Online Appendix for

"The Heterogeneous Responses of Consumption between Poor and Rich to Government Spending Shocks"

Eunseong Ma[*]

September 27, 2018

---
[*]Department of Economics, Texas A&M University, College Station, TX 77843, e-mail: masilver85@tamu.edu.

A   Empirical Evidence

I empirically investigate the distributional effects of government spending shocks on consumption across the income distribution using micro data. Related work is Anderson, Inoue and Rossi (2016) and De Giorgi and Gambetti (2012).

Data and Approach

I use the CEX, which is conducted by the Bureau of Labor Statistics (BLS), to collect information on income, consumption and age across individual households.[1] I mainly use quarterly data which span from the first quarter of 1980 to the third quarter of 2008 (right before zero lower bound (ZLB) periods).[2] Following Heathcote, Perri and Violante (2010), the measure of non-durable consumption includes food and beverages, tobacco, apparel and services, personal care, gasoline, public transportation, household operation, medical care, entertainment, reading material and education. The definition of the non-durable goods is similar to that of Anderson, Inoue and Rossi (2016) and De Giorgi and Gambetti (2012).[3] Annual income is defined as before-tax income, which is the sum of wages, salaries, business and farm income, financial income, and transfers.[4] The measure of age is defined as the age of a head of each household. Income and non-durable consumption for households are real per capita values: they are divided by family size (the number of family members), deflated by CPI-U series, and seasonally adjusted by X-12-ARIMA. It is important what measure is used for government spending shocks. I use the SPF forecast errors to identify government spending policy shocks. The measure of SPF forecast errors is defined as the difference between actual federal spending growth and the one-quarter-ahead SPF forecasted growth.[5] Regarding the measure of government spending, real per capita total government spending

---

[1]The CEX is rotating panel data where individuals are interviewed for four consecutive quarters at most.

[2]The data for the period before 2007 are from Heathcote, Perri and Violante (2010).

[3]Anderson, Inoue and Rossi (2016) define non-durable consumption as expenditures on food, alcoholic beverages, tobacco, utilities, personal care, household operations, public transportation, gas and motor oil, and miscellaneous expense, and De Giorgi and Gambetti (2012) use food (including alcohol and tobacco), heating fuel, public and private transport (including gasoline), and personal care as the non-durable consumption following Attanasio and Weber (1995).

[4]In the CEX, unfortunately, information on household-level income is only available in the first and fourth survey, which means that income for the second and third surveys is the same as that of the first, while information on consumption is available on a quarterly basis. Since income data are only used to construct income groups as in Anderson, Inoue and Rossi (2016), I believe that the effects of the measurement errors for income may not affect main results seriously.

[5]I use the SPF forecast errors instead of using the defense spending news shocks developed by Ramey (2011) since the news shocks do not show enough explanatory power for government spending in the sample period I focus on (Anderson, Inoue and Rossi, 2016).

is used.

In order to study the effects of a government spending shock, I consider a three-variable Vector Autoregressive (VAR) model as in Anderson, Inoue and Rossi (2016), including the SPF forecast errors, government spending, and consumption. Consider the following reduced-form VAR,

$$X_t = B(L)X_{t-1} + \varepsilon_t, \tag{A.1}$$

where X_t is a vector including the SPF forecast errors, the logged real government spending, and logged real consumption across different socioeconomic groups; $B(L)$ is a polynomial in the lag operator; $\varepsilon_t \sim N(0, \Sigma)$ are reduced-form innovations. The structural innovations, $v_t \sim N(0, I)$, are defined by an orthonormal rotation of the reduced-form residuals:

$$v_t = A_0 \varepsilon_t, \tag{A.2}$$

where $A_0^{-1}(A_0^{-1})' = \Sigma$. I order the SPF forecast errors first and consumption last in the VAR and identify the matrix A_0^{-1} as the Cholesky decomposition of Σ. Constant terms, quadratic trend terms, and four lags are included in the VAR as in Ramey (2011) and Anderson, Inoue and Rossi (2016).[6] As discussed in Anderson, Inoue and Rossi (2016), by including the shocks measures in a VAR where the shock is ordered first, we can ensure that the shock is uncorrelated with past information contained in the other variables in the VAR, and other variables response to the shocks contemporaneously.

