

# Technical Appendix

(not for publication)

## B Proofs

### B.1 Proof of $\alpha_2 \leq 1$ given $NV_{B_1, \alpha=1}^{nr} \leq 0$

We prove this result by contradiction. We show that if  $\alpha_2 > 1$ , then  $NV_{B_1, \alpha=1}^{nr} > 0$ .

When  $\alpha_2 > 1$ ,  $B_1$ 's liquidation value at date 1 is more than enough to repay its depositors and  $L_1$  at date 1 even when  $L_1$  recalls all of its interbank loans at date 1. Thus we have

$$V_{B_1, liq} = \lambda L(1 + \hat{R}) - \frac{1}{2}\gamma(L(1 + \hat{R}))^2 > (D_0 - x) + x = D_0. \quad (\text{B1})$$

When  $NV_{B_1, \alpha=1}^{nr} > 0$ ,  $B_1$  has more than enough assets to repay its depositors at date 2 even when  $L_1$  recalls all of its interbank loans. Thus we have

$$x = \lambda l(1 + \hat{R}) - \frac{1}{2}\gamma(l(1 + \hat{R}))^2, \quad (\text{B2})$$

$$NV_{B_1, \alpha=1}^{nr} = (L - l(x))(1 + \hat{R}) - (D_0 - x) > 0. \quad (\text{B3})$$

A simple transformation of Eq. (B3) gives us  $(L - l(x))(1 + \hat{R}) + x > D_0$ . Thus, proving that  $NV_{B_1, \alpha=1}^{nr} > 0$  is equivalent to proving that  $(L - l(x))(1 + \hat{R}) + x > D_0$ .

Using Eq. (B1), we find that proving that  $(L - l(x))(1 + \hat{R}) + x > D_0$  is equivalent to proving that  $(L - l(x))(1 + \hat{R}) + x > V_{B_1, liq}$ . The proof is as follows. Using Eq. (B1) and Eq. (B2), we get

$$\begin{aligned} & (L - l(x))(1 + \hat{R}) + x - V_{B_1, liq} \\ = & (L - l(x))(1 + \hat{R}) + \left[ \lambda l(x)(1 + \hat{R}) - \frac{1}{2}\gamma(l(x)(1 + \hat{R}))^2 \right] - \left[ \lambda L(1 + \hat{R}) - \frac{1}{2}\gamma(L(1 + \hat{R}))^2 \right] \\ = & (1 + \hat{R})(1 - \lambda)(L - l(x)) + (L^2 - l(x)^2)\frac{1}{2}\gamma((1 + \hat{R}))^2 > 0, \end{aligned} \quad (\text{B4})$$

because  $L > l(x)$ ,  $0 < \lambda < 1$ ,  $\gamma > 0$ , and  $1 + \hat{R} > 0$ . Thus we prove that if  $\alpha_2 > 1$ , then  $NV_{B_1, \alpha=1}^{nr} > 0$ . So, by contradiction, if  $NV_{B_1, \alpha=1}^{nr} \leq 0$ , then  $\alpha_2 \leq 1$ .

The intuition behind this result is as follows. When  $\alpha_2 > 1$ ,  $B_1$ 's liquidation value at date 1 is more than enough to repay its creditors, when all of its deposits are withdrawn and all of its interbank loans are recalled at date 1. Now suppose that all of  $B_1$ 's interbank loans are still recalled at date 1, but its depositors wait until period 2. Then  $B_1$ 's asset value at date 2 is definitely higher than in the case where its depositors withdraw at date 1. This is because the long-term project that would otherwise be liquidated to repay depositors at date 1 can now be carried over to date 2. Since liquidation is costly by assumption, less liquidation always leads to a higher asset value. On the other hand, with a zero interest rate, depositors still withdraw the same amount at date 2 as they would have at date 1. Thus  $B_1$ 's assets at date 2 must be more than enough to repay its depositors when its depositors withdraw at date 2 and all of its interbank loans are recalled, implying that  $NV_{B_1, \alpha=1}^{nr} > 0$ . So, by contradiction, when  $NV_{B_1, \alpha=1}^{nr} \leq 0$ , we must have  $\alpha_2 \leq 1$ . ■

## B.2 Proof of proposition 1

Appendix A.2 gives  $L_1$ 's payoff when  $0 \leq \alpha_1 < \alpha_2 \leq 1$ . The following points elaborate on all the other possible cases, which are illustrated by Fig. B1.

First,  $\alpha_1 \in [0, 1]$  (or equivalently  $NV_{B_1, \alpha=1}^{nr} \leq 0$  and  $NV_{B_1, \alpha=0}^{nr} \geq 0$ ) and  $\alpha_2 \leq \alpha_1$  (Fig. B1 (a)). In this case, when  $\alpha \in [0, \alpha_1]$ , bank  $L_1$ 's payoff,  $\Pi$ , is given by Eq. (A2). This is because when  $\alpha \leq \alpha_1$ , a no-run equilibrium is feasible for  $B_1$ , and  $B_1$  is solvent and able to repay all the creditors at date 2. When  $\alpha \in [\alpha_1, 1]$ ,  $L_1$ 's payoff is given by Eq. (A7), which we proved is strictly increasing in  $\alpha$ . This is because when  $\alpha > \alpha_1$ ,  $B_1$ 's depositors will withdraw at date 1. In addition,  $B_1$  has no assets left at date 2 since  $\alpha > \alpha_1 > \alpha_2$ .

Note that when  $\alpha_1 \in [0, 1]$ ,  $L_1$ 's payoff from its interbank loans is maximized at  $\alpha = 0$ . This is because when  $\alpha_1 \in [0, 1]$ , a no-run equilibrium is always feasible for  $B_1$  at  $\alpha = 0$ .

Second,  $NV_{B_1, \alpha=1}^{nr} > 0$  (Fig. B1 (b)). In this case,  $L_1$ 's payoff,  $\Pi$ , is given by Eq. (A2) over  $\alpha \in [0, 1]$ . This is because  $B_1$ 's net asset value at date 2 is positive even when  $L_1$  recalls all of its interbank loans. Thus,  $B_1$  depositors will not withdraw at date 1 for all  $\alpha \in [0, 1]$ , and  $L_1$ 's interbank loans will be fully repaid for all  $\alpha \in [0, 1]$ .

Third,  $NV_{B_1, \alpha=0}^{nr} < 0$ . In this case,  $B_1$  does not have enough assets to repay its liabilities at date 2, even when  $L_1$  does not recall any interbank loans at date 1. So  $B_1$ 's

depositors will always withdraw at date 1 for  $\alpha \in [0, 1]$ . In the case of  $\alpha_2 \in (0, 1]$  (Fig. B1 (c)), when  $\alpha \in [0, \alpha_2]$ ,  $L_1$ 's payoff is given by Eq. (A3), which we proved is strictly decreasing in  $\alpha$ . When  $\alpha \in [\alpha_2, 1]$ ,  $L_1$ 's payoff is given by Eq. (A7), which we proved is strictly increasing in  $\alpha$ . As a result,  $L_1$ 's payoff is maximized either at  $\alpha = 0$  or at  $\alpha = 1$ . In the case of  $\alpha_2 \leq 0$  (Fig. B1 (d)),  $L_1$ 's payoff is given by Eq. (A7) over  $\alpha \in [0, 1]$ , which we proved is strictly increasing in  $\alpha$  and is maximized at  $\alpha = 1$ .<sup>1</sup>

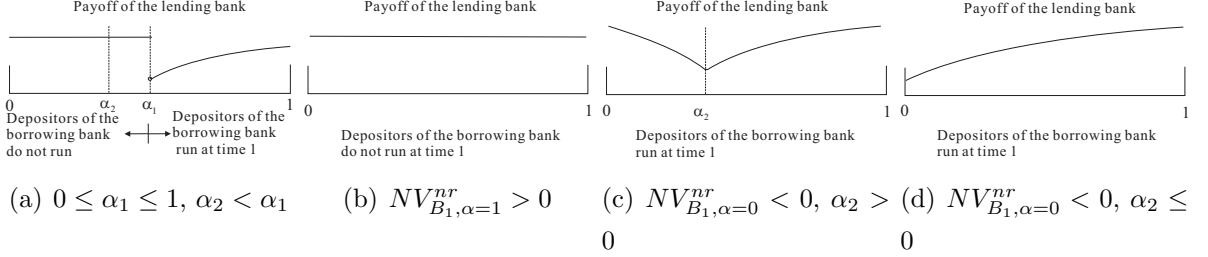


Figure B1: The lending bank's payoff from interbank loans in other cases.

