A new measure of equity and cash flow duration: The duration-based explanation of the value premium revisited

Online appendix

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APPENDIX A. DISCOUNT RATE DURATION

Cash flow duration is an equity duration that applies a constant, exogenous discount rate to the cash flows of all firms. In doing so, cash flow duration allows disentangling cross-sectional differences in the firms' cash flow patterns from discount rate effects.

In analogy to cash flow duration, it is also possible to define another duration measure that applies a constant, exogenous cash flow growth rate to all firms. The idea behind this third duration measure, which we call *discount rate duration*, is to isolate cross-sectional differences in discount rates across firms from cash flow growth effects.

This appendix provides a comprehensive analysis of discount rate duration, and examines its relation to the equity and cash flow duration concepts presented in the paper.

A.1. Discount rate duration

DEFINITION A.1 (Discount rate duration): *Assume that the firms' cash flows are expected grow at an identical growth rate $\bar{g}$. Let $E_0[\tilde{c}_t]$ denote so-obtained expected stream of cash flows per share at time t, and k the share's implied equity yield. Then discount rate duration D_0^{DR} is defined as*

$$D_0^{DR} = \frac{1}{\widetilde{P}_0} \int_{t=0}^{\infty} t\, E_0[\tilde{c}_t] e^{-kt} dt, \tag{A.1}$$

where

$$\widetilde{P}_0 = \int_{t=0}^{\infty} E_0[\tilde{c}_t] e^{-kt} dt.$$

Different from equity duration, discount rate duration is not obtained from company-specific earnings forecasts, but uses a uniform, exogenously specified growth rate $\bar{g}$ to estimate the firms' expected cash flows. Discount rate duration can hence be interpreted as equity duration with exogenous cash flow growth rate.

EXAMPLE 1 *continued: In the Gordon (1962) model, discount rate duration is given by*

$$D_0^{DR} = \frac{1}{k - \overline{g}}.$$

Similar to equity duration, discount rate duration depends on the firm's equity yield or expected rate of return. Companies with low expected returns have a long discount rate duration, since the weight attributed to distant cash flows is greater. However, by using an exogenously specified cash flow growth rate, discount rate duration does not depend on the firm's actual cash flow growth rate, and therefore isolates cross-sectional differences in expected returns from the firms' heterogeneity in cash flow growth.

To estimate discount rate duration, we first estimate for each firm expected earnings per share up to year 5 by applying a uniform growth rate to current earnings. We set this growth rate at $\overline{g} = 18\%$, which corresponds to the sample average earnings growth rate of

17.7%. If current earnings are negative, we construct a hypothetical earnings estimate by multiplying last available book value of equity with the average industry return on equity over the last 10 years, using the methodology by Gebhardt et al. (2001). The long term growth in residual incomes is assumed to be $\overline{g}^l = 2.2\%$, which is the average long-term bond yield less 3 percent over the examined time horizon. Then we estimate a hypothetical share price $\widetilde{P}_0$ by discounting the so-obtained expected earnings at the firms' implied yield. Finally, discount rate duration is the product of the slope coefficient of the pricing formula and $-(1+k)/\widetilde{P}_0$.

A.2. Descriptive statistics

Panel A of table A.1 reports the descriptive statistics. The mean discount rate duration is around 19.6 years, close to the firms' average equity duration. This similarity results from the assumed earnings growth rate of 18%, which is close to the average growth rate used to estimate the firms' equity duration. Relative to equity duration, discount rate duration has a higher variance of 9.2 years. The higher variance is a consequence of the positive correlation between expected growth and expected return, as documented earlier (see table 2 of the paper). When estimating equity duration, the effect of a higher discount rate is partially compensated by a higher expected cash flow growth rate, see equation (5) in the paper, thereby reducing its cross-sectional variation.