Empirical Results

I present the main empirical results of the effects of government spending shocks on consumption across various dimensions of inequality. I mainly focus on the responses of average non-durable consumption in the five groups (quintiles) of individuals sorted by income levels. Additionally, empirical results based on age and consumption quintiles, as proxies for income groups, will be discussed.

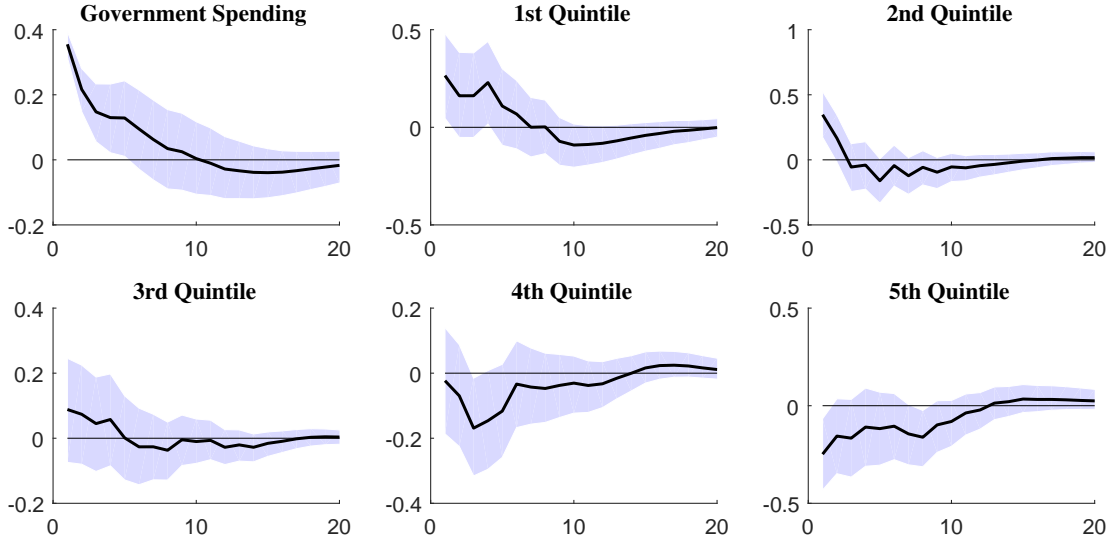[6]The results are robust to different lag order lengths including 2, 3, and 5 lags.

Figure A.1: Responses of Consumption across Income Quintiles

Note: Responses of logged real per capita government spending and logged average real per capita consumption across the income quintiles to a government spending shock. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile. The shaded regions are the 68 percent confidence bands generated by Monte Carlo simulations.

Figure A.1 exhibits the responses of average consumption across the income quintiles to government spending shocks.[7] As shown in Figure A.1, there are substantial differences in the responses of consumption across income groups to spending shocks: consumption increases for the poor while it decreases for the rich when government expenditure rises. Estimated peak and cumulative multipliers across the income quintiles are summarized in Table A.1.[8] The peak multipliers for the first three quintiles are positive while they are negative for the top two quintiles. For example, the peak multiplier for the lowest income quintile is 0.15 whereas that of the highest is -0.41. In other words, when government increases its spending by one dollar, poorest consumers increase their consumption by 15 cents, but consumption for the richest decreases by 41 cents on average. These findings are robust when the cumulative multipliers are used as a measure of the effects of government spending shocks.[9] The fourth and fifth columns of Table A.1 report the cumulative multipliers for a one-year and two-year horizon, respectively. Both integral multipliers indicate that consumption of the first three lowest income quintiles rises while that of the last two income groups falls after positive spending shocks.[10] The results are also robust when the full sample (1980:I-2015:III), which

---

[7]I smooth the impulse-response functions using centered moving averages with three periods.