Considering all the combinations of  $\alpha_1$  and  $\alpha_2$ , we find when  $NV_{B_1, \alpha=0}^{nr} \geq 0$ , which includes both the case of  $NV_{B_1, \alpha=0}^{nr} \geq 0$  and  $NV_{B_1, \alpha=1}^{nr} \leq 0$  and the case of  $NV_{B_1, \alpha=1}^{nr} > 0$ ,<sup>2</sup> a no-run equilibrium is feasible for  $B_1$  at  $\alpha = 0$ . Thus at  $\alpha = 0$ ,  $L_1$  receives the maximum interbank loan payoff,  $x$ . So  $\alpha = 0$  produces the highest payoff. When  $NV_{B_1, \alpha=0}^{nr} < 0$ , a no-run equilibrium is not feasible for  $B_1$ , even when  $L_1$  does not recall any interbank loans ( $\alpha = 0$ ). Thus,  $B_1$ 's depositors will always withdraw at date 1. Our analysis reveals that in this case, when  $\alpha_2 > 0$ ,  $L_1$ 's payoff is first strictly decreasing in  $\alpha$ , and then strictly increasing in  $\alpha$ . When  $\alpha_2 \leq 0$ , it is strictly increasing in  $\alpha$ . So the optimal solution is either  $\alpha = 0$  or  $\alpha = 1$ . ■

### B.3 Proof of corollary 1

Proof of result (1): we first prove that  $B_1$ 's depositors will never initiate a bank run when  $R_{shock} \leq R_1^s$ .  $B_1$ 's depositors will never initiate a run if a no-run equilibrium is feasible. The no-run equilibrium is always feasible when  $B_1$  is solvent conditional on its depositors

<sup>1</sup>When  $NV_{B_1, \alpha=0}^{nr} < 0$ , we need only to consider the case where  $\alpha_2 \leq 1$ , because we prove that if  $NV_{B_1, \alpha=1}^{nr} \leq 0$ , then  $\alpha_2 \leq 1$ . Note that when  $NV_{B_1, \alpha=0}^{nr} < 0$ , we must have  $NV_{B_1, \alpha=1}^{nr} < 0$ , because  $NV_{B_1}^{nr}$  is non-increasing in  $\alpha$  over  $\alpha \in [0, 1]$

<sup>2</sup>Note that because  $NV_{B_1}^{nr}$  is non-increasing in  $\alpha$  over  $\alpha \in [0, 1]$ , when  $NV_{B_1, \alpha=1}^{nr} > 0$ ,  $NV_{B_1, \alpha=0}^{nr}$  must be greater than zero too.

not withdrawing at date 1, that is, when  $L(1 + \hat{R}) \geq D_0$ . Define  $R_1^s$  as the level of  $R_{shock}$  at which  $L(1 + \hat{R}) = D_0$  (note that  $\hat{R} = R - R_{shock}$ ). Thus  $B_1$ 's depositors will never initiate a run if  $R_{shock} \leq R_1^s$ .

We next prove that  $B_1$ 's depositors will always initiate a run if  $R_{shock} > R_1^s$ . This is because in this case,  $L(1 + \hat{R}) < D_0$  and a no-run equilibrium is infeasible. Thus we prove that  $B_1$ 's depositors follow a trigger strategy of  $R_1^s$ .

Proof of result (4): we first prove that  $R_{2,c}^s \geq R_1^s$ . If  $R_{shock} \leq R_1^s$ ,  $B_1$  is solvent and  $L_1$ 's interbank loans can be fully repaid. As a result,  $L_1$  will never recall its interbank loans. Thus,  $R_{2,c}^s \geq R_1^s$ .

Provided that  $B_1$  experiences a run at date 1 and that  $L_1$  does not, proposition 1 reveals that  $L_1$  decides whether to recall its interbank loans or not by comparing its payoff differential between  $\alpha = 1$  and  $\alpha = 0$ . We denote this differential by  $\Phi = P_{L_1, \alpha=1} - P_{L_1, \alpha=0}$ , where  $P_{L_1, \alpha=1}$  and  $P_{L_1, \alpha=0}$  are  $L_1$ 's interbank loan payoffs when  $\alpha = 1$  and  $\alpha = 0$ , respectively.  $L_1$  will recall its interbank loans ( $\alpha = 1$ ) if and only if  $\Phi > 0$ .

Note that

$$P_{L_1, \alpha=0} = \max(0, (L - l_{B_1})(1 + \hat{R})), \quad (\text{B5})$$

where  $l_{B_1}$  is given by

$$\lambda l_{B_1}(1 + \hat{R}) - \frac{1}{2}\gamma[l_{B_1}(1 + \hat{R})]^2 = D_0 - x. \quad (\text{B6})$$

That is, by choosing  $\alpha = 0$ ,  $L_1$  will either receive zero if  $B_1$  has no assets left for period 2 after experiencing a run, or seize  $B_1$ 's remaining asset at date 2,  $(L - l_{B_1})(1 + \hat{R})$ , if  $B_1$  has any positive assets left for period 2 after experiencing a run.

Note that

$$\frac{\partial(L - l_{B_1})(1 + \hat{R})}{\partial \hat{R}} = L - l_{B_1} - (1 + \hat{R}) \frac{\partial l_{B_1}}{\partial \hat{R}}. \quad (\text{B7})$$

From Eq. (B6), we find

$$\frac{\partial l_{B_1}}{\partial \hat{R}} = \frac{\gamma l_{B_1}^2 (1 + \hat{R}) - \lambda l_{B_1}}{\lambda(1 + \hat{R}) - \gamma l_{B_1}(1 + \hat{R})^2} = -\frac{l_{B_1}}{1 + \hat{R}}. \quad (\text{B8})$$

Thus  $\frac{\partial(L - l_{B_1})(1 + \hat{R})}{\partial \hat{R}} = L > 0$ . That is, when  $P_{L_1, \alpha=0}$  is positive, it is strictly increasing in  $\hat{R}$ , or equivalently strictly decreasing in  $R_{shock}$  since  $\hat{R} = R - R_{shock}$ . Define the level

of  $\hat{R}$  at which  $l_{B_1} = L$  as  $\hat{R}^*$ . Note that at  $\hat{R}^*$ ,  $P_{L_1, \alpha=0} = (L - l_{B_1})(1 + \hat{R}) = 0$ . It is straightforward to see that when  $\hat{R} \geq \hat{R}^*$ ,  $P_{L_1, \alpha=0}$  is strictly increasing in  $\hat{R}$ . When  $\hat{R} < \hat{R}^*$ , it stays at zero.

Note that

$$P_{L_1, \alpha=1} = \frac{x}{D_0} V_{B_1, liq}. \quad (\text{B9})$$

That is, by choosing  $\alpha = 1$ ,  $L_1$  shares  $B_1$ 's liquidation value proportionally with its depositors, since  $B_1$ 's liquidation value can not fully repay its liabilities. Recall that  $V_{B_1, liq} = \frac{x}{D_0} \left( \lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [L(1 + \hat{R})]^2 \right)$  is given by Eq. (1).

First, consider  $P_{L_1, \alpha=0} = 0$  at  $R_1^s$ , that is,  $B_1$  has no assets left for period 2 after experiencing a run at  $R_1^s$ .<sup>3</sup> It implies that  $P_{L_1, \alpha=0} = 0$  for all  $R_{shock} > R_1^s$ .<sup>4</sup> Meanwhile, note that  $P_{L_1, \alpha=1}$  is always positive. Thus  $\Phi > 0$  for all  $R_{shock} > R_1^s$  in this case. Thus  $L_1$  will choose  $\alpha = 1$  for all  $R_{shock} > R_1^s$ , implying  $R_{2,c}^s = R_1^s$ .

Second, consider  $P_{L_1, \alpha=0} > 0$  at  $R_1^s$ , that is,  $B_1$  has some positive assets left for period 2 after experiencing a run at  $R_1^s$ . In this case,  $P_{L_1, \alpha=0} = (L - l_{B_1})(1 + \hat{R})$ , and we have

$$\Phi = P_{L_1, \alpha=1} - P_{L_1, \alpha=0} = \frac{x}{D_0} \left( \lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [L(1 + \hat{R})]^2 \right) - (L - l_{B_1})(1 + \hat{R}). \quad (\text{B10})$$

Thus

$$\frac{\partial \Phi}{\partial \hat{R}} = \frac{x}{D_0} [\lambda - \gamma(1 + \hat{R})L]L - L, \quad (\text{B11})$$

since we just proved  $\frac{\partial(L - l_{B_1})(1 + \hat{R})}{\partial \hat{R}} = L$ . Note that  $\frac{\partial \Phi}{\partial \hat{R}} < 0$  because both  $\frac{x}{D_0}$  and  $\lambda - \gamma(1 + \hat{R})L$  are between 0 and 1 by assumption. Hence, we prove that  $\frac{\partial \Phi}{\partial \hat{R}} < 0$ , or equivalently  $\frac{\partial \Phi}{\partial R_{shock}} > 0$  when  $P_{L_1, \alpha=0} > 0$ .

There are two possible situations when  $P_{L_1, \alpha=0} > 0$  at  $R_1^s$ . In the first situation,  $\Phi \geq 0$  at  $R_1^s$ , implying that  $\Phi > 0$  for all  $R_{shock} > R_1^s$  because we just proved  $\Phi$  is strictly increasing in  $R_{shock}$  in this case. Thus  $R_{2,c}^s = R_1^s$ . In the second situation,  $\Phi < 0$  at  $R_1^s$ . Because  $\Phi$  is strictly increasing in  $R_{shock}$ , there exists a unique threshold level of  $R_{shock}$ ,  $R_{2,c}^s \in (R_1^s, R)$  above which  $\Phi > 0$  and below which  $\Phi < 0$ . Note that  $\Phi$  will become

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<sup>3</sup>Note that here  $P_{L_1, \alpha=0}$  at  $R_1^s$  is a hypothetical payoff conditional on  $B_1$  experiencing a run, which will not occur in equilibrium. In equilibrium, a no-run equilibrium is feasible and  $B_1$  will never experience a run. Throughout the rest of the proofs, this argument is applied to all the variables calculated at  $R_1^s$ .

