with a mild distortion goes from 7.5% to 6.8%, and with a severe distortion drops to 5.3%. Another stark result is that the wealth share at the top of the distribution increases, and it increases disproportionately towards the wealthiest. The top 1% gain more than the next 4%, and the top 5% gain more than the next 5%. For example, the wealth ratio of the top 1% over the top 5% is $\frac{30\%}{54\%} = 0.56$ in the benchmark economy. This ration becomes

[61] A financial market similar to countries in the middle 20% of GDP per capita distribution.
[62] A financial market similar to countries at the bottom 10% of GDP per capita distribution.



(a) Bottom 10%

(b) Middle 20%

Figure 4.6: Loan rate distributions - distorted

$\frac{36\%}{59\%} = 0.61$ with mild distortion, and $\frac{45\%}{65\%} = 0.69$ with severe distortion. This indicates an increase in the concentration of wealth at the top when financial frictions increase. This is the issue of wealth concentration at the top that I have mentioned throughout the paper, which other models are not successful at producing.

Table 5 shows implications of financial distortions on prices and entrepreneurs' financing. The broken link between loan rates and deposit rates can be seen in the first two rows of the table. As we can see, when financial frictions increase, the deposit rates drop, and the loan rates increase on average. This would discourage savings by poor individuals. However, the wealthiest individuals in the economy can enjoy cheap borrowing to start or expand their businesses. This is consistent with both model implications and empirical evidence on wealth concentration. The wages drop as a result of financial frictions. However, the drop in wages is small, which would put only a small upward pressure on savings, easily offset by the downward effect from higher loan rates. The percentage of entrepreneurs who do not apply for loans also increases as a result of financial frictions. In the U.S., only 4% of the non-applicants are constrained and produce under the desired capacity. About a third of the entrepreneurs in the U.S. economy use internal funds for production. In the severely distorted economy, 71% of entrepreneurs do not apply for loans, 68% of the whom are financially constrained.

In another exercise, I change the parameters one at a time while keeping all other parameters at the benchmark U.S. level. Table 6 shows the results of this exercise. We observe that persistence of the creditworthiness can cause a drop of 12% when distorted to the lowest level

Table 4: Distorted U.S. financial market

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
γ		0.2	0.5	0.9
ζ		20	4	1
ρ_κ		0.98	0.88	0.71

collateral rate	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%			Distorted Bottom 10%		
moments			Data	:	Model	Data	:	Model
mean		138	194	:	192	245	:	279
std(bottom half)		14	33	:	23	30	:	35
skewness		2.7	2.6	:	2.3	2.1	:	2.1

Moments	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
top1% wealth%	30	30	36	45
top5% wealth%	54	54	59	65
top20% wealth%	81	79	83	87
top40% wealth%	94	90	91	94
% Entrepreneurs	7.5	7.5	6.8	5.3
TFP (rel. U.S.)	1	1	0.87	0.57

Top panel shows the financial friction parameters and their values for the benchmark U.S. and their distorted values. Distorted Middle 20% (Bottom 10%) means that the U.S. financial market is replaced with that of the countries in the middle 20% (bottom 10%) of the GDP per capita distribution.
Middle panel shows the relevant moments of the collateral rate distribution. std(bottom half) is the standard deviation of the collateral distribution below median.
Bottom panel shows the aggregate moments of undistorted and distorted U.S. economy. top1% wealth% is the wealth share of top 1% wealthiest individuals, and so on. % Entrepreneurs is the fraction of entrepreneurs in the population. TFP (rel. U.S.) is the TFP relative to benchmark U.S. economy.

Table 5: Financial distortions impact on prices and entrepreneurs' financing

	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
avg. loan interest rates	0.062	0.083	0.122
deposit rates	0.04	0.028	0.009
wage (relative to U.S.)	1	0.98	0.95
no loan (% of entp.)	34	42	71
constrained (% of no-loan)	4	20	68

Distorted Middle 20% (Bottom 10%) means that the U.S. financial market is replaced with that of the countries in the middle 20% (bottom 10%) of the GDP per capita distribution. wage (relative to U.S.) is the wage relative to undistorted U.S. economy. no loan (% of entp.) denotes the fraction of entrepreneurs in my model who do not get any loans. constrained (% of no-loan) denotes the fraction of those entrepreneurs who do not get any loans while producing under their desired capacity.

Table 6: Isolated effects of distortions

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
γ only		0.2	0.5	0.9
TFP (rel. U.S.)	1	1	0.97	0.91
ζ only		20	4	1
TFP (rel. U.S.)	1	1	0.99	0.95
ρ_κ only		0.98	0.88	0.71
TFP (rel. U.S.)	1	1	0.95	0.88
γ and ζ				
TFP (rel. U.S.)	1	1	0.94	0.81
γ and ρ_κ				
TFP (rel. U.S.)	1	1	0.90	0.68
ζ and ρ_κ				
TFP (rel. U.S.)	1	1	0.92	0.78

Top panel shows the effect of changing financial friction sources on TFP one at a time. Bottom panel shows the effect of changing financial friction sources two by two on TFP. Distorted Middle 20% (Bottom 10%) means that the U.S. financial market is replaced with that of the countries in the middle 20% (bottom 10%) of the GDP per capita distribution. TFP (rel. U.S.) is relative to banchmark U.S. economy.

of the previous exercise, while the depth of the decision trees can cause a 5% reduction, and ownership transfer cost can cause a 9% drop in TFP. If we add up these numbers, we only get a 26% drop through the isolated effects on TFP. This implies a relatively large amplification effect that happens when all three frictions are at work simultaneously. This can have important policy implications because reducing single friction not only can improve the TFP as a result of its direct effect, it can also improve a great deal through the amplification effects that happen in the presence of multiple frictions. Table 6 also shows the joint effects of two frictions at a time. Similar amplification effects are also evident from the joint effects of two frictions.