[8]The peak multiplier for a group j is computed as $max_h sign\left(\frac{\Delta log C_{t+h}^j}{\Delta log G_t}\right)\left|\frac{\Delta log C_{t+h}^j}{\Delta log G_t}\right|\frac{\overline{C^j}}{\overline{G}}$ where $\frac{\overline{C^j}}{\overline{G}}$ is the average ratio of consumption of the group j to government spending.

[9]The integral multiplier is defined as the sum of the responses of consumption divided by the sum of changes in government expenditure.

[10]For robustness check, I also consider the responses of median consumption in each income quintile to government spending shocks, and the results are similar to those of the average consumption case.

Table A.1: Estimated Multipliers across Income Quintiles

Variable	Peak	68 Percent C.I.	1 Year Integral	2 Year Integral	Full Sample
1st Quintile	0.15	$[0.03, 0.24]$	0.19	0.17	0.07
2nd Quintile	0.26	$[0.15, 0.36]$	0.13	0.01	0.18
3rd Quintile	0.08	$[-0.08, 0.20]$	0.10	0.05	0.11
4th Quintile	-0.19	$[-0.40, -0.02]$	-0.19	-0.22	-0.19
5th Quintile	-0.41	$[-0.81, -0.10]$	-0.47	-0.61	-0.22

Note: Estimated consumption multipliers across the income quintiles. "Peak" denotes the peak multipliers, "68 Percent C.I" denotes the 68 percent confidence interval for the peak multipliers, and "1 Year Integral" denotes the one-year cumulative multipliers. The sample periods are 1980:I-2008:III other than "Full Sample." "Full Sample" reports the peak multipliers for the period 1980:I-2015:III. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile. Confidence bands are generated by Monte Carlo simulations.

includes the ZLB period, is used for the estimation of the multipliers (reported in the last column of Table A.1).[11] These empirical results are consistent with those in Anderson, Inoue and Rossi (2016) and De Giorgi and Gambetti (2012).

I also consider other individual characteristics such as age and consumption as a robustness check. Age and consumption can be viewed as proxy variables for individual income. Figure A.2 summarizes the effects of government spending shocks on consumption across age and consumption quintiles. As far as the responses of consumption across age groups are concerned, as shown in the upper panel of Figure A.2, consumption increases for the youngest and the oldest individuals, and it decreases for middle-aged individuals when government expenditure increases. Considering that income profiles over ages are inverted U-shaped or hump-shaped, these findings are consistent with those of income quintiles. The bottom panel of Figure A.2 shows that consumption in the lower consumption quintiles increases while consumption in the top decreases, which confirms that the poor consume more and the rich consume less in response to an increase in government expenditure based on the assumption that consumption is a good proxy for income.

B   Sensitivity Analyses

I check to see if the results are robust to different values of key parameters: tax progressivity, τ, output elasticities of public spending, γ, and the tax policy parameter, ω. Since whether the model with smaller values of the parameters can account for the main findings is a key issue in these analyses, I only consider the parameter values which are less than the baseline values.[12]

---

[11]"Full Sample" reports the peak multipliers.