<sup>4</sup>Recall that we just proved that  $P_{L_1, \alpha=0}$  is non-increasing in  $R_{shock}$ .

positive as long as  $R_{shock}$  is sufficiently high, which ensures the existence of  $R_{2,c}^s$ . This is because at any given level of  $D_0 - x$ , we can always find a level of  $R_{shock}$  below  $1 + R$  that is sufficiently high such that  $B_1$  has no assets left for date 2. As a result,  $\Phi > 0$  because  $\alpha = 1$  always yields a positive payoff, while  $\alpha = 0$  yields a zero payoff. Thus we prove that  $L_1$  will follow a trigger strategy of  $R_{2,c}^s > R_1^s$  in the second situation.

In sum, we prove result (4), that is,  $L_1$  follows a trigger strategy of  $R_{2,c}^s \geq R_1^s$ , conditional on  $L_1$  not experiencing a run.

Proof of result (3): Note that  $R_3^s \geq R_1^s$ , because when  $R_{shock} \leq R_1^s$ , a no-run equilibrium is feasible, and  $L_1$ 's depositors never initiate a run. So we need only to consider the region of  $R_{shock} > R_1^s$ , where a no-run equilibrium is infeasible. In this region,  $B_1$  always experiences a run, and  $L_1$  will experience a run if and only if  $NV_{L_1,t=2} < 0$ , where  $NV_{L_1,t=2}$  is  $L_1$ 's maximum net asset value at date 2, conditional on  $L_1$  not experiencing a run. More specifically,  $NV_{L_1,t=2} = \max(NV_{L_1,t=2,\alpha=0}, NV_{L_1,t=2,\alpha=1})$ . Here

$$NV_{L_1,t=2,\alpha=0} = P_{L_1,\alpha=0} + L(1 + R) - D_0 - x, \quad (\text{B12})$$

where  $P_{L_1,\alpha=0} = \max(0, (L - l_{B_1})(1 + \hat{R}))$  is given by Eq. (B5). It is  $L_1$ 's net asset value at date 2 when  $L_1$  chooses  $\alpha = 0$ .

We also have

$$NV_{L_1,t=2,\alpha=1} = P_{L_1,\alpha=1} + L(1 + R) - D_0 - x, \quad (\text{B13})$$

where  $P_{L_1,\alpha=1} = \frac{x}{D_0} \left( \lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [L(1 + \hat{R})]^2 \right)$  is given by Eq. (B9). It is  $L_1$ 's net asset value at date 2 when  $L_1$  chooses  $\alpha = 1$ .

For  $P_{L_1,\alpha=0}$ , our previous analysis reveals that there are two possible situations. In the first situation, it is zero for all  $R_{shock} > R_1^s$ . In the second situation, it is positive when  $R_{shock} > R_1^s$  is below a threshold level and remains zero above the threshold level. In addition, it is strictly decreasing in  $R_{shock}$  when it is positive.

For  $P_{L_1,\alpha=1}$ , we find that  $\frac{\partial P_{L_1,\alpha=1}}{\partial \hat{R}} = \frac{x}{D_0} [\lambda - \gamma(1 + \hat{R})L] > 0$  since  $\lambda - \gamma(1 + \hat{R})L > 0$  by assumption. Thus it is strictly decreasing in  $R_{shock}$ . In addition, it is always positive.

Define  $P_{L_1} = \max(P_{L_1,\alpha=0}, P_{L_1,\alpha=1})$ . Thus  $NV_{L_1,t=2} = \max(NV_{L_1,t=2,\alpha=0}, NV_{L_1,t=2,\alpha=1}) = P_{L_1} + L(1 + R) - D_0 - x$ . Note that our analysis above implies that  $P_{L_1}$  is strictly decreasing in  $R_{shock}$  for  $R_{shock} > R_1^s$ . This is because  $P_{L_1}$  is always positive and will never equal  $P_{L_1,\alpha=0}$  when it is zero. Thus  $P_{L_1}$  equals either  $P_{L_1,\alpha=1}$  or  $P_{L_1,\alpha=0}$  when it is positive. In

both cases,  $P_{L_1}$  is strictly decreasing in  $R_{shock}$ . Because  $L(1+R) - D_0 - x$  is independent of  $R_{shock}$ ,  $NV_{L_1,t=2}$  is also strictly decreasing in  $R_{shock}$ . Thus, there is a unique level of  $R_{shock}$ ,  $R_3^s$ , above which  $NV_{L_1,t=2} < 0$ .<sup>5</sup> Note that it is possible that  $NV_{L_1,t=2} < 0$  for all  $R_{shock} > R_1^s$ . In this case,  $R_3^s = R_1^s$ .

Thus we prove result (3), that is,  $L_1$ 's depositors follow a trigger strategy of  $R_3^s \geq R_1^s$ .

Proof of results (2) and (5): When  $R_{2,c}^s \leq R_3^s$ ,  $L_1$  does not experience a run until  $R_{shock} > R_3^s$ , in which case we proved that  $L_1$  will follow a trigger strategy of  $R_{2,c}^s$ . However, when  $R_{2,c}^s > R_3^s$ ,  $L_1$  will experience a run and be forced to recall all of its interbank loans when  $R_{shock} > R_3^s$  in equilibrium. Thus, in this case  $R_2^s = R_3^s$ . The above analysis implies that  $R_2^s = \min(R_{2,c}^s, R_3^s)$ . Thus we prove result (5).

Proof of result (6): When  $R_{shock} \leq R_1^s$ , a no-run equilibrium is always feasible for  $B_1$ , in which case we proved that  $B_1$  and  $L_1$  are solvent and do not experience a run, and that  $L_1$  does not recall its interbank loans. This implies that  $R_1^s \leq \min(R_2^s, R_3^s)$ . Result (5) implies that  $R_2^s \leq R_3^s$ . Thus we prove result (6). ■

## B.4 Proof of corollary 2

Fig. B2 illustrates corollary 2, that is, how  $R_{2,c}^s$  changes in  $x$ . We prove it as follows.

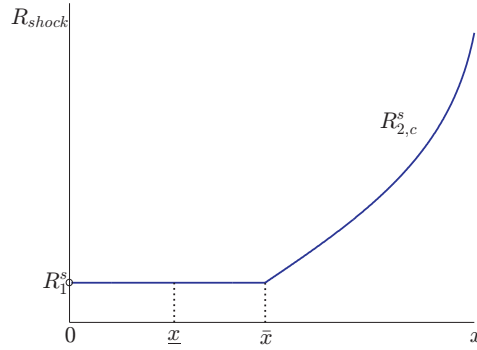


Figure B2: How  $R_{2,c}^s$  changes in  $x$ .

<sup>5</sup>Note that our assumption of  $e_0 < \frac{1-R}{1+R}D_0$  ensures that at least for some values of  $x \in (0, D_0)$ ,  $NV_{L_1,t=2} < 0$  when  $R_{shock}$  is sufficiently high such that  $R_3^s$  exists. To see this, note that this condition implies that  $L(1+R) - D_0 - x = (e_0 + D_0)(1+R) - D_0 - x < 0$  at  $x = D_0$ . Thus for some values of  $x < D_0$ ,  $L(1+R) - D_0 - x$  becomes negative. Meanwhile, when  $R_{shock}$  approaches  $1+R$ ,  $P_{L_1}$  approaches zero. Thus at some values of  $x$  and  $R_{shock}$ ,  $NV_{L_1,t=2} < 0$  and  $R_3^s$  exists.

Define  $\underline{x}$  as the level of  $x$  at which  $V_{B_1,liq}(R_1^s) = D_0 - x$ , where  $V_{B_1,liq}(R_1^s)$  is  $B_1$ 's liquidation value when  $R_{shock} = R_1^s$ . Thus below  $\underline{x}$ ,  $V_{B_1,liq}(R_1^s) < D_0 - x$ , and above  $\underline{x}$ ,  $V_{B_1,liq}(R_1^s) > D_0 - x$ . Note that a positive  $\underline{x}$  must exist because at  $R_1^s$ ,  $L(1 + \hat{R}) = D_0$ , that is,  $B_1$ 's total assets equal its total liabilities without any liquidation. Thus  $B_1$ 's liquidation value at  $R_1^s$ ,  $V_{B_1,liq}(R_1^s) < L(1 + \hat{R}) = D_0$  since liquidation is costly. This condition ensures the existence of  $\underline{x}$  above 0.

First, we prove that when  $x \in (0, \underline{x}]$ ,  $R_{2,c}^s = R_1^s$ . To see this, note that when  $x \leq \underline{x}$ ,  $V_{B_1,liq} < D_0 - x$  for all  $R_{shock} > R_1^s$ . This is because when  $x \leq \underline{x}$ ,  $V_{B_1,liq}(R_1^s) \leq D_0 - x$ , and  $V_{B_1,liq}$  is strictly decreasing in  $R_{shock}$ . Thus when  $x \in (0, \underline{x}]$ ,  $B_1$  has no assets left for period 2 after experiencing a run for all  $R_{shock} > R_1^s$ . It implies that  $\Phi > 0$  for all  $R_{shock} > R_1^s$ , and  $R_{2,c}^s = R_1^s$ . This is because in this case  $\alpha = 0$  yields a zero payoff, while  $\alpha = 1$  always yields a positive payoff.