4.4 A Model Without Default

As I have discussed in the paper, the model without default would be isomorphic to a standard Buera and Shin (2013)-type model of entrepreneurship, where the persistence of productivity plays a crucial role in driving the results. I have created a version of my model without default and used a similar calibration strategy. I have used two different values for the persistence of productivity shocks: a high persistence of productivity, similar to the benchmark model with default, and a low one. The level of financial frictions in this model is governed by a collateral constraint parameter related to measure of external dependency. See Appendix B.2 for an explanation of model environment with no default. The results of this exercise with high persistence of productivity are reported in table 7. As we can see, when we distort the financial markets using this model, we only get about 7% TFP losses. Also, the drop in the share of entrepreneurs is not as large. In the model without default, the share of wealth held by the wealthiest, say top 1%, 5%, etc., increases, but this increase is not as large as in the model with default. Another important point is that in the model without default, the share of wealth at the top does not go to the wealthiest, i.e., the top 1% share does not increase much compared to the share of next 4%, and similarly for the other segments at the top of the wealth distribution.

Table 7: Model without default: high persistence, $\rho = 0.97$

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
Ext. Fin. GDP		2.5	0.5	0.1
Moments	**U.S. Data**	**Benchmark Model (U.S.)**	**Distorted Middle 20%**	**Distorted Bottom 10%**
top1% wealth%	30	27	27	28
top5% wealth%	54	59	59	60
top20% wealth%	81	94	95	95
top40% wealth%	94	99	99	99
% Entrepreneurs	7.5	8.2	8.1	7.9
TFP (rel. U.S.)	1	1	0.96	0.93

Effect of financial frictions on economic aggregates. Level of financial frictions is measured by external finance to GDP. Distorted Middle 20% (Bottom 10%) means that the U.S. financial market is replaced with that of the countries in the middle 20% (bottom 10%) of the GDP per capita distribution. top1% wealth% is the wealth share of top 1% wealthiest individuals, and so on. % Entrepreneurs is the fraction of entrepreneurs in the population. TFP (rel. U.S.) is the TFP relative to benchmark U.S. economy.

The results of the exercise with relatively low persistence of productivity are reported in table 8. In this case, the distorted financial market produces larger TFP losses. In the case with lower persistence of productivity, the TFP losses are comparable to losses generated from the benchmark model with default. Similarly, we observe a substantial drop in the fraction of entrepreneurs, from 8.6% to 4.1%.

5 Conclusion

I develop a model of entrepreneurship with default and heterogeneity in collateral and loan rates. My model generates relatively large losses from financial frictions while consistent with empirical evidence on a high persistence of productivity and a declining self-financing motive. My model is also consistent with evidence on wealth concentration at the top of the wealth distribution. That is, consistent with my empirical observations, when financial frictions increase in my model, wealth becomes more and more concentrated. The version of my model without default

Table 8: Model without default: low persistence, $\rho = 0.70$

Parameters	U.S. Data	Benchmark Model (U.S.)	Distorted Middle 20%	Distorted Bottom 10%
Ext. Fin. GDP		2.5	0.5	0.1
Moments	**U.S. Data**	**Benchmark Model (U.S.)**	**Distorted Middle 20%**	**Distorted Bottom 10%**
Top 1%	30	26	32	41
Top 5%	54	57	69	83
Top 20%	81	90	97	98
Top 40%	94	97	99	99
% Entrepreneurs	7.5	8.6	6.4	4.1
TFP (rel. U.S.)	1	1	0.76	0.55

Effect of financial frictions on economic aggregates. Level of financial frictions is measured by external finance to GDP. Distorted Middle 20% (Bottom 10%) means that the U.S. financial market is replaced with that of the countries in the middle 20% (bottom 10%) of the GDP per capita distribution. top1% wealth% is the wealth share of top 1% wealthiest individuals, and so on. % Entrepreneurs is the fraction of entrepreneurs in the population. TFP (rel. U.S.) is the TFP relative to benchmark U.S. economy.

fails to generate large losses from financial frictions if it has high persistence of individual productivity shocks supported with empirical evidence.

My model can also disentangle the effects of financial frictions due to enforceability and informational frictions. Instead of a single parameter calibrated to reproduce external dependence, used frequently in the literature, I use the collateral rate distribution to identify financial frictions in the economy. Also, I can analyze the isolated effects of the sources of financial frictions and their amplifying effects. As my results indicate, when acting simultaneously, the amplifying impact of these financial friction sources on TFP is significantly large. This suggests that improving financial markets might prove valuable even if the improvement occurs in one dimension or is related to one source of financial friction.

For future research, I plan to introduce entry costs in the financial frictions model with default. Adding an entry cost would add extra value and help us analyze the extensive margin effects in the financial frictions model. Also, different physical adjustment costs, such as capital adjustment costs, would be useful. They can help us analyze another source of financial friction related to differences in collateral valuation by financial intermediaries and entrepreneurs. There is ample empirical evidence for such differences in valuation in the firm-level data of the World Bank's Enterprise Survey. A simple capital adjustment cost makes capital more valuable for the entrepreneurs. This makes the repurchase value greater than the book value of capital. The latter is the value used by a financial intermediary. Since this valuation difference varies across countries, it can be studied as another source of financial friction.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. 2017. "Income and wealth distribution in macroeconomics: A continuous-time approach." National Bureau of Economic Research.

Aizenman, Joshua, Yothin Jinjarak, and Donghyun Park. 2015. "Financial development and output growth in developing Asia and Latin America: A comparative sectoral analysis." National Bureau of Economic Research.

Amaral, Pedro S, and Erwan Quintin. 2010. "Limited enforcement, financial intermediation, and economic development: a quantitative assessment." *International Economic Review*, 51(3): 785–811.

Arcand, Jean Louis, Enrico Berkes, and Ugo Panizza. 2015. "Too much finance?" *Journal of Economic Growth*, 20(2): 105–148.

Banerjee, Abhijit V, and Andrew F Newman. 1993. "Occupational choice and the process of development." *Journal of political economy*, 101(2): 274–298.

Beck, Thorsten, Asli Demirgüç-Kunt, and Ross Levine. 2000. "A new database on the structure and development of the financial sector." *The World Bank Economic Review*, 14(3): 597–605.

Berger, Allen N, and Gregory F Udell. 1990. "Collateral, loan quality and bank risk." *Journal of Monetary Economics*, 25(1): 21–42.

Bermejo, Vicente J, Miguel A Ferreira, Daniel Wolfenzon, and Rafael Zambrana. 2018. "Entrepreneurship and economic conditions: Evidence from regional windfall gains."

Bernanke, Ben, and Mark Gertler. 1989. "Agency costs, net worth, and business fluctuations." *American Economic Review*, 79(1): 14–31.