[12]For example, one may argue that the benchmark parameter value of γ is too large.
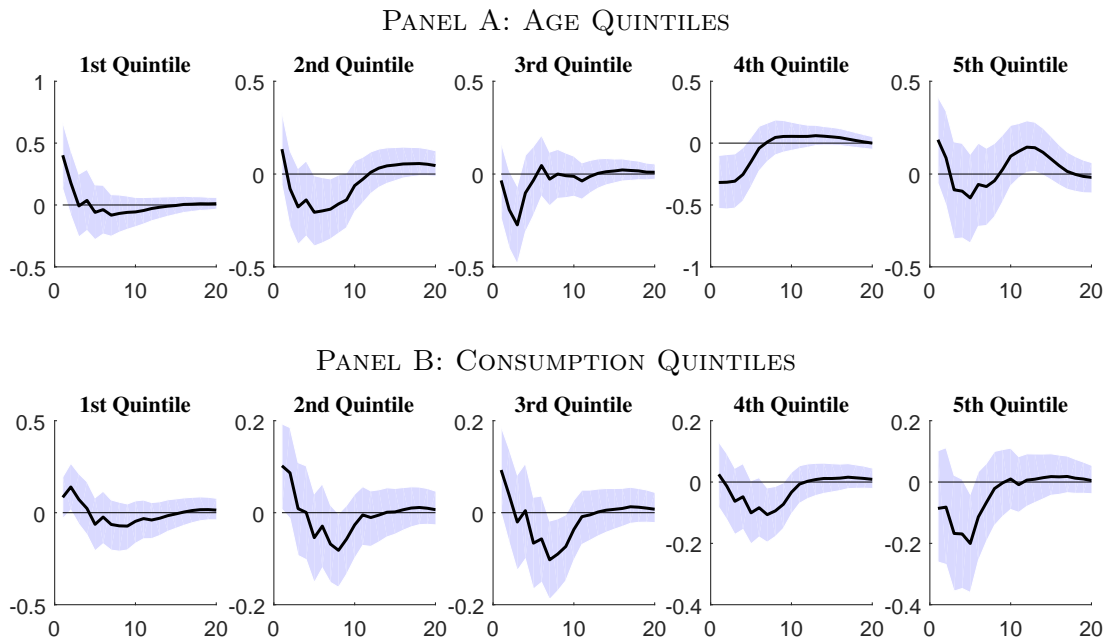
Figure A.2: Responses of Consumption across Age and Consumption Quintiles

Note: Response of the logged average real per capita consumption across age quintiles (Panel A) and consumption quintiles (Panel B) to a government spending shock. The sample periods are 1980:I-2008:III. "1st Quintile" denotes the youngest or the lowest consumption quintile, and "5th Quintile" denotes the oldest or the highest consumption quintile. The shaded regions are the 68 percent confidence bands generated by Monte Carlo simulations.

The first panel of Figure A.3 shows the responses of consumption across income quintiles to government spending shock with different indexes of tax progressivity, τ. The heterogeneous effects of the government expenditure on consumption across income distribution are consistently found with any positive indexes of tax progressivity.[13] The effects of government spending on consumption are still positive for households in the lower income quintiles and negative for ones in the upper quintiles. Next, the consumption responses across the income quintiles to government spending shocks with different values of output elasticities of public spending are shown in the second panel of Figure A.3. With any values of γ, the heterogeneous effects of the government spending on consumption across the income distribution are reasonably generated: the effects of government spending on consumption are positive for the poor (except the second quintile) and negative for the top two quintiles.[14] Lastly, the third

---

[13]A change in τ can affect the steady-state distributions of income and wealth. In turn, it will affect aggregate labor supply elasticity (e.g., Chang and Kim (2006)) and income and wealth mobility. Besides, progressivity elasticity of government spending can change with different progressivity since I did not recalibrate ω to match the moment. Hence, the interaction between all these factors can make the effects of the different progressivity non-monotone.