Figure A.1 displays the average discount rate duration from 1992 to 2010. The aggregate discount rate duration increases over the sample period from around 15 years to 25 years. This increase reflects the decrease in the firms' expected returns, and is especially pronounced from 2000 to 2004. The pattern mirrors the evolution in nominal U.S. interest rates, which declined sharply from 6.5% to 1% around the 2001 recession.

A.3. Equity, cash flow, and discount rate duration

Panel B of table A.1 reports the correlation statistics, where we use the natural logarithm of firm size and the B/M ratio to reduce their skewness. For a given monthly cross-section, there is a high correlation between the firms' equity and discount rate durations. This result implies that cross-sectional differences in the firms' expected cash flow growth rates have little impact on the firms' equity durations. The main source of variation in equity duration thus originates from cross-sectional differences in expected returns.

Further, there is a negative correlation between discount rate duration and cash flow duration. Again, this results from the positive association between expected returns and expected growth rates. Companies with higher growth rates have longer cash flow durations, but as their expected returns are higher as well, their discount rate durations are shorter. Given the strong association between equity and discount rate duration, the correlation pattern of discount rate duration with other firm risk characteristics is similar to those of equity duration, as reported in panel A of table 2 in the paper.

Although the results suggest that the main determinant of equity duration are cross-sectional differences in the firms' expected returns (and hence, discount rate durations), the above analysis does not allow for a clean separation between discount rate effects and growth effects, since both components are not independent from each other. To shed further light on the determinants of equity duration, panel C reports the partial and semi-partial correlations of equity duration with each of the two constraint duration measures, together with the squared correlations.

The table shows that, even if the other duration measure did not vary, discount rate duration has a larger explanatory power for equity duration than cash flow duration. The squared semi-partial correlation of cash flow duration, i.e., the decrease of the R^2 value when removing cash flow duration as explanatory variable for equity duration is small.[1]

[1]Additional tests show that the relation between discount rate duration and subsequent stock returns is similar to equity duration, and are hence not reported here. The results are available on request.

Table A.1: Discount rate duration

Panel A: Descriptive statistics

	Mean	Std. dev.	25% centile	50% centile	75% centile
Discount rate duration	19.60	9.20	14.70	17.73	21.81

Panel B: Correlation statistics

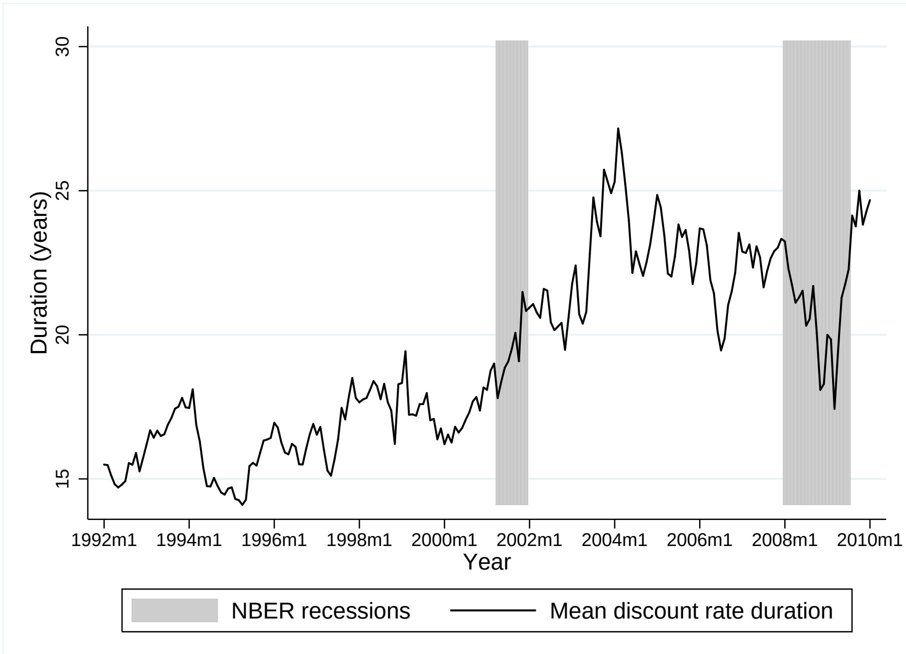
	Discount rate duration
Equity duration	0.980
Cash flow duration	−0.125
Expected return	−0.789
Expected growth	−0.111
Market beta	0.031
B/M ratio	−0.104
Firm size	0.119
Price momentum	0.155