Bloom, Nicholas, Benn Eifert, Aprajit Mahajan, David McKenzie, and John Roberts. 2013. "Does management matter? Evidence from India." *The Quarterly Journal of Economics*, 128(1): 1–51.

Buera, Francisco J, and Yongseok Shin. 2013. "Financial frictions and the persistence of history: A quantitative exploration." *Journal of Political Economy*, 121(2): 221–272.

Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin. 2011. "Finance and development: A tale of two sectors." *American economic review*, 101(5): 1964–2002.

Buera, Francisco J, Joseph P Kaboski, and Yongseok Shin. 2015. "Entrepreneurship and financial frictions: A macrodevelopment perspective." *economics*, 7(1): 409–436.

Cagetti, Marco, and Mariacristina De Nardi. 2006. "Entrepreneurship, frictions, and wealth." *Journal of political Economy*, 114(5): 835–870.

Caselli, Francesco, and Nicola Gennaioli. 2013. "Dynastic management." *Economic Inquiry*, 51(1): 971–996.

Chatterjee, Satyajit, and Burcu Eyigungor. 2020. "The firm size and leverage relationship and its implications for entry and concentration in a low interest rate world."

Chatterjee, Satyajit, Dean Corbae, Kyle P Dempsey, and José-Víctor Ríos-Rull. 2020. "A Quantitative Theory of the Credit Score." National Bureau of Economic Research.

Čihák, Martin, Asli Demirgüç-Kunt, Erik Feyen, and Ross Levine. 2012. "Benchmarking financial systems around the world." *World Bank Policy Research Working Paper*, , (6175).

Cooley, Thomas F, and Vincenzo Quadrini. 2001. "Financial markets and firm dynamics." *American economic review*, 91(5): 1286–1310.

Dabla-Norris, Ms Era, and Mr Narapong Srivisal. 2013. *Revisiting the link between finance and macroeconomic volatility.* International Monetary Fund.

David, Joel M, Hugo A Hopenhayn, and Venky Venkateswaran. 2016. "Information, misallocation, and aggregate productivity." *The Quarterly Journal of Economics*, 131(2): 943–1005.

De Mel, Suresh, David McKenzie, and Christopher Woodruff. 2008. "Returns to capital in microenterprises: evidence from a field experiment." *The quarterly journal of Economics*, 123(4): 1329–1372.

De Nardi, Mariacristina, Giulio Fella, and Gonzalo Paz-Pardo. 2020. "Nonlinear household earnings dynamics, self-insurance, and welfare." *Journal of the European Economic Association*, 18(2): 890–926.

Fafchamps, Marcel, David McKenzie, Simon R Quinn, and Christopher Woodruff. 2011. "When is capital enough to get female microenterprises growing? Evidence from a randomized experiment in Ghana." National Bureau of Economic Research.

Gentry, William M, and R Glenn Hubbard. 2004. "Entrepreneurship and household saving." *The BE Journal of Economic Analysis & Policy*, 4(1).

Goldsmith, Raymond William. 1969. "Financial structure and development."

Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez. 2017. "Capital allocation and productivity in South Europe." *The Quarterly Journal of Economics*, 132(4): 1915–1967.

Hopenhayn, Hugo A. 1992. "Entry, exit, and firm dynamics in long run equilibrium." *Econometrica: Journal of the Econometric Society*, 1127–1150.

Hsieh, Chang-Tai, and Peter J Klenow. 2009. "Misallocation and manufacturing TFP in China and India." *The Quarterly journal of economics*, 124(4): 1403–1448.

Hurst, Erik, and Annamaria Lusardi. 2004. "Liquidity constraints, household wealth, and entrepreneurship." *Journal of political Economy*, 112(2): 319–347.

Jalilian, Hossein, and Colin Kirkpatrick. 2005. "Does financial development contribute to poverty reduction?" *Journal of development studies*, 41(4): 636–656.

Jeong, Hyeok, and Robert M Townsend. 2007. "Sources of TFP growth: occupational choice and financial deepening." *Economic Theory*, 32(1): 179–221.

Jo, In Hwan, and Tatsuro Senga. 2019. "Aggregate consequences of credit subsidy policies: Firm dynamics and misallocation." *Review of Economic Dynamics*, 32: 68–93.

King, Robert G, and Ross Levine. 1993. "Finance and growth: Schumpeter might be right." *The quarterly journal of economics*, 108(3): 717–737.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit cycles." *Journal of political economy*, 105(2): 211–248.

Klenow, Peter J, and Andres Rodriguez-Clare. 1997. "The neoclassical revival in growth economics: Has it gone too far?" *NBER macroeconomics annual*, 12: 73–103.

Levine, Ross. 2005. "Finance and growth: theory and evidence." *Handbook of economic growth*, 1: 865–934.

Lindh, Thomas, and Henry Ohlsson. 1996. "Self-employment and windfall gains: evidence from the Swedish lottery." *The Economic Journal*, 106(439): 1515–1526.

Madsen, Jakob B, Md Rabiul Islam, and Hristos Doucouliagos. 2018. "Inequality, financial development and economic growth in the OECD, 1870–2011." *European Economic Review*, 101: 605–624.

Matsuyama, Kiminori, Mark Gertler, and Nobuhiro Kiyotaki. 2007. "Aggregate implications of credit market imperfections [with comments and discussion]." *NBER Macroeconomics Annual*, 22: 1–81.

McKenzie, David. 2015. *Identifying and spurring high-growth entrepreneurship: Experimental evidence from a business plan competition.* The World Bank.

McKenzie, David, and Christopher Woodruff. 2008. "Experimental evidence on returns to capital and access to finance in Mexico." *The World Bank Economic Review*, 22(3): 457–482.

McKinnon, Ronald I. 1973. *Money and capital in economic development.* Brookings Institution Press.

Midrigan, Virgiliu, and Daniel Yi Xu. 2014. "Finance and misallocation: Evidence from plant-level data." *American economic review*, 104(2): 422–58.

Moll, Benjamin. 2014. "Productivity losses from financial frictions: Can self-financing undo capital misallocation?" *American Economic Review*, 104(10): 3186–3221.

Nykvist, Jenny. 2008. "Entrepreneurship and liquidity constraints: Evidence from Sweden." *Scandinavian Journal of Economics*, 110(1): 23–43.

Paulson, Anna L, and Robert Townsend. 2004. "Entrepreneurship and financial constraints in Thailand." *Journal of Corporate Finance*, 10(2): 229–262.