[14]The reason why the second quintile shows the negative sign is as follows. In the model economy, households in the second group own a relatively large amount of wealth compared to other poor income groups. With large γ, the effects of productive government spending are large, so they can increase their consumption and investment at the same time since after-tax interest rates also rise after a shock. However, when γ is small, they want to save more and consume less since the intertemporal substitution effect is much larger
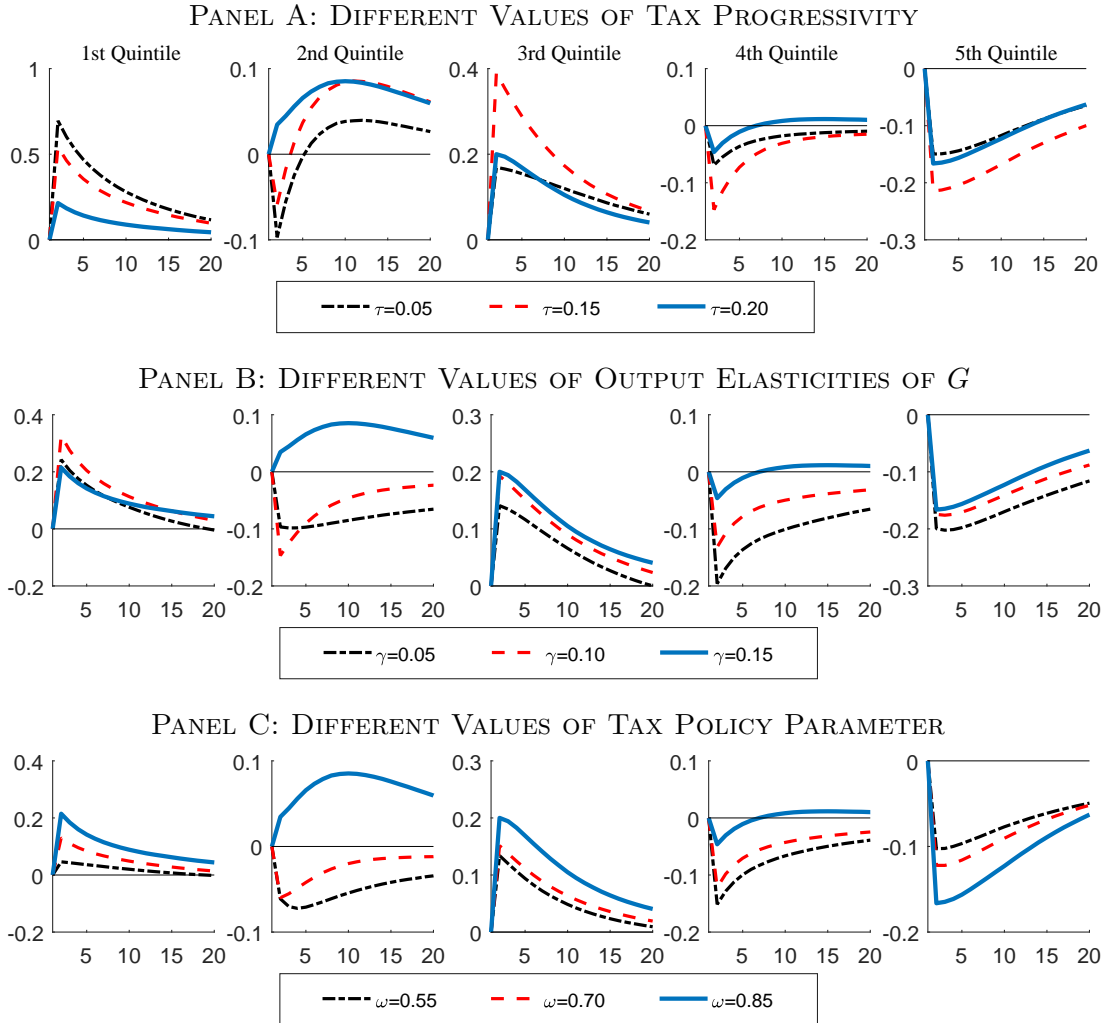
Figure A.3: Responses of Consumption across Income Quintiles with Different Parameter Values

Note: Responses of average consumption across the income quintiles to a government spending shock with different parameter values. All variables are logged. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

panel of Figure A.3 reports the consumption dynamics across the income distribution after a spending shock with different measures of tax policy parameter, ω. Overall, the different effects of the government expenditure on consumption across the income quintiles are also well-replicated with different values of ω.[15]

C   Computational Procedures

Steady-state (Stationary) Economy

I use the algorithm suggested by Rios-Rull (1997) to find the stationary measure, μ_{ss}. The steps are as follows.

Step 1. Have guesses for endogenous parameters such as β, χ, G_{ss}, and λ.

Step 2. Construct grids for individual state variables, such as asset holdings, a, and logged individual labor productivity, $\hat{x} = \ln x$, where the number of grids for a and $\hat{x}$ are denoted by n_a and n_x, respectively. I use $n_a = 201$ and $n_x = 19$. The range of a is $[-2, 200]$. More asset grid points are assigned on the lower asset range using a convex function. $\hat{x}$ is equally spaced in the range of $[-3\sigma_{\hat{x}}, 3\sigma_{\hat{x}}]$, where $\sigma_{\hat{x}} = \sigma_x / \sqrt{1 - \rho_x^2}$.

Step 3. Approximate the transition probability matrices for individual labor productivity, $\mathbb{P}_x$, using Tauchen (1986).