When  $x > \underline{x}$ ,  $V_{B_1,liq}(R_1^s) > D_0 - x$ , implying that  $B_1$  has positive assets left for period 2 after experiencing a run at  $R_1^s$ . Define  $\Phi(R_1^s)$  as the value of  $\Phi$  when  $R_{shock} = R_1^s$ . In this case  $\Phi(R_1^s)$  is given by Eq. (B10). Note that Eq. (B10) is also applied to the case of  $x = \underline{x}$ , because  $l_{B_1} = L$  at  $\underline{x}$  such that  $P_{L_1, \alpha=0} = (L - l_{B_1})(1 + \hat{R}) = 0$ . We will prove that  $\Phi(R_1^s)$  has three properties: (1)  $\Phi(R_1^s)$  is strictly decreasing in  $x$  when  $x \geq \underline{x}$ . (2)  $\Phi(R_1^s) > 0$  at  $\underline{x}$ . (3) When  $x$  is sufficiently large,  $\Phi(R_1^s)$  will become negative.

We first prove that  $\Phi(R_1^s)$  is strictly decreasing in  $x$  when  $x \geq \underline{x}$ . According to Eq. (B10), we have

$$\frac{\partial \Phi}{\partial x} = \left[ \frac{1}{D_0} (\lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [L(1 + \hat{R})]^2) - \frac{1}{\lambda - \gamma l_{B_1}(1 + \hat{R})} \right], \quad (\text{B14})$$

because  $\frac{\partial l_{B_1}}{\partial x} = -\frac{1}{(1 + \hat{R})(\lambda - \gamma l_{B_1}(1 + \hat{R}))}$ .

We can prove that  $\frac{\partial \Phi}{\partial x} < 0$  by proving that  $\frac{\lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [(1 + \hat{R})L]^2}{D_0} < 1$  and  $\frac{1}{\lambda - \gamma l_{B_1}(1 + \hat{R})} > 1$ . First, since  $B_1$  is insolvent, its liquidation value,  $\lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [L(1 + \hat{R})]^2$  is always below its total liabilities,  $D_0$ . Thus we prove that  $\frac{\lambda L(1 + \hat{R}) - \frac{1}{2} \gamma [(1 + \hat{R})L]^2}{D_0} < 1$ . Second, by assumption,  $0 < \lambda - \gamma l_{B_1}(1 + \hat{R}) < 1$ . Thus we prove that  $\frac{1}{\lambda - \gamma l_{B_1}(1 + \hat{R})} > 1$ . Since this result is applied to the case of  $R_{shock} = R_1^s$ , we prove that  $\Phi(R_1^s)$  is strictly decreasing in  $x$  when  $x \geq \underline{x}$ .

Next we prove that  $\Phi(R_1^s) > 0$  at  $\underline{x}$ . This is because by definition  $V_{B_1,liq}(R_1^s) = D_0 - x$  at  $\underline{x}$ , implying that  $P_{L_1, \alpha=0} = 0$ . On the other hand,  $P_{L_1, \alpha=1}$  is always positive. Thus



$\Phi(R_1^s) = P_{L_1, \alpha=1} - P_{L_1, \alpha=0} > 0$  at  $\underline{x}$ .

Third, we prove that when  $x$  is sufficiently large,  $\Phi(R_1^s)$  will become negative. To see this, consider the extremely case where  $x$  approaches  $D_0$ . That is,  $B_1$ 's deposits approach zero. In this case,  $L_1$ 's interbank loan payoff with  $\alpha = 1$  approaches  $B_1$ 's liquidation value, while  $L_1$ 's interbank loan payoff with  $\alpha = 0$  approaches  $B_1$ 's asset value at date 2 without any liquidation. Since liquidation is costly, it is straightforward to see that  $\alpha = 0$  yields a higher payoff, that is,  $\Phi(R_1^s) < 0$ .

These three properties of  $\Phi(R_1^s)$  imply that there exists a unique level of  $x$ ,  $\bar{x} \in (\underline{x}, D_0)$ , such that  $\Phi(R_1^s) = 0$  at  $\bar{x}$ . In addition, when  $x \in [\underline{x}, \bar{x})$ ,  $\Phi(R_1^s) > 0$  and when  $x \in (\bar{x}, D_0)$ ,  $\Phi(R_1^s) < 0$ . Note that property (1) ensures the uniqueness of  $\bar{x}$ , property (2) ensures the existence of  $\bar{x}$  above  $\underline{x}$ , and property (3) ensures the existence of  $\bar{x}$  below  $D_0$ .

Now we prove that when  $x \in [\underline{x}, \bar{x}]$ ,  $R_{2,c}^s = R_1^s$ . To see this, note that when  $x \in [\underline{x}, \bar{x}]$ ,  $\Phi(R_1^s) \geq 0$ , implying that  $\Phi > 0$  for all  $R_{shock} > R_1^s$  because we proved in corollary 1 that  $\Phi$  is strictly increasing in  $R_{shock}$ . Thus  $R_{2,c}^s = R_1^s$ . It also implies that  $R_{2,c}^s$  is continuous in  $x$  at  $\underline{x}$ .

Finally, we prove that  $x \in [\bar{x}, D_0)$ ,  $R_{2,c}^s$  is strictly increasing in  $x$ . In addition,  $R_{2,c}^s$  is continuous in  $x$  at  $\bar{x}$ . We prove it as follows.

When  $x \in (\bar{x}, D_0)$ ,  $\Phi(R_1^s) < 0$ . In this case,  $R_{2,c}^s$  is determined by the level of  $R_{shock}$  at which  $\Phi = 0$ , where  $\Phi$  is given by Eq. (B10). Note that at  $R_{2,c}^s$  where  $\Phi = 0$ ,  $B_1$  must have positive assets left for period 2 after experiencing a run. Thus Eq. (B10) can be applied. To see this, note that at  $\Phi = 0$ ,  $P_{L_1, \alpha=0} = P_{L_1, \alpha=1} > 0$ . We can prove that in this case,  $R_{2,c}^s$  is strictly increasing in  $x$ . To see this, note that according to the Implicit Function Theorem,

$$\frac{\partial \hat{R}_{2,c}^s}{\partial x} = -\frac{\frac{\partial \Phi}{\partial x}}{\frac{\partial \Phi}{\partial \hat{R}}}, \quad (\text{B15})$$

where  $\hat{R}_{2,c}^s = R - R_{2,c}^s$ . We just proved that  $\frac{\partial \Phi}{\partial x} < 0$ . Recall in corollary 1, we proved  $\frac{\partial \Phi}{\partial \hat{R}} < 0$ . Thus,  $\frac{\partial \hat{R}_{2,c}^s}{\partial x} < 0$ , or equivalently  $\frac{\partial R_{2,c}^s}{\partial x} > 0$ .

Note that  $R_{2,c}^s$  is continuous at  $\bar{x}$ . This is because Eq. (B10) is continuous in both  $x$  and  $\hat{R}$ , implying that  $R_{2,c}^s$  must be continuous at  $\bar{x}$  as well.

In sum, we prove that there exist two threshold levels of  $x$  and  $\bar{x}$ , where  $0 < \underline{x} < \bar{x} < D_0$  such that when  $x \in (0, \underline{x}]$ ,  $B_1$  has no assets left for period 2 after experiencing a run for

all  $R_{shock} > R_1^s$ . Thus  $\Phi > 0$  for all  $R_{shock} > R_1^s$ , and  $R_{2,c}^s = R_1^s$ . When  $x \in (\underline{x}, \bar{x}]$ ,  $B_1$  has positive assets left for period 2 after experiencing a run. Again,  $\Phi > 0$  for all  $R_{shock} > R_1^s$  in this case, and  $R_{2,c}^s = R_1^s$ . When  $x \in (\bar{x}, D_0)$ ,  $B_1$  has positive assets left for period 2 after experiencing a run, and  $\Phi < 0$  at  $R_1^s$ . In this case,  $R_{2,c}^s$  is strictly increasing in  $x$ . In addition,  $R_{2,c}^s$  is continuous in  $x$ . Thus we prove corollary 2. ■

## B.5 Proof of corollary 3

Define  $\hat{x}$  as the value of  $x$  at which  $L(1 + R) - D_0 - x = 0$ . Thus  $\hat{x} = L(1 + R) - D_0$ . We first prove that when  $x \leq \hat{x}$ ,  $R_3^s$  does not exist. To see this, note that when  $x \leq \hat{x}$ ,  $L(1 + R) - D_0 - x \geq 0$ . Recall that  $NV_{L_1, t=2} = P_{L_1} + L(1 + R) - D_0 - x$ , where  $L_1$ 's interbank loan payoff,  $P_{L_1} = \max(P_{L_1, \alpha=1}, P_{L_1, \alpha=0})$  is always positive. Thus when  $x \leq \hat{x}$ ,  $L_1$ 's net asset value is always positive, and  $L_1$  will never experience a run. That is,  $R_3^s$  does not exist.

Second, consider the case when  $x > \hat{x}$ . In this case,  $L_1$  will become insolvent ( $NV_{L_1, t=2} < 0$ ) at a sufficiently high level of  $R_{shock}$ . Thus  $R_3^s$  exists. Because  $L_1$  optimally chooses the recall strategy to maximize its net asset value,  $R_3^s = \max\{R_3^s(\alpha = 1), R_3^s(\alpha = 0)\}$ , where  $R_3^s(\alpha = 1)$  is the  $R_3^s$  chosen when  $L_1$ 's net asset value is determined by  $\alpha = 1$  and  $R_3^s(\alpha = 0)$  is the  $R_3^s$  chosen when  $L_1$ 's net asset value is determined by  $\alpha = 0$ . That is, the actual  $R_3^s$  is always determined by the recall strategy that yields a higher net asset value and, consequently, a higher level of  $R_3^s$ .