Pawasutipaisit, Anan, and Robert M Townsend. 2011. "Wealth accumulation and factors accounting for success." *Journal of econometrics*, 161(1): 56–81.

Quadrini, Vincenzo. 1999. "The importance of entrepreneurship for wealth concentration and mobility." *Review of income and Wealth*, 45(1): 1–19.

Rajan, Raghuram, and Luigi Zingales. 1998. "Financial development and growth." *American Economic Review*, 88(3): 559–586.

Rajan, Raghuram G, and Luigi Zingales. 2004. *Saving capitalism from the capitalists: Unleashing the power of financial markets to create wealth and spread opportunity.* Princeton University Press.

Restuccia, Diego, and Richard Rogerson. 2008. "Policy distortions and aggregate productivity with heterogeneous establishments." *Review of Economic dynamics*, 11(4): 707–720.

Restuccia, Diego, and Richard Rogerson. 2017. "The causes and costs of misallocation." *Journal of Economic Perspectives*, 31(3): 151–74.

Ruiz-Garcıa, Juan Carlos. 2020. "Financial Frictions, Firm Dynamics and the Aggregate Economy: Insights from Richer Productivity Processes."

Schäfer, Dorothea, Oleksandr Talavera, and Charlie Weir. 2011. "Entrepreneurship, windfall gains and financial constraints: Evidence from Germany." *Economic Modelling*, 28(5): 2174–2180.

Shaker Akhtekhane, Saeed. 2017. "Firm Entry and Exit in Continuous Time."

Shaker Akhtekhane, Saeed. 2020. "Impact of Entry Costs on Aggregate Productivity: Financial Development Matters."

Svirydzenka, Katsiaryna. 2016. "Introducing a new broad-based index of financial development."

Taylor, Mark P. 2001. "Self–employment and windfall gains in Britain: evidence from panel data." *Economica*, 68(272): 539–565.

Townsend, Robert. 2010. "Financial structure and economic welfare: Applied general equilibrium development economics." *Annu. Rev. Econ.*, 2(1): 507–546.

Appendix

A Empirics

A.1 Data Description

I use firm-level data from the World Bank's Enterprise Survey. The sample covers the years 2008 to 2020, with more than 160,000 observations in the entire sample. The firms surveyed from 148 countries, where some countries participated in one year and some in two or three years. There are a total of 285 country-year combinations in the sample. On average more than 550 firms are surveyed in each country-year sample. Some important variables that I use are questions regarding the firms' loan applications. Questions about whether they applied for loans or lines of credit in the last fiscal year, and a list of reasons for not applying if they did not apply for loans. The reasons are listed in table 9.

Table 9: Reasons for not applying for loans

Reason for not applying for loans or lines of credit
No need for a loan - establishment had sufficient capital
Interest rates were not favorable
Application procedures were complex
Collateral requirements were too high
Did not think it would be approved
Size of loan and maturity were insufficient
other

The main variables that I use are the most recent loan value and the collateral value for the most recent loan. Using these two variables, I create a collateral rate variable defined as the collateral value as a percentage of the loan value. I will use this variable to create cross-country observations related to collateral distribution within countries. Also, this data set will vary across years. Therefore I will use different indicators extracted from detailed collateral rate observations and reduce the sample to 285 country-year observations.

Using firm-level observations of collateral rates, I create multiple measures related to collateral rate distributions for each country-year. These measures vary from the simple mean, standard deviation, and some higher moments to other inter-quantile moments, as well as more complex measures of divergence and entropy of distributions. The measures I have extracted for each country-year are shown in table 10. For the complex measures such as distance, di-

Table 10: Features extracted from collateral rate observations for each country-year.

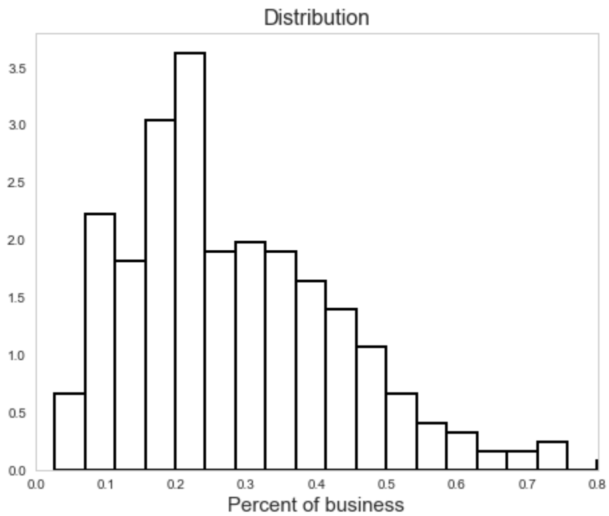
Collateral rate distributional features
mean
standard deviation
skewness
kurtosis
1st quartile
median
3rd quartile
standard deviation above median
standard deviation below median
inter-quartile range between 1st and median
inter-quartile range between median and 3rd
Jensen-Shannon Distance
Kolmogrov-Smirnov Distance
Mann-Whitney rank test
Cressie-Read power divergence statistic
Renyi entropy

vergence, and entropy, I have used the collateral rate distribution in the entire sample as a benchmark comparison point to the collateral rate distributions across all countries, and I have used the test statistic obtained from these tests as my extracted feature. Also, note that I have used multiple other measures, but I do not report them as they were not significant in explaining TFP and GDP per capita.

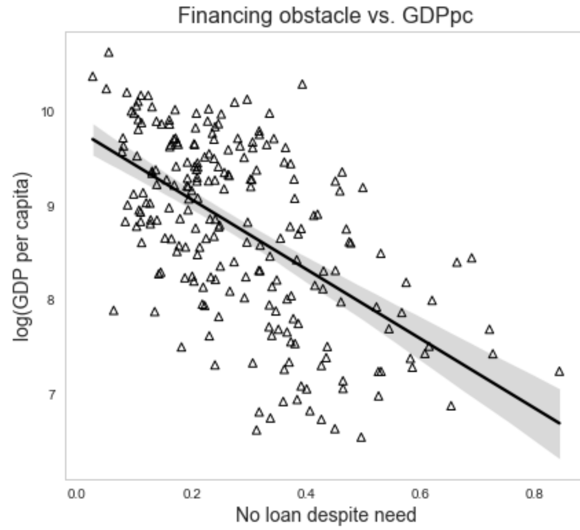
After extracting the distributional features and other cross-country variables such as the reasons that deter firms from applying to loans, I have combined the obtained cross country data set with other standard cross-country data sets such as World Bank's WDI, Penn World Tables, International Monetary Fund's Financial Development Index, Credit Suisse Institute's Wealth Distribution and UNU-WIDER's World Income Inequality Data.