Step 4. Solve the individual value functions at each grid point. In this step, I obtain the optimal decision rules for saving $a'(a, x)$ and hours worked $h(a, x)$, the value functions $V^E(a, x)$, $V^N(a, x)$, and $V(a, x)$. The detailed steps are as follows:

(a) Compute the wage rate in the steady state, w_{ss}, through the firm's first-order condition, $w_{ss} = (1 - \alpha) \left(\frac{\alpha}{r_{ss} + \delta} \right)^{\frac{\alpha}{1-\alpha}} G_{ss}^{\frac{\gamma}{1-\alpha}}$, where the steady-state real interest rate, r_{ss}, is chosen to be 0.01.

(b) Make an initial guess for the value function, $V_0(a, x)$ for all grid points.

(c) Solve the consumption-saving problem for each employment status:

$$V_1^E(a, x) = \max_{a' \geq \bar{b}} \left\{ \ln \left(w_{ss} x \bar{h} + (1 + r_{ss})a - T(w_{ss} x \bar{h} + r_{ss}a) - a' \right) \right.$$

___

than the income effect.

[15]Similarly, when ω is small, the second quintile reduces consumption since the intertemporal substitution effect is larger than the income effect.

$$-\chi \frac{\overline{h}^{1+1/\phi}}{1+1/\phi} + \beta \sum_{x'=1}^{n_x} \mathbb{P}_x(x'|x) V_0(a', x') \Bigg\},$$

and

$$V_1^N(a, x) = \max_{a' \geq \overline{b}} \left\{ \left\{ \ln\left((1 + r_{ss})a - T(r_{ss}a) - a'\right) + \beta \sum_{x'=1}^{n_x} \mathbb{P}_x(x'|x) V_0(a', x') \right\} \right\}.$$

(d) Compute $V_1(a, x)$ as $V_1(a, x) = \max\left\{ V_1^E(a, x), V_1^N(a, x) \right\}.$

(e) If V_0 and V_1 are close enough for each grid point, and go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (c).

Step 5. Obtain the time-invariant measure, μ_{ss} with finer grid points for assets. Using cubic spline interpolation, compute the optimal decision rules for asset holdings with the new grid points. μ_{ss} can be computed using the new optimal decision rules and $\mathbb{P}_x$.

Step 6. Compute aggregate variables using μ_{ss}. If the computed rental price for capital, the employment rate, government spending-ouput ratio, the average income tax rate become sufficiently close to the targeted ones, then the steady-state economy is found. Otherwise, reset the endogenous parameters, and go back to Step 4.

Dynamic Economy

In order to solve a dynamic economy, the distribution across households, μ, which affects prices, should be tracked. Instead, following Krusell and Smith (1998), I use the first moment of the distribution and the forecasting function for it to solve a dynamic economy.

Step 1. Construct grids for aggregate state variables such as aggregate capital, K, and government spending, G, where the number of grids for K and G are denoted by n_K and n_g, respectively. I use $n_K = 9$ and $n_g = 9$. The range of K is $[0.9K_{ss}, 1.1K_{ss}]$, where K_{ss} is the steady-state mean capital. g, defined as $g = \ln G - \ln G_{ss}$, is equally spaced in the range of $[-3s_g, 3s_g]$, where $s_g = \sigma_g/\sqrt{1 - \rho_g^2}$. The grids for individual state variables are the same as those in the steady-state (stationary) economy.

Step 2. Parameterize coefficients for forecasting functions for the next-period capital, the wage rate, and tax progressivity:

$$\ln K' = a_0 + a_1 \ln K + a_2 \ln G, \tag{A.3}$$

$$\ln w = b_0 + b_1 \ln K + b_2 \ln G, \tag{A.4}$$

9

$$\tau = d_0 + d_1 \ln K + d_2 \ln G. \tag{A.5}$$

Given the wage rate w, the real interest rate, r, is computed from the firm's profit maximization: $r = G^{\frac{\gamma}{\alpha}} \alpha \left(\frac{w}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta$.