Panel C: Partial and semi-partial correlation of equity duration

	Cash flow duration	Discount rate duration
Partial correlation	25.67%	98.03%
Squared partial correlation	6.59%	96.11%
Semi-partial correlation	3.87%	96.83%
Squared semi-partial correlation	0.15%	93.77%

Panel A presents the mean, standard deviation, and quartiles of the firms' discount rate duration, as derived from the two-stage RIM (Claus and Thomas, 2001), see equation (12) in the paper. Panel B presents the correlation statistics, calculated as the mean of the monthly cross-sectional correlations. We use the natural logarithm of firm size and B/M ratio to reduce their skewness. Panel C presents the partial and semi-partial correlations between equity duration and both cash flow and discount rate duration. The sample period is from January 1992 to January 2010. Observations: 278,717.

Figure A.1: Aggregate U.S. discount rate duration (1992-2010)



The graph plots the monthly mean (equal-weight) discount rate duration in the United States from 1992 to 2010. The shaded areas indicate NBER recessions.

APPENDIX B. DETAILS OF SIMULATION STUDY

We simulate a cross-section of $N = 1,000$ firms with time-varying firm-specific discount rates $k_{n,t}$ and cash flows $e_{n,t}$ for a time horizon of $T = 200$ years.

B.1. Discount rates

Discount rates $k_{n,t}$ in year t of firm n are assumed to be lognormal, consisting of two components, a deterministic time-varying market term structure of equity yields $\mu_{k,t}$, and a firm-specific (cross-sectional) component ε_n. The log term structure of equity yields is assumed to be mean-reverting from short-term market equity yields $\mu_{k,1}$ to its long-run level, $\mu_{k,\infty}$,

$$\mu_{k,t} = \kappa(\mu_{k,t-1} - \mu_{k,\infty}) + \mu_{k,\infty}, \qquad t = 2, \ldots, 200.$$

The relation between $\mu_{k,1}$ and $\mu_{k,\infty}$ determines the slope of the yield curve. If $\mu_{k,1} < \mu_{k,\infty}$, the equity yield curve is upward-sloping; in case $\mu_{k,1} > \mu_{k,\infty}$ it is downward-sloping. Finally, $\mu_{k,1} = \mu_{k,\infty}$ implies a flat equity yield curve. We consider all three cases.

The log of the firm-specific discount rate component ε_n is normally distributed with mean 0 and standard deviation σ_k,

$$\varepsilon_n \sim \mathcal{N}(0, \sigma_k^2), \qquad n = 1, \ldots, 1000.$$

Combining the cross-sectional and the times-series dimension gives the $N \times T$ matrix of log discount rates $\ln \mathbf{K}$,

$$\ln \mathbf{K} = \mathbf{1}_N \boldsymbol{\mu}_k + \boldsymbol{\varepsilon} \mathbf{1}_T'.$$

B.2. Cash flows

The firms' cash flows $e_{n,t}$ are assumed to be lognormal as well. Log first year earnings are given by

$$\ln e_{n,1} \sim \mathcal{N}(\mu_e, \sigma_e^2), \qquad n = 1, \ldots, 1000.$$

Short-term earnings growth rates $g_{n,t}^s$ up to year 5 are determined by two independent random components, a random component across all firms $\theta_{n,t}$, and a firm-specific (cross-sectional) component η_n. Both components are lognormally distributed,

$$\begin{aligned} \ln \theta_{n,t} &\sim \mathcal{N}(\mu_g, \sigma_g^2), \qquad n = 1, \ldots, 1000 \text{ and } t = 1, \ldots, 4. \\ \ln \eta_n &\sim \mathcal{N}(0, \sigma_{g,i}^2), \qquad n = 1, \ldots, 1000. \end{aligned}$$