One important variable that I have used in the paper is the TFP measure for each country-year observation. I have calculated this measure from Penn World Tables data using Klenow and Rodriguez-Clare (1997) method. The TFP measure I create, together with the GDP per capita measure from World Bank's WDI (similarly to the one reported in Penn World Tables), are the main development indicators used in this paper.

I have used the IMF's financial development index data for financial development indica-



(a) Distribution: no loan despite need



(b) Financing obstacle vs. GDP per capita

Figure A.1: Variation and relevance of financing constraints

tors, which contains the measures for depth, accessibility, and efficiency of financial markets and institutions.

A.2 Collateral and Misallocation

Here, I explore evidence on financial frictions and collateral constraints using firm-level data from the World Bank's Enterprise Survey.

In the entire sample of businesses, only 27% of the firms have applied for loans or lines of credit in the last fiscal year. About 60% of the remaining firms (the 73% that did not apply for loans) did not need any loans because they already had sufficient capital. The rest, which is almost one-third of all firms in the sample, did not apply for loans despite their needs. This is substantial and significant in terms of access and allocation of capital. It would cause severe misallocation in the intensive margin as one-third of the businesses in the sample are under-financed and are operating under their desired capacity. Note that these measures are related to the businesses in all countries in the sample pooled together, and obviously, there is variation across countries regarding the percentage of the businesses that cannot get the needed funds. Figure A.1 illustrates the variation of this measure across countries as well as its relationship with GDP per capita.

Figure A.1a shows a large variation across countries for the percentage of businesses that could not find the capital they need. As we can see, the average is about 30%, but the distribution is heavily right-skewed, meaning that some countries are extremely far to the right. In those countries, financing is out of reach for most businesses, which would create massive misallocation and impact economic aggregates. Figure A.1b shows the relationship between the percent of businesses that could not get the needed financing and log GDP per capita for the countries in the sample. This relationship is strong, and it suggests the possibility of the extreme misallocation effects caused by financial constraints and their impact on economic development.

In the survey, the businesses are also asked about the underlying reasons that kept them from applying for the needed financing. The reasons listed are related to loan rates, collateral, application complexity, loan size and maturity, and their expectations about loan approvals. Again, in the entire sample, the breakdown of the reasons for not applying is reported in table 11. This indicates that interest rates and collateral are responsible for more than half of the firms deciding not to apply for loans. This is a conservative estimate since the other reasons for not applying, such as 'complex procedures', 'didn't think it would be approved' and 'others' are also very likely related to interest rates and collateral. For example, it can be argued that determining the right amount of collateral and, as a result, loan rates add significantly to the complexity of the loan application process. Similarly, not having enough assets to collateralize the loans might be why those businesses did not have any hope for their loan approval. Also, note that the data only contains operating businesses, and as a result, we can only observe the financing restrictions in the intensive margin. There might well be a large unobserved extensive margin related to those high productivity individuals that want to start a business but cannot do so because of the mentioned reasons. In order to have a grasp on the effects of these financial constraints, consider, for example, a country where, consistent with the observations in figure A.1a, some 60% or 70% of the firms do not have access to the funds they need mainly because of the loan rates and collateral. On top of that, many individuals would want to start businesses but cannot do so for the same reasons. This would cause extreme misallocation of capital (and, as a result, labor), which would have strong effects on output, TFP, investment, employment, etc. This also re-confirms and gives context to the discussion in section 2.3 about the relationship between financial development and wealth concentration at the top and amongst entrepreneurs. The reason is related to existing entrepreneurs' growth prospects and the potential entrants' hopes for a future entry. In an environment that a large portion of firms does not have access to the needed funds, only the wealthy can grow and expand (or start) their businesses, which would further widen the gap between wealthy and

Table 11: Reasons for not applying for loans despite need (entire sample)

Reason for not applying	% of non-applicants
unfavorable interest	35%
complex application procedures	20%
too high collateral requirement	16%
didn't think it would be approved	7%
insufficient size and maturity	5%
other	17%

the poor, and more inequality and more concentration would follow. The bottom line of this discussion is: although financial constraints might seem a minor issue in developed economies, they are extreme and can do serious damage to the development prospects of under-developed economies.

The percentages reported in table 11 are for the entire sample. Different reasons might be dominant for different countries in the sample, and the orders might change from one country to another. Related to this, a question of interest is: which one of the mentioned reasons (in table 11) that deter business from financing is more relevant for economic development? It is simply not possible to do a thorough analysis and assess causal links from the mentioned reasons to economic development without an extensive data set on multiple factors affecting each.[63] However, we can gauge the significance of these reasons regarding economic development relative to each other while controlling for some relevant factors for which data is available.

I will utilize Random Forest Classification technique and will use it to extract feature importance indexes for the mentioned reasons in table 11 based on how well they can explain TFP and GDP per capita. Random Forest Classification is a highly non-linear technique, and as the name suggests, it is for classifying discrete categorical variables using any given explanatory variable (also called a feature in that context). Since I want to use different variables and assess their strength in explaining continuous dependent variables (TFP and GDP per capita), I split the dependent variables into fine quantiles (say 10, 20, etc.) and then use the classifier to measure each explanatory variable's strength in classifying each quantile. The results of this exercise indicate the following ranking: 1. too high collateral, 2. not applied because had enough capital, 3. interest rate, 4. complex application procedure, 5. didn't think it would be approved, and 6. insufficient size and maturity.[64] In a similar exercise, I have run

[63] It would also be out the context of this paper.

[64] In addition to random forest, I have also used extra trees classifier, which works similar to the random forest, and have got the same ordering.

a regression with TFP/GDP per capita as my dependent variables and these deterrent reasons as the explanatory variables. I have also controlled for financial development index and some other financial development indicators from the International Monetary Fund, such as depth, access, and efficiency indices of financial markets and institutions. I have found that both collateral and loan rates are significant as deterrents for business financing.