Next we will prove the properties of the  $R_3^s(\alpha = 1)$  curve and the  $R_3^s(\alpha = 0)$  curve. We first examine the  $R_3^s(\alpha = 1)$  curve.

When  $x \leq \hat{x}$ ,  $NV_{L_1, t=2, \alpha=1}$  will never be negative and  $R_3^s(\alpha = 1)$  does not exist. When  $x > \hat{x}$ ,  $R_3^s(\alpha = 1)$  will exist. Recall that  $NV_{L_1, t=2, \alpha=1}$  is given by Eq. (B13). Thus

$$\frac{\partial NV_{L_1, t=2, \alpha=1}}{\partial x} = \frac{V_{B_1, liq}}{D_0} - 1 < 0, \quad (\text{B16})$$

because  $V_{B_1, liq} < D_0$ . This is because provided that  $B_1$  experiences a run, we have  $V_{B_1, liq} < L(1 + \hat{R}) < D_0$ . That is,  $B_1$ 's long-term project return is below its total liabilities,  $D_0$ , implying that its liquidation value is also below its total liabilities.

When  $x > \hat{x}$ , there are two situations. First, for all the values of  $x$ ,  $NV_{L_1, t=2, \alpha=1} \geq 0$  at  $R_1^s$ . In this case,  $R_3^s(\alpha = 1)$  is determined by  $NV_{L_1, t=2, \alpha=1} = 0$ . Since  $NV_{L_1, t=2, \alpha=1}$

is continuous, strictly decreasing in  $x$  and strictly decreasing in  $R_{shock}$ , according to the Implicit Function Theorem,  $R_3^s(\alpha = 1)$  is strictly decreasing in  $x$  for all the values of  $x$ . This situation is illustrated by Fig. B3(a).

Second, there exists a threshold level of  $x$ ,  $\tilde{x}$ , below which  $NV_{L_1,t=2,\alpha=1} > 0$  at  $R_1^s$  and above which  $NV_{L_1,t=2,\alpha=1} < 0$  at  $R_1^s$ . In this case, when  $x < \tilde{x}$ ,  $R_3^s(\alpha = 1)$  is determined by  $NV_{L_1,t=2,\alpha=1} = 0$  and is strictly decreasing in  $x$ . When  $x \geq \tilde{x}$ ,  $R_3^s(\alpha = 1) = R_1^s$ , because  $NV_{L_1,t=2,\alpha=1}$  is negative for all  $R_{shock} > R_1^s$ . Note that  $R_3^s(\alpha = 1)$  is continuous at  $\tilde{x}$ : when  $x$  approaches  $\tilde{x}$  from below,  $NV_{L_1,t=2,\alpha=1}$  at  $R_1^s$  approaches zero, implying that  $R_3^s(\alpha = 1)$  converges to  $R_1^s$ . This situation is illustrated by Fig. B3(b).

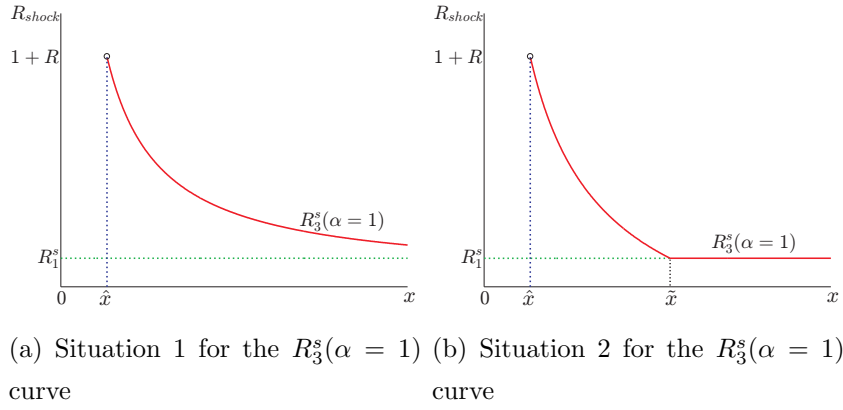


Figure B3: How  $R_3^s(\alpha = 1)$  changes in  $x$ .

In addition, note that in both situations,  $R_3^s(\alpha = 1)$  converges to  $1 + R$  when  $x$  approaches  $\hat{x}$  from above. To see this, note that at  $\hat{x}$ ,  $L(1 + R) - D_0 - x = 0$ , implying that  $L_1$ 's minimum asset value would be zero only if we allowed  $R_{shock}$  to be  $1 + R$  such that the negative shock led to a zero return for  $B_1$ 's long-term project and  $P_{L_1} = 0$ .

We next examine the  $R_3^s(\alpha = 0)$  curve. Similarly, when  $x \leq \hat{x}$ ,  $R_3^s(\alpha = 0)$  does not exist. When  $x > \hat{x}$ , recall that  $NV_{L_1,t=2,\alpha=0}$  is given by Eq. (B12). In this case,  $P_{L_1,\alpha=0}$  and subsequently  $NV_{L_1,t=2,\alpha=0}$  depend crucially on  $\underline{x}$ . When  $x \leq \underline{x}$ ,  $B_1$  has no assets left for period 2 at  $R_1^s$  after experiencing a run, and  $P_{L_1,\alpha=0} = 0$  at  $R_1^s$ . When  $x > \underline{x}$ ,  $B_1$  has positive assets left for period 2 at  $R_1^s$  after experiencing a run, and  $P_{L_1,\alpha=0} = (L - l_{B_1})(1 + \hat{R})$  at  $R_1^s$ .

Thus when  $x > \hat{x}$ , there are two situation. *In the first situation*,  $\hat{x} \geq \underline{x}$ . This situation is illustrated by Fig. B4(a). In this case, for all the values of  $x > \hat{x}$ ,  $B_1$

has positive assets left for period 2 at  $R_1^s$  after experiencing a run, and  $NV_{L_1,t=2,\alpha=0} = (L - l_{B_1})(1 + \hat{R}) + L(1 + R) - D_0 - x$  at  $R_1^s$ . It implies that at  $R_3^s(\alpha = 0)$ ,  $B_1$  must also have positive assets left for period 2 after experiencing a run, and the same formula for  $NV_{L_1,t=2,\alpha=0}$  at  $R_1^s$  is applied. To see this, note that when  $x > \hat{x}$ ,  $L(1 + R) - D_0 - x < 0$ . On the other hand, at  $R_3^s(\alpha = 0)$ ,  $NV_{L_1,t=2,\alpha=0} = 0$ , implying that  $(L - l_{B_1})(1 + \hat{R})$  must be positive. In this case,

$$\frac{\partial NV_{L_1,t=2,\alpha=0}}{\partial x} = -\frac{\partial l_{B_1}}{\partial x}(1 + \hat{R}) - 1, \quad (\text{B17})$$

where

$$\frac{\partial l_{B_1}}{\partial x} = -\frac{1}{1 + \hat{R}} \frac{1}{\lambda - \gamma[l_{B_1}(1 + \hat{R})]} \quad (\text{B18})$$

according to Eq. (B6). Thus we have

$$\frac{\partial NV_{L_1,t=2,\alpha=0}}{\partial x} = \frac{1}{\lambda - \gamma[l_{B_1}(1 + \hat{R})]} - 1 > 0, \quad (\text{B19})$$

because  $0 < \lambda - \gamma[l_{B_1}(1 + \hat{R})] < 1$  by assumption. Thus we prove that  $NV_{L_1,t=2,\alpha=0}$  is strictly increasing in  $x$  when  $B_1$  has positive assets left for period 2. Recall that in corollary 1, we proved that  $NV_{L_1,t=2,\alpha=0}$  is strictly decreasing in  $R_{shock}$  when  $B_1$  has positive assets left for period 2. Thus according to the Implicit Function Theorem,  $R_3^s(\alpha = 0)$  is strictly increasing in  $x$ . Note that at  $x = \hat{x}$ ,  $NV_{L_1,t=2,\alpha=0} = P_{L_1,\alpha=0} \geq 0$  at  $R_1^s$ , implying that the  $R_3^s(\alpha = 0)$  curve will converge to a level of  $R_{shock} \geq R_1^s$  when  $x$  approaches  $\hat{x}$  from above.

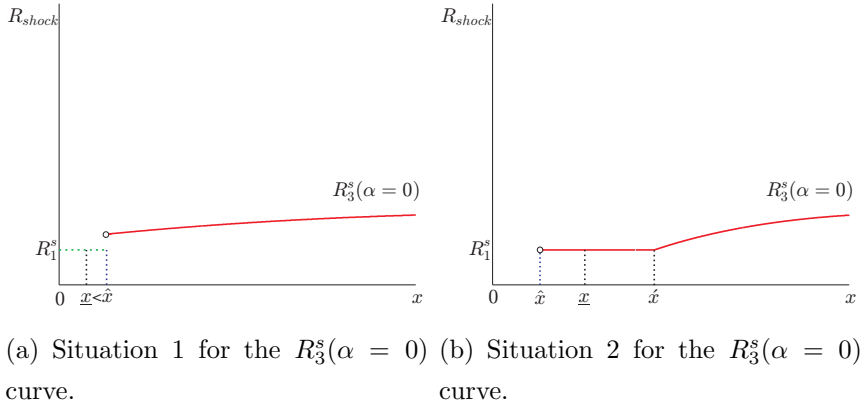


Figure B4: How  $R_3^s(\alpha = 0)$  changes in  $x$ .