Combining these two components gives the $N \times 4$ matrix of short-term log earnings growth rates $\ln \mathbf{G^s}$,

$$\ln \mathbf{G^s} = \boldsymbol{\Theta} + \boldsymbol{\eta}_n \mathbf{1}_4'.$$

Following the residual income model (12), earnings beyond year 5 are determined by an assumed long-term growth in residual incomes. Residual incomes in year 5 are derived from expected earnings and book values of equity, which are calculated using the clean-surplus assumption. Initial book values of equity are normalized to 1, and payout ratios are fixed at 50%.

B.3. Estimating duration

The simulated data are used to calculate the firms' share prices. We then apply our estimation approach to share price and simulated cash flow data to estimate the firms' expected returns, equity and cash flow durations, and compare the results with the estimates obtained from the actual data. The simulation parameters governing discount rates, earnings and earnings growth rates are chosen such that the means and standard deviations obtained from the simulated data match those of the actual data. Furthermore, in the scenario assuming an upward-sloping equity yield curve, the slope of the curve is chosen such that the correlation between expected returns and cash flow duration matches the correlation observed in the data.

Panel A of table B.1 presents the simulation parameters. Panel B compares the estimates obtained from the three simulations to those from the actual data. The results show that the scenario assuming an upward-sloping equity yield curve is best in explaining the estimates obtained from the actual data.

Table B.1: Details of simulation study

Panel A: Parameters of the model

Simulation parameters	upward-sloping yield curve	flat yield curve	downward-sloping yield curve
$\mu_{k,1}$	-2.555	-2.390	-2.251
$\mu_{k,\infty}$	-2.315	-2.390	-2.329
σ_k	0.335	0.326	0.302
μ_e	-0.29	-0.33	-0.49
σ_e	1.56	1.04	0.66
μ_g^s	0.056	0.048	0.048
σ_g	0.307	0.327	0.332
$\sigma_{g,i}$	0.307	0.327	0.332
Implied short-term equity yields	8.22%	9.66%	10.02%
Implied long-term equity yields	10.45%	9.66%	9.35%
Long-term growth in residual income, g^l		2.60%	
Payout ratio, p		50%	
Book value of equity, b_0		1	
Mean reversion of market equity yields, κ		0.8	

Panel B: Estimates from simulated and actual data

	Model (upward-sloping)	Model (flat)	Model (downward-sloping)	Data
Means				
Equity duration	19.17	19.01	18.98	19.11
Cash flow duration	15.59	15.39	15.32	16.81
Expected return	9.83%	9.83%	9.83%	9.83%
Expected growth	17.83%	17.75%	17.80%	17.72%
B/M ratio	0.33	0.33	0.33	0.33
Standard deviation				
Equity duration	9.38	9.79	9.88	7.57
Cash flow duration	2.71	3.07	3.19	2.06
Expected return	3.19%	3.19%	3.19%	3.20%
Expected growth	40.21%	43.21%	44.02%	44.20%
B/M ratio	0.66	0.66	0.66	0.66
Correlation statistics				
EQ duration/CF duration	0.102	0.288	0.399	-0.092
EQ duration/expected return	-0.746	-0.757	-0.782	-0.778
EQ duration/expected growth	0.116	0.197	0.264	-0.088
EQ duration/BM-ratio	-0.006	-0.270	-0.402	-0.087
CF duration/expected return	0.224	-0.002	-0.215	0.224
CF duration/expected growth	0.750	0.765	0.738	0.088
CF duration/BM-ratio	-0.474	-0.428	-0.672	-0.158

Panel A summarizes the simulation parameters for each of the three scenarios considered, together with the model-implied equity yield curves. Panel B compares the means, standard deviations, and correlation statistics of the estimates obtained from the three simulations to those from the actual data.

References

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