Step 3. Using the forecasting functions of A.3, A.4, and A.5, solve the individual value functions at each grid point. In this step, I obtain the optimal decision rules for asset holdings, $a'(a, x, K, G)$, hours worked, $h(a, x, K, G)$, and the value function, $V(a, x, K, G)$. The detailed steps are similar to those in Step 4 of the steady-state economy.

Step 4. Given the forecasting functions of A.3, A.4, and A.5, and the value functions, solve the optimization problem of individuals for 3,500 periods with finer grid points for assets holding. The detailed steps are as follows.

(a) Set initial values for K, G, and $\mu(a, x)$.

(b) Obtain the value function, $\widetilde{V}(a, x)$, for finer asset grids, which is evaluated at the aggregate state variables using the value function obtained in Step 3 and forecasting the functions of A.3, A.4, and A.5.[16]

(c) Set $\widehat{\tau}$ as a guess for progressivity. Then, $\widehat{\lambda}$ is obtained from Equation 2.

(d) Given $\widehat{\tau}$ and $\widehat{\lambda}$, obtain the decision rule for employment, $h(a, x)$, using the forecasting function of A.4 and $\widetilde{V}(a, x)$.[17] Then, compute tax payments, $T(wxh(a, x)+ra)$, from Equation 1.

(e) Check a balanced budget for the government: $G = \int T(wxh(a, x) + ra)d\mu$. If the government runs a balanced budget, then go to the next step. Otherwise, reset $\widehat{\tau}$, and go to Step (c). The progressivity that government runs a balanced budget is denoted by τ^*.

(f) Set $\widehat{w}$ as a guess for the wage rate. Then, $\widehat{r} = G^{\frac{\gamma}{\alpha}} \alpha \left(\frac{\widehat{w}}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta$ from the firm's profit maximization.

(g) Under $\widehat{w}$, obtain the decision rule for labor supply, $h(a, x)$, using the forecasting function of A.5 and $\widetilde{V}(a, x)$.

(h) Check labor market clearing: $L = G^{\gamma} \left\{(1 - \alpha)/\widehat{w}\right\}^{1/\alpha} K = \int h(a, x)xd\mu$. If the labor market clears, then go to the next step. Otherwise, reset $\widehat{w}$, and go back to Step (f). The wage rate that clears the labor market is denoted by w^*.

---

[16] I use cubic spline interpolation for off grid points.

[17] Given the forecasting function for the wage rate, the rental price for capital r can be computed by: $r = G^{\frac{\gamma}{\alpha}} \alpha \left(\frac{w}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta$.

(i) Using $\tilde{V}(a,x)$, τ^*, and w^*, obtain the decision rules for consumption, $c(a,x)$, employment, $h(a,x)$, and asset holdings, $a'(a,x)$.

(j) Compute aggregate variables: $C = \int c(a,x)d\mu$, $L = \int h(a,x)xd\mu$, $K' = \int a'(a,x)d\mu$, $H = \int h(a,x)d\mu$, $Y = K^{\alpha}L^{1-\alpha}G^{\gamma}$, and $I = Y - C - G$.

(k) Obtain the next-period distribution, $\mu'(a,x)$, using $\mathbb{P}_x$ and $a'(a,x)$.

Step 5. Obtain the new coefficients for the forecasting functions using the simulated data with an OLS regression.[18] If the new coefficients are close enough to the previous ones, the simulation is done. Otherwise, update the coefficients and go back to Step 3. I check the goodness of fit for the forecasting functions using R^2. High accuracy is obtained such that:

$$\ln K' = 0.013379 + 0.991539 \ln K - 0.002313 \ln G, \ R^2 = 0.9994,$$

$$\ln w = -0.177162 + 0.342779 \ln K + 0.127333 \ln G, \ R^2 = 0.9973,$$

$$\tau = 1.464993 - 0.255302 \ln K + 1.008161 \ln G, \ R^2 = 0.9991.$$

D   Impulse Response Functions

In a heterogeneous agent economy, obtaining impulse-response functions (IRFs) is computationally demanding since the distribution across characteristics of households (μ) needs to be tracked in each simulation period. To solve this issue, I approximate the cross-sectional distribution using the mean asset as in Krusell and Smith (1998). I generate a 3,500-periods time series using value functions and policy functions, which are obtained from the individual optimization problems. With the simulated data of 1000 periods, I run VAR with one-period-lagged variables and a constant term in which a government spending shock is ordered first and other variables of interest are ordered next. I also use different approaches to compute IRFs, but the three different approaches generate qualitatively similar IRFs.