In the second situation,  $\hat{x} < \underline{x}$ . This situation is illustrated by Fig. B4(b): When  $x \in (\hat{x}, \underline{x}]$ ,  $B_1$  has no assets left for period 2 at  $R_1^s$ , implying that  $B_1$  has no assets left

for all  $R_{shock} > R_1^s$ . As a result,  $P_{L_1, \alpha=0} = 0$  and  $NV_{L_1, t=2, \alpha=0} < 0$  for all  $R_{shock} > R_1^s$ , implying  $R_3^s(\alpha = 0) = R_1^s$ . When  $x \in (\underline{x}, D_0)$ ,  $B_1$  has positive assets left for period 2 at  $R_1^s$ . In this case, there exists a threshold level of  $x$ ,  $\hat{x}$ , at which  $NV_{L_1, t=2, \alpha=0} = 0$  at  $R_1^s$ . When  $x \in (\underline{x}, \hat{x}]$ ,  $NV_{L_1, t=2, \alpha=0} \leq 0$  at  $R_1^s$ , implying that  $NV_{L_1, t=2, \alpha=0} < 0$  for all  $R_{shock} > R_1^s$ . Thus  $R_3^s(\alpha = 0) = R_1^s$ . Note that at  $x = \underline{x}$ ,  $NV_{L_1, t=2, \alpha=0} < 0$  at  $R_1^s$  because  $P_{L_1, \alpha=0} = 0$  at  $R_1^s$  and  $L(1+R) - D_0 - \underline{x} < 0$ . When  $x \in (\hat{x}, D_0)$ , our analysis on the first situation is applied, that is,  $R_3^s(\alpha = 0)$  is strictly increasing in  $x$ . Note that  $R_3^s(\alpha = 0)$  is continuous at  $\hat{x}$ . This is because when  $x$  approaches  $\hat{x}$  from above,  $NV_{L_1, t=2, \alpha=0}$  at  $R_1^s$  approaches zero, implying  $R_3^s(\alpha = 0)$  will converge to  $R_1^s$ . In addition, note that  $NV_{L_1, t=2, \alpha=0}$  at  $R_1^s$  will become positive when  $x$  is sufficiently large such that  $\hat{x}$  exists. To see this, consider the extreme case where  $x$  approaches  $D_0$ . In this case, when  $\alpha = 0$ ,  $L_1$ 's interbank loan payoff approaches  $D_0$  at  $R_1^s$ , and  $NV_{L_1, t=2, \alpha=0}$  approaches  $L(1+R) - x = (e_0 + D_0)(1+R) - x$ , which must be positive because  $x < D_0$ .

In sum, we prove that when  $x > \bar{x}$ , the  $R_3^s(\alpha = 1)$  curve starts from  $1+R$ , strictly decreases in  $x$  if it is above  $R_1^s$ , and stays at  $R_1^s$  once it reaches  $R_1^s$ .<sup>6</sup> On the other hand, there are two possible cases for the  $R_3^s(\alpha = 0)$  curve. First, there is a threshold level of  $x$ ,  $\hat{x}$ , below which it stays at  $R_1^s$  and above which it is strictly increasing in  $x$ . Second, it starts at some  $R_{shock} > R_1^s$  and is strictly increasing in  $x$ .

In addition, note that when  $x$  is sufficiently large,  $R_3^s(\alpha = 0) > R_3^s(\alpha = 1)$ . The proof is similar to the one in corollary 1 that  $\Phi(R_1^s)$  will become negative when  $x$  is sufficiently large. As long as  $B_1$  has positive assets left for period 2 after experiencing a run, which is the case when  $R_3^s(\alpha = 0)$  is chosen, consider the extreme case when  $x$  approaches  $D_0$ .  $L_1$ 's payoff at  $\alpha = 0$  will approach  $B_1$ 's asset value at date 2 without liquidation, while  $L_1$ 's payoff at  $\alpha = 1$  will approach  $B_1$ 's liquidation value at date 1. Obviously the former is higher. As a result,  $R_3^s(\alpha = 0) > R_3^s(\alpha = 1)$  when  $x$  is sufficiently high.

Based on the above analysis, we conclude that there are two possible situations. In the first situation, the  $R_3^s(\alpha = 1)$  curve reaches  $R_1^s$  after the  $R_3^s(\alpha = 0)$  curve rises from  $R_1^s$ . In this case, there exists a unique intersection between the  $R_3^s(\alpha = 0)$  curve and the  $R_3^s(\alpha = 1)$  curve at which  $x = x^*$ . When  $x < x^*$ ,  $R_3^s(\alpha = 0) < R_3^s(\alpha = 1)$ . While when  $x > x^*$ ,  $R_3^s(\alpha = 0) > R_3^s(\alpha = 1)$ . Since the actual  $R_3^s$  is always given by  $\max(R_3^s(\alpha = 0), R_3^s(\alpha = 1))$  when  $L_1$ 's net asset value is maximized, we find that

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<sup>6</sup>It is possible that  $R_3^s(\alpha = 1)$  never reaches  $R_1^s$  for all the values of  $x$ .

when  $x < x^*$ ,  $R_3^s = R_3^s(\alpha = 1)$  and is strictly decreasing in  $x$ ; while when  $x > x^*$ ,  $R_3^s = R_3^s(\alpha = 0)$  and is strictly increasing in  $x$ . Note that at  $x^*$ ,  $R_3^s(\alpha = 1) = R_3^s(\alpha = 0)$ . Thus  $L_1$ 's net asset values at  $\alpha = 1$  and  $\alpha = 0$  are both zero at  $R_3^s$ , implying that  $\Phi = 0$ , that is,  $L_1$ 's payoffs from interbank loan recall when  $\alpha = 0$  and  $\alpha = 1$  are the same. Thus at  $x^*$ ,  $R_{2,c}^s = R_3^s$ . This situation is illustrated by Fig. 4 in our paper.

In the second situation, the  $R_3^s(\alpha = 1)$  curve reaches  $R_1^s$  before the  $R_3^s(\alpha = 0)$  curve rises from  $R_1^s$ . In this case, there is a unique region of  $x$  below which  $R_3^s(\alpha = 0) < R_3^s(\alpha = 1)$  and above which  $R_3^s(\alpha = 0) > R_3^s(\alpha = 1)$ . This case is similar to the first one except that now the unique level of  $x$  is replaced by a unique region of  $x$  in which  $R_3^s = R_1^s$ . This situation is illustrated by Fig. B5.<sup>7</sup>

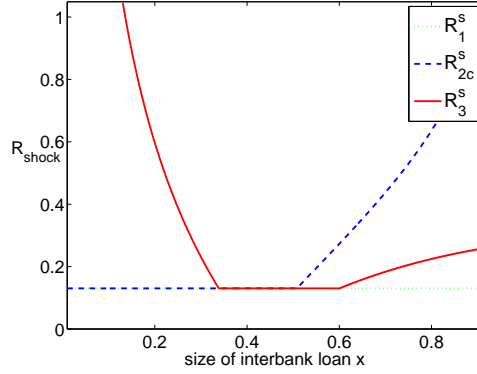


Figure B5: The equilibrium results of the perfect information case when  $\lambda = 0.8$ .

Moreover, in both situations, when  $x$  is below  $x^*$ , at  $R_3^s = R_3^s(\alpha = 1)$ ,  $L_1$ 's optimal recall strategy is  $\alpha = 1$  without experiencing a run. Since  $L_1$  will optimally choose  $\alpha = 1$  without experiencing a run if and only if  $R_{shock} > R_{2,c}^s$ , it implies  $R_3^s > R_{2,c}^s$ . Similarly, when  $x$  is above  $x^*$ , at  $R_3^s = R_3^s(\alpha = 0)$ ,  $L_1$ 's optimal recall strategy is  $\alpha = 0$  without experiencing a run, implying  $R_3^s < R_{2,c}^s$ . ■

<sup>7</sup>It turns out that the first situation tends to happen when the liquidation cost is low, while the second situation tends to happen when the liquidation cost is high. This is because a higher liquidation cost will lower  $L_1$ 's payoff from interbank loans, inducing the  $R_3^s(\alpha = 1)$  curve to reach  $R_1^s$  at a smaller  $x$  and the  $R_3^s(\alpha = 0)$  curve to rise from  $R_1^s$  at a larger  $x$ .