The regression results for TFP against the deterring reasons for financing is reported in table 12. In this regression, TFP is the dependent variable, and the deterring reasons are explanatory variables.[65] Deterring reasons for any country-year is measured by the percent of firms that did not apply for loans because of that reason. This percentage is among the firms that did not apply for loans, not all firms for that country-year. I have controlled for the financial markets and institutions indicator from the IMF data.

A.3 Other Indicators of Financial Development

Here I will briefly review different types of financial development indicators used in the literature and discuss their differences with collateral distribution features introduced in this paper.

A.3.1 External Dependence and Similar Measures

The size of the financial sector relative to output has been a measure of financial development traditionally. See Goldsmith (1969) and McKinnon (1973) for instance. This is a measure of financial depth. Also, King and Levine (1993) use the ratio of private credit to total domestic credit as well as the ratio of private sector credit to GDP as measures of financial market development, emphasizing the importance of credit distribution between private and state-owned firms. External dependence index of Rajan and Zingales (1998) is probably the most widely used measure of financial development in the recent literature. External finance measure encompasses private sector credit and private bond market as well as stock market capitalization.[66]

A.3.2 Multi-factor Indicators

More comprehensive list of financial development indicators introduced by Beck, Demirgüç-Kunt and Levine (2000). Their indicators include size (depth), activity, and efficiency of dif-

[65] Similar results were obtained when I use GDP per capita as the dependent variable.

[66] Following Rajan and Zingales (1998) many researchers used ratio of external finance to GDP as an indicator of financial development: Buera, Kaboski and Shin (2011), Buera and Shin (2013), Moll (2014), Shaker Akhtekhane (2020) just to name few in a related subject to this paper's.

Table 12: Regression: deterring reasons vs TFP

	Dependent variable: log(TFP)		
	(1)	(2)	(3)
No Loan (collateral)	-4.182***	-2.575***	-2.388***
	(0.787)	(0.889)	(0.748)
No Loan (rates)	-2.116***	-2.324***	-1.094**
	(0.479)	(0.462)	(0.426)
No Loan (complexity)		-1.811***	-0.286
		(0.639)	(0.568)
No Loan (no approval hope)		-0.177	1.579
		(1.653)	(1.43)
No Loan (size/maturity)		1.168	-0.654
		(2.563)	(2.212)
No Loan (other reasons)		-2.985***	-2.428***
		(0.727)	(0.637)
Fin. Inst. Index			1.467***
			(0.266)
Fin. Markets Index			0.576***
			(0.211)
constant	6.736***	6.996***	6.033***
	(0.076)	(0.089)	(0.161)
Observations	173.0	173.0	168.0
Adjusted R^2	0.282	0.371	0.561
Note:			*p<0.1; **p<0.05; ***p<0.01

ferent types of financial intermediaries and markets. Also, Čihák et al. (2012) introduced a data set on the characteristics of financial systems, including a comprehensive list of indicators: size, access, efficiency, and stability. Following Čihák et al., the International Monetary Fund (IMF) introduced the Financial Development Index Database, which is comprehensive, and at the same time brief, list of financial development indicators. IMF's financial development index includes depth, access, and efficiency for both financial institutions and markets. A break down of the different components of the index is as the following:

i. Depth
   - Financial Institutions: 1. Private-sector credit (% of GDP), 2. Pension fund assets (% of GDP), 3. Mutual fund assets (% of GDP), 4. Insurance premiums, life and non-life (% of GDP).
   - Financial Markets: 1. Stock market capitalization to GDP, 2. Stocks traded to GDP, 3. International debt securities government (% of GDP), 4. Total debt securities of nonfinancial corporations (% of GDP), 5. Total debt securities of financial corporations (% of GDP).

ii. Access to financing
   - Financial Institutions: 1. Branches (commercial banks) per 100,000 adults, 2. ATMs per 100,000 adults.
   - Financial Markets: 1. Percent of market capitalization outside of the top 10 largest companies, 2. Total number of issuers of debt (domestic and external, nonfinancial corporations, and financial corporations).

iii. Efficiency
   - Financial Institutions: 1. Net interest margin, 2. Lending-deposits spread, 3. Non-interest income to total income, 4. Overhead costs to total assets, 5. Return on assets, 6. Return on equity.
   - Financial Markets: Stock market turnover ratio (stocks traded/capitalization)

See Svirydzenka (2016) for a discussion on the financial development index.

A.4 Why Use Collateral Rate Distribution?

There are some advantages to using the distribution of collateral rates as an indicator of financial development. First, this object (distribution of collateral rates) is obtained as an output of my model. Second, this distribution can be summarized by a few features or moments that can directly be matched in the model. Third, one can argue that the collateral rates distribution contains information related to economic development that is not accounted for by other financial development measures, mainly because it is only related to firms' financing. In contrast, other multi-factor measures include elements related to, for example, household financing. Also, to show the relevance of collateral rate distribution to TFP or GDP per capita, I condense the features extracted from the collateral distribution to create a one-dimensional variable to illustrate it visually. I use the features shown in table 10 and apply Linear Discriminant analysis (LDA) to reduce the dimensionality of these features in the most related way to TFP and GDP per capita. To use LDA, I split TFP (same for GDP per capita) into ten quantiles. Then I apply LDA to get the most variation from the features of the collateral distribution to achieve the best classification of the ten deciles of the TFP distribution. LDA is a widely used method for targeted dimensionality reduction in machine learning.

Given the relevance of collateral rate distribution, I choose three simple moments as my main indicators. I choose the mean, standard deviation at the bottom half, and skewness of the collateral distribution. One main reason is that these measures are straightforward and easily calculated as the outputs of my model. Another reason is that these measures are significant when regressing against TFP or GDP per capita. Table 13 shows this regression results controlling for several other indicators of financial markets and institutions from the IMF's Financial Development data. As we can see, the mean collateral rate is less significant and becomes insignificant when controlling for other indicators. However, I will keep this indicator because it is simple and easily related to my model's outcome.

A.5 Wealth Concentration vs Financial Development

Inequality amongst entrepreneurs and the share of aggregate wealth held by the wealthiest matters for economic development. From a modeling perspective, the importance of inequality amongst entrepreneurs stems from the standard assumption of a decreasing returns-to-scale (DRS) technology. In a DRS environment, a more even distribution of resources implies higher TFP and GDP per capita.