I discuss and compare three different ways to compute the IRFs from the model economy. Approach I and Approach II are based on estimation using the simulated data, while Approach III employs a method directly simulating IRFs using the estimated forecasting the functions of A.3, A.4, and A.5 and the decision rules.

---

[18]I drop the first 500 periods to eliminate the impact of the arbitrary choice of initial aggregate state variables.

11

- Approach I: With the simulated data of 1000 periods, I run VAR with one-period-lagged variables and a constant term in which a government spending shock is ordered first and other variables of interest are ordered next.

- Approach II: By construction, government spending shocks are exogenous in the model economy. Hence, using the 1000 periods of simulated data, I estimate a series of regressions for each horizon for each variable. Specifically, I regress $\ln y$ on a constant and $\ln x$, where y is a variable of interest and x is a vector $[G_t\, G_{t-1}\, ...G_{t-19}]$ and obtain the estimates. With an exogenous series of government spending and the estimates, I draw the IRFs.

- Approach III: Using the estimated forecasting functions for future capital, the wage rate, and tax progressivity and the obtained decision rules, I compute the responses of variables of interest. The steps are summarized as follows.

  1. Using the forecasting functions, solve the individual value functions at each grid point.

  2. Assume the series of government spending: I assume that spending is at steady state value for 500 periods, unexpectedly increases by one percent in a period of 501, and decreases following the AR(1) process with no error term.

  3. Given the forecasting functions, the value functions, and the series of government spending, solve the optimization problem individuals under finer grid points for assets.

  4. Compute aggregate and disaggregate variables.

Figure A.4 and Figure A.5 show the IRFs for key aggregate variables and consumption across the income quintiles using the three approaches. The three approaches generate qualitatively similar results. In particular, the estimation methods (Approach I and Approach II) produce almost identical results by construction.
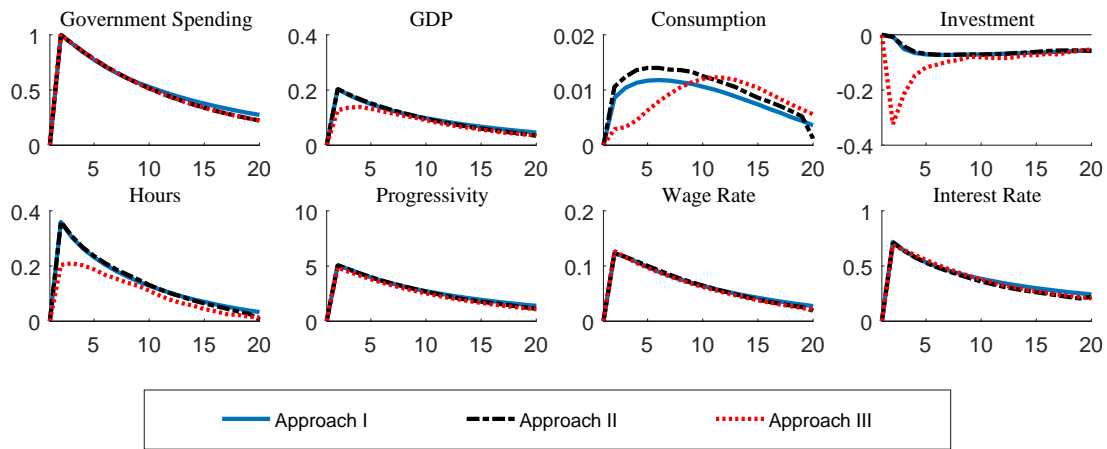
Figure A.4: Impulse-responses of Aggregate Variables

Note: Responses of aggregate variables to a government spending shock using the three approaches. All variables other than the interest rate are logged.