## B.6 Optimal choices for banks $L_1$ and $L_2$ under imperfect information at a given market rate

First, we define a variable  $Z$  as the total resources that the lending bank collects to repay its depositors at date 1, which is given by

$$Z = \lambda l(1 + R) - \frac{1}{2}\gamma[l(1 + R)]^2 + F(\alpha x), \quad (\text{B20})$$

where  $\lambda l(1 + R) - \frac{1}{2}\gamma[l(1 + R)]^2$  are the proceeds that the bank receives from liquidating  $l$  of the long-term project, and  $F(\alpha x)$  is the proceeds that the bank receives from recalling  $\alpha x$  of its interbank loans. When the borrowing bank's net asset value conditional on its depositors not withdrawing at date 1,  $NV^{nr} \geq 0$ , or when  $NV^{nr} < 0$ , but its liquidation value,  $V_{liq} \geq D_0 - x + \alpha x$ ,  $F(\alpha x) = \alpha x$ . When for the borrowing bank,  $NV^{nr} < 0$  and  $V_{liq} < \alpha x + D_0 - x$ ,  $F(\alpha x) = \frac{\alpha x}{D_0 - x + \alpha x} V_{liq}$ .

If  $Z < D_0 + x$ , the lending bank will roll over a positive amount of  $D_0 + x - Z > 0$  of deposits, and its net asset value at date 2 is given by

$$NV = (L - l)(1 + R) - (D_0 + x - Z)(1 + \hat{r}) + H((1 - \alpha)x), \quad (\text{B21})$$

where  $(L - l)(1 + R)$  is the proceeds the bank receives at date 2 from the unliquidated long-term project,  $(D_0 + x - Z)(1 + \hat{r})$  is the repayment to depositors, and  $H((1 - \alpha)x)$  is the proceeds from the remaining interbank loans. When for the borrowing bank,  $NV^{nr} \geq 0$ ,  $H((1 - \alpha)x) = (1 - \alpha)x$ . When for the borrowing bank,  $NV^{nr} < 0$  but  $V_{liq} > D_0 - x + \alpha x$ ,  $H((1 - \alpha)x)$  equals the asset value of the borrowing bank at date 2. When for the borrowing bank,  $NV^{nr} < 0$  and  $V_{liq} < \alpha x + D_0 - x$ ,  $H((1 - \alpha)x) = 0$ .

If  $Z \geq D_0 + x$ , the lending bank chooses not to roll over any deposits, and its net asset value is given by

$$NV = (L - l)(1 + R) + Z - (D_0 + x) + H((1 - \alpha)x). \quad (\text{B22})$$

It is difficult to give a general analytical solution to the above problem. We focus on the more interesting case where  $Z < D_0 + x$  (that is, the lending bank chooses to roll over a positive amount of deposits) in equilibrium. In this case, given that in equilibrium  $\hat{r} > 0$ , and  $Z < D_0 + x$ , we find that: (1) The lending banks will liquidate their long-term projects if, and only if,  $1 + \hat{r} > \frac{1}{\lambda}$ . (2) Provided that  $0 \leq \alpha_1 < \alpha_2 \leq 1$ , the optimal

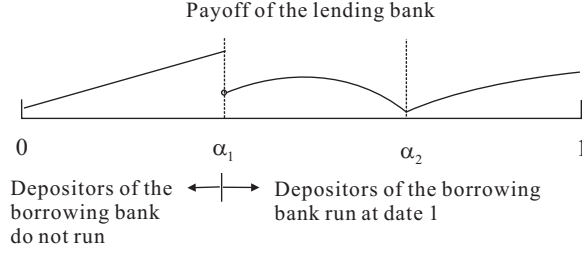


Figure B6: An example of the lending bank's payoff from recalling  $\alpha x$  of interbank loans under imperfect information

amount of recalled interbank loans,  $\alpha x$ , can be chosen from three locally optimal points in the three regions of  $[0, \alpha_1]$ ,  $[\alpha_1, \alpha_2]$ , and  $[\alpha_2, 1]$ , respectively. The bank will recall at least  $\alpha_1 x$  of its interbank loans.

We can prove the above results as follows. Let  $l$  denote the units of the long-term project that a bank liquidates to repay deposits. A bank will choose an optimal  $l$  to maximize the associated payoff

$$[\lambda l(1+R) - \frac{1}{2}\gamma(l(1+R))^2](1+\hat{r}) + (L-l)(1+R). \quad (\text{B23})$$

The first term is debt reduction achieved by using the liquidated goods to repay deposits, and the second term is the value of the unliquidated long-term project. Note that the first-order derivative of the first term w.r.t  $l$ ,  $(1+R)[(\lambda - \gamma l(1+R))(1+\hat{r})]$ , is the marginal benefit from liquidation. The first-order derivative of the second term w.r.t  $l$ ,  $-(1+R)$  is the marginal cost from liquidation. A bank will never liquidate its long-term project if its marginal cost exceeds its marginal benefit. When  $1+\hat{r} < \frac{1}{\lambda}$ ,  $(1+R)[(\lambda - \gamma l(1+R))(1+\hat{r})] < (1+R)(\lambda(1+\hat{r})) < 1+R$  because  $\gamma > 0$  by assumption. Thus we prove result (1).

A lending bank's decision of  $\alpha$  can be analyzed in a similar way to the perfect information case. Fig. B6 illustrates the intuition behind this decision. We can still separate  $\alpha$  into three regions of  $[0, \alpha_1]$ ,  $(\alpha_1, \alpha_2]$ , and  $[\alpha_2, 1]$ . The reactions of the borrowing banks' depositors given  $\alpha$  are the same as in the perfect information case. The payoff for the lending bank is different, however, because the market rate for deposits is now  $1+\hat{r}$ , instead of zero.

Let  $\Pi^i$  be the total interbank loan payoff in terms of date 2 value. Note when the market rate  $\hat{r}$  is positive, the bank will use the proceeds from the recall to repay its deposits. Thus, in terms of date 2 value, the payoff from recalling  $\alpha x$  of interbank loans equals the proceeds from the recall multiplied by  $1+\hat{r}$ .



When  $\alpha \in [\alpha_2, 1]$ ,  $\Pi^i = \frac{\alpha x}{\alpha x + (D_0 - x)}(\lambda L(1 + \hat{R}) - \frac{1}{2}\gamma[L(1 + \hat{R})]^2)(1 + \hat{r})$ , which is strictly increasing in  $\alpha$ . So the locally optimal point is  $\alpha = 1$ , the upper bound of this region.

When  $\alpha \in [0, \alpha_1]$ ,  $\Pi^i = (1 - \alpha)x + \alpha x(1 + \hat{r})$ , which is strictly increasing in  $\alpha$ . So the locally optimal point is  $\alpha_1$ , the upper bound of this region.

When  $\alpha \in (\alpha_1, \alpha_2]$ ,

$$\Pi^i = \alpha x(1 + \hat{r}) + (L - l)(1 + \hat{R}), \quad (\text{B24})$$

$$\alpha x + D_0 - x = \lambda l(1 + \hat{R}) - \frac{1}{2}\gamma[l(1 + \hat{R})]^2. \quad (\text{B25})$$

In this case, the borrowing bank experiences a run, and the lending bank owns the borrowing bank's remaining assets at date 2. It turns out that

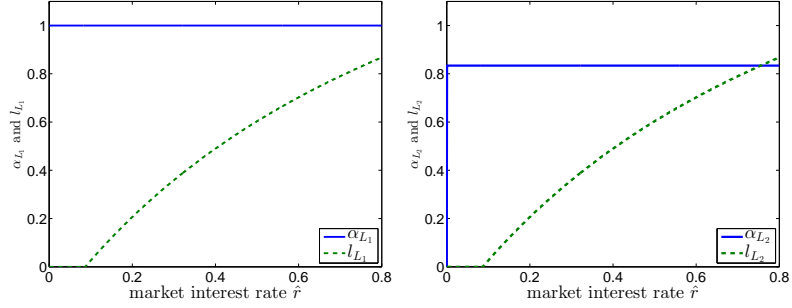
$$\frac{\partial \Pi^i}{\partial \alpha} = x(1 + \hat{r}) - \frac{x}{\lambda - \gamma(1 + \hat{R})l}. \quad (\text{B26})$$

Let  $l(\alpha_1)$  and  $l(\alpha_2)$  denote the liquidated long-term project at  $\alpha_1$  and  $\alpha_2$  respectively. Note that  $l$  is strictly increasing in  $\alpha$  in this region according to Eq.(A5). When  $l(\alpha_1) \geq \frac{\lambda - \frac{1}{1 + \hat{r}}}{\gamma(1 + \hat{R})}$ ,  $1 + \hat{r} < \frac{1}{\lambda - \gamma(1 + \hat{R})l}$  for all the values of  $\alpha$  in this region, and  $\Pi^i$  is strictly decreasing in  $\alpha$  in this region. So the locally optimal point is  $\alpha_1$ , the lower bound of this region. When  $l(\alpha_2) \leq \frac{\lambda - \frac{1}{1 + \hat{r}}}{\gamma(1 + \hat{R})}$ ,  $1 + \hat{r} \geq \frac{1}{\lambda - \gamma(1 + \hat{R})l}$  for all the values of  $\alpha$  in this region, and  $\Pi^i$  is strictly increasing in  $\alpha$  in this region. So the locally optimal point is  $\alpha_2$ , the upper bound of this region. When  $l(\alpha_1) < \frac{\lambda - \frac{1}{1 + \hat{r}}}{\gamma(1 + \hat{R})} < l(\alpha_2)$ ,  $\Pi^i$  is concave when  $\alpha \in (\alpha_1, \alpha_2]$ , and there is an optimal level of  $\alpha \in (\alpha_1, \alpha_2)$  that maximizes  $\Pi^i$ . Thus we prove result (2).