Regarding the wealth share held by the wealthiest, we should note that agents in this group are either entrepreneurs or fund most of the economy's production. If wealth becomes highly concentrated (say at the top 1%), the wealthiest agents will more likely become entrepreneurs[67]

[67] This is consistent with the findings of Hurst and Lusardi (2004).

Table 13: Regression: features of collateral distribution vs TFP

	Dependent variable: log(TFP)		
	(1)	(2)	(3)
Collateral mean	-0.001*	0.0	-0.0
	(0.0)	(0.0)	(0.0)
Collateral std. below median	-0.01***	-0.012***	-0.006**
	(0.003)	(0.002)	(0.003)
Collateral skewness	0.081***	0.031**	0.043**
	(0.021)	(0.014)	(0.02)
FinInstit.Efficiency.Index		0.382*	
		(0.218)	
FinInstitAccess.Index		1.349***	
		(0.135)	
FinInstitDepth.Index		0.587***	
		(0.192)	
FinMarketsAccess.Index			0.753***
			(0.235)
FinMarketsDepth.Index			0.41
			(0.309)
FinMarketsEfficiency.Index			0.146
			(0.177)
const	6.424***	5.62***	6.161***
	(0.113)	(0.142)	(0.116)
Observations	159.0	156.0	156.0
R2	0.193	0.626	0.318
Adjusted R2	0.178	0.611	0.291
Note:			*p<0.1; **p<0.05; ***p<0.01

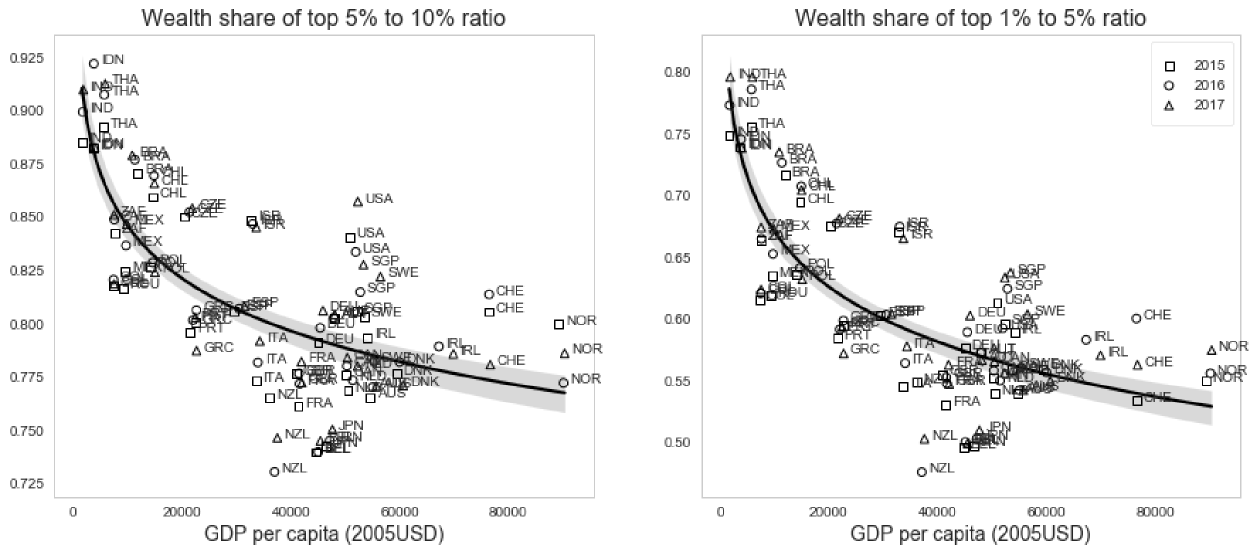


Figure A.2: Wealth concentration vs. GDP per capita.

and take a large portion of funding away from productive projects to their own, perhaps less productive ones.

I have used GDP per capita data from World Bank's WDI to check the relationship between wealth concentration at the top and GDP per capita. See figure A.2. These figures, along with the high negative correlation of wealth concentration with financial development index mentioned in the paper, are robust for different wealth and income groups. Wealth concentration measures also show strong negative correlations with TFP.

Regarding the wealth concentration across entrepreneurs, I have used the Herfindahl-Hirschman Index (HHI) as a proxy and analyzed its relationship with financial development. In table 14, I report the regression results of the HHI against the financial development index. In another specification, I control for GDP per capita and GDP per worker, and in the third specification, I control for factors like human capital, average hours worked, and investment as a ratio of GDP. These controls are taken from Penn World Tables. In all specifications, we see a negative and significant coefficient for financial development. This strengthens the arguments laid out in the paper regarding the relationship between financial development and wealth inequality amongst entrepreneurs.

Table 14: Regression results HHI vs financial development

	Dependent variable: HHI		
	(1)	(2)	(3)
Fin. Dev. Index	-1.581***	-1.297**	-1.7***
	(0.387)	(0.557)	(0.624)
GDP per capita		0.0	0.0
		(0.0)	(0.0)
GDP per woker		-0.0	-0.0
		(0.0)	(0.0)
Investment/GDP			-1.036
			(1.074)
Average hours			0.001**
			(0.0)
Human Capital Index			0.132
			(0.241)
const	-1.742***	-1.777***	-3.993***
	(0.113)	(0.115)	(1.008)
Observations	190.0	182.0	72.0
R2	0.082	0.088	0.215
Adjusted R2	0.077	0.073	0.142
Note:			*p<0.1; **p<0.05; ***p<0.01