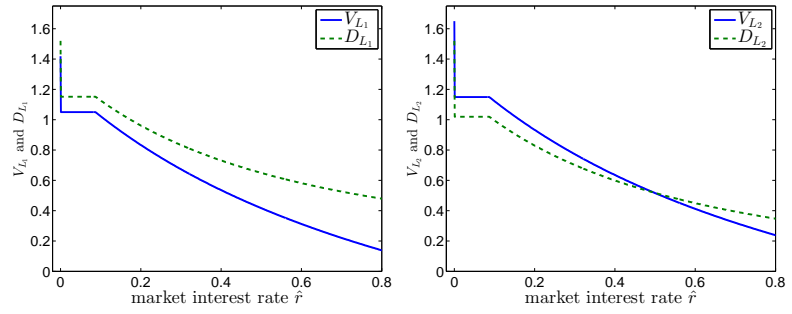
Similar to the perfect information model, other cases with different combinations of  $\alpha_1$  and  $\alpha_2$  are simply special examples of our case above. We can find these payoffs in a similar way. ■

## C Numerical examples: optimal choices for banks $L_1$ and $L_2$ and the determination of $\Gamma(\hat{r})$

Panels (a) and (b) of Fig. C1 illustrate the optimal choices of banks  $L_1$  and  $L_2$  on interbank loan recall and long-term project liquidation at  $R_{shock} = 0.32$  for different levels of  $\hat{r}$ . At this  $R_{shock}$  level, bank  $L_1$  always chooses to recall all the interbank loans from bank  $B_1$  for any  $\hat{r} \geq 0$ . Bank  $L_2$  will always recall  $\alpha_1^{L_2} x = 0.8334x$  of interbank loans when  $\hat{r} > 0$ ,

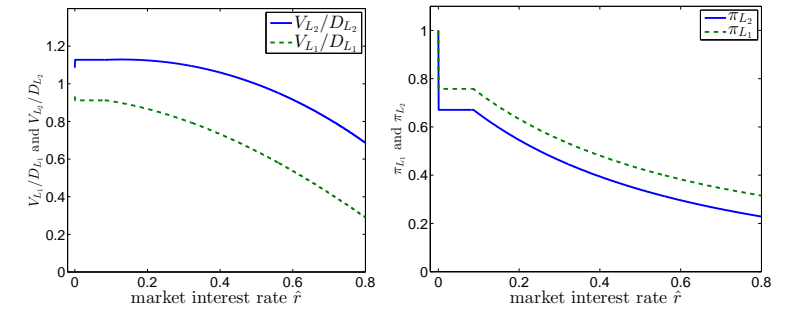


(a) Long-term project liquidation  $L_1$  (b) Long-term project liquidation and interbank loan recall of bank  $L_2$



(c)  $V_{L_1}$  and  $D_{L_1}$

(d)  $V_{L_2}$  and  $D_{L_2}$



(e) Maximum feasible return

(f) Rollover probability

Figure C1: Optimal choices of banks  $L_1$  and  $L_2$  at  $R_{shock} = 0.32$

where  $\alpha_1^{L_2}$  is determined by our previous analysis on  $\alpha_1$ . Both  $L_1$  and  $L_2$  start to liquidate long-term projects when  $1 + \hat{r} > \frac{1}{\lambda} \approx 1.087$ . Given the parameter values in our numerical example,  $l = \frac{\lambda - \frac{1}{1+\hat{r}}}{\gamma(1+R)}$  (because  $\frac{1}{\lambda} < 1 + \hat{r} < \frac{1}{\lambda - \gamma L(1+R)}$ ) is strictly increasing in  $\hat{r}$ .

Panels (c) and (d) of Fig. C1 illustrate how  $V$  and  $D$  of the lending banks change in  $\hat{r}$ . For both banks  $L_1$  and  $L_2$ , a downward jump of  $V$  and  $D$  occurs when  $\hat{r}$  changes from zero to positive. For bank  $L_1$ , when  $\hat{r} = 0$ , the bank is indifferent between keeping the proceeds from recalling the interbank loan and using the proceeds to repay its depositors. We assume that the bank will keep the proceeds. When  $\hat{r} > 0$ , the bank will use the proceeds to repay its depositors at date 1, causing a downward jump of both  $V_{L_1}$  and  $D_{L_1}$ . Similarly, when  $\hat{r}$  becomes positive, bank  $L_2$  will recall  $\alpha_1^{L_2}x$  of the interbank loan and use the proceeds to repay its depositors, causing a downward jump of  $V_{L_2}$  and  $D_{L_2}$ . When  $1 + \hat{r} > \frac{1}{\lambda}$ ,  $V$  and  $D$  decrease in  $\hat{r}$ . This is because, as  $\hat{r}$  becomes higher, the banks will liquidate more long-term projects to repay its depositors at date 1.

Panel (e) of Fig. C1 illustrates how  $\frac{V}{D}$  changes in  $\hat{r}$ . When  $\hat{r}$  turns from zero to positive, the repayment to depositors by bank  $L_2$  will cause  $\frac{V_{L_2}}{D_{L_2}}$  to jump upward, while the repayment to depositors by bank  $L_1$  will cause  $\frac{V_{L_1}}{D_{L_1}}$  to jump downward. This is because in this example, at  $\hat{r} = 0$ , we have  $\frac{V_{L_2}}{D_{L_2}} > 1$  and  $\frac{V_{L_1}}{D_{L_1}} < 1$ . It is straightforward to show that  $\frac{V-Z}{D-Z}$  is strictly increasing in  $Z$  when  $\frac{V}{D} > 1$ , and is strictly decreasing in  $Z$  when  $\frac{V}{D} < 1$ , where  $Z$  is the cash used to repay the depositors, with  $0 < Z < \min(V, D)$ . So here repaying the depositors increases the maximum rate available to bank  $L_2$  depositors, but reduces the maximum rate available to bank  $L_1$  depositors. When  $1 + \hat{r} < \frac{1}{\lambda}$ , both  $\frac{V_{L_2}}{D_{L_2}}$  and  $\frac{V_{L_1}}{D_{L_1}}$  remain constant. When  $1 + \hat{r} > \frac{1}{\lambda}$ , both  $\frac{V_{L_2}}{D_{L_2}}$  and  $\frac{V_{L_1}}{D_{L_1}}$  are decreasing in  $\hat{r}$ . This is because the marginal cost of liquidating long-term projects is increasing, and a decrease in one additional unit of  $V$  leads to a less and less decrease in  $D$ .

Panel (f) of Fig. C1 illustrates how  $\pi$  changes in  $\hat{r}$ . There is a downward jump in both  $\pi_{L_1}$  and  $\pi_{L_2}$  when  $\hat{r}$  turns positive, caused by the repayment to depositors explained before. Except for the jump at  $\hat{r} = 0$ , both  $\pi_{L_2}$  and  $\pi_{L_1}$  remain constant when  $1 + \hat{r} \leq \frac{1}{\lambda}$ . When  $1 + \hat{r} > \frac{1}{\lambda}$ , both  $\pi_{L_2}$  and  $\pi_{L_1}$  are decreasing in  $\hat{r}$ , because both banks liquidate more long-term projects to repay their depositors.

Next we give a detailed explanation for the movement of  $\Gamma(\hat{r})$  in Fig. 5. The equilib-

rium condition of  $\Gamma(\hat{r})$  ((6)) can be written as

$$1 = \frac{1}{2} [\pi_{L_2}(1 + \Gamma(\hat{r})) + (1 - \pi_{L_2})] + \frac{1}{2} \left[ \pi_{L_1} \frac{V_{L_1}}{D_{L_1}} + (1 - \pi_{L_1}) \right]$$

$\Gamma(\hat{r})$  has a small upward jump when  $\hat{r}$  turns positive. As we explained before, when  $\hat{r}$  turns positive, there is a downward jump in both  $\pi_{L_2}$  and  $\pi_{L_1}$ . A lower probability that deposits will be rolled over by the good bank,  $\pi_{L_2}$ , will induce a higher  $\Gamma(\hat{r})$ , while a lower  $\pi_{L_1}$  will induce a lower  $\Gamma(\hat{r})$ . In addition, the maximum rate from the bad bank  $\frac{V_{L_1}}{D_{L_1}}$  is lower, while the maximum rate from the good bank  $\frac{V_{L_2}}{D_{L_2}}$  is higher. The lower  $\frac{V_{L_1}}{D_{L_1}}$  will induce a higher  $\Gamma(\hat{r})$ , but the higher  $\frac{V_{L_2}}{D_{L_2}}$  has no effect on  $\Gamma(\hat{r})$ . This is because, as long as  $\frac{V_{L_2}}{D_{L_2}} > 1 + \hat{r}$ , depositors receive only the promised interest rate of  $1 + \hat{r}$  from the good bank. The overall effect is a small upward jump in  $\Gamma(\hat{r})$ . When  $0 < \hat{r} < \frac{1}{\lambda}$ ,  $\Gamma(\hat{r})$  remains constant because there are no changes in the choices of the two banks. When  $\hat{r} > \frac{1}{\lambda}$ ,  $\Gamma(\hat{r})$  is increasing in  $\hat{r}$ . This is because banks start to liquidate their long-term projects, incurring liquidation costs. As a result,  $\frac{V_{L_1}}{D_{L_1}}$  decreases, causing depositors to require a higher interest rate,  $\Gamma(\hat{r})$ , from the good bank to compensate for the higher expected loss to the bad bank.