B Model

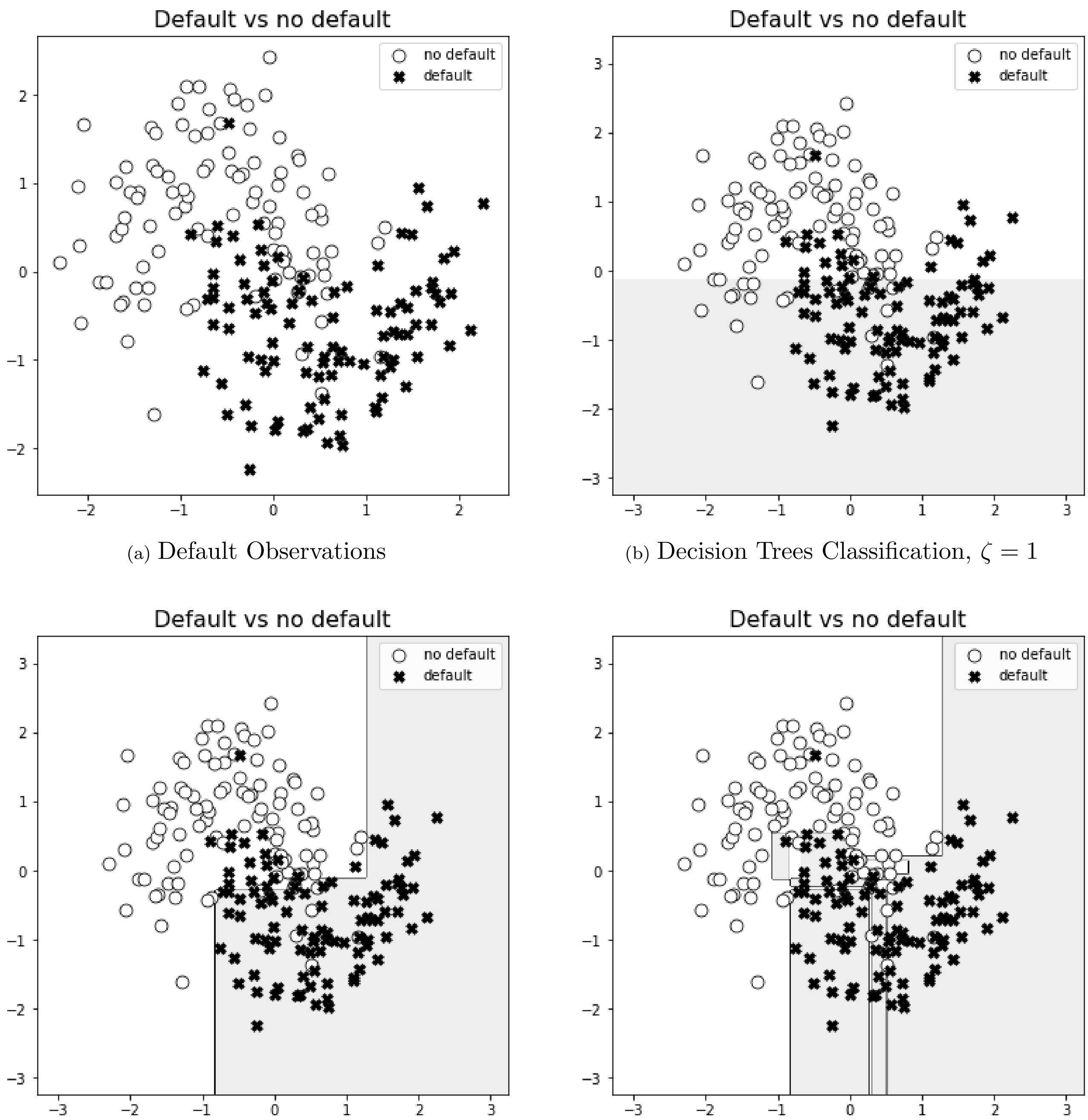
B.1 Assessment of Default Risk: Decision Trees

Here I will use a simple example to clarify the method used by the financial intermediary to evaluate loan applicants' default probabilities. The financial intermediary's information set is (b, d, x, κ). To illustrate the method visually, I will use a hypothetical example in a two-dimensional space (instead of the four-dimensional space in the actual problem).

In the two-dimensional space, we have some observations for defaulters and some for non-defaulters. Also, note these observations will be different across observations of ϵ. For a given ϵ we will have observations scattered across the financial intermediary's information space. In figure B.1a I have denoted defaulters by (✖) and non-defaulters by ($\circ$). The decision trees classification for different values of depth is shown in figure B.1, panels (b, c, d). As we can see, the classification becomes more accurate when the depth increases. However, if we increase the depth too much, we may encounter issues with over-fitting. Since this is a low dimensional and simple example, we can see some minor over-fitting issues at a depth of 10. However, given the dimension and complexity of the original problem, higher levels of depth work fine. Given the classification, we can assign probabilities to the financial intermediary's information space and take the expectation over different realizations of ϵ to obtain the default probability in the entire information space. Note that other classification methods also work in this environment, and they are more accurate than decision trees. I use the decision trees because I want a range of assessment abilities for the financial intermediary, from very inefficient to more efficient assessments. Other complex methods generally become very accurate as depth increases slightly and do not provide much room for such variation in the assessment. I have tried to use Random Forest instead of decision trees, but I do not get much variation in the efficiency of assessment by changing the depth parameter of Random Forest, as it becomes very close to the most efficient case at a depth of 2 or 3.

B.2 Model Without Default

Here, I describe the environment and formulation of the version of my model without default. Without default, there is no role for creditworthiness as an individual state variable. Also, since there is no default, there is no need for the unknown part of the productivity. Therefore, the state variables will be wealth and the persistent (and known) part of the productivity. There is no role for a collateral choice because it is tied to default risks. As a consequence, distinguishing



(a) Default Observations

(b) Decision Trees Classification, $\zeta = 1$

(c) Decision Trees Classification, $\zeta = 4$

(d) Decision Trees Classification, $\zeta = 10$

Figure B.1: Default assessment with different depth levels of decision trees down payment and wealth would also be unnecessary. Finally, loan rates and deposit rates will be the same. The environment I just described is isomorphic to a Buera and Shin (2013) type model.

The source of financial frictions in this environment is the collateral constraint which is related to contracts' limited enforceability. Entrepreneurs can borrow up to a fraction of their assets. Therefore, entrepreneurs' capital will be constrained as a result of the borrowing limit proportional to their wealth.

$$k \leq \frac{1}{\phi} a,$$

where ϕ is the financial constraint parameter, which determines the amount entrepreneurs can borrow. ϕ ranges from 1, no enforceability (autarky), to 0, perfect enforceability (unlimited borrowing). Also, note that, $\frac{1}{\phi}$ is directly related to the external dependence measure.

In the absence of default and loan choices, the only decisions will be related to occupational choices and consumption-saving decisions. Since I formulate my model in continuous time, I will use Ornstein-Uhlenbeck process for productivity. I use high persistence as well as low persistence for productivity shocks. ϕ determines the level of financial frictions in the economy. I order to distort the benchmark economy, I increase ϕ closer to 1, to match the value of external dependence for low income countries. See tables 7 and 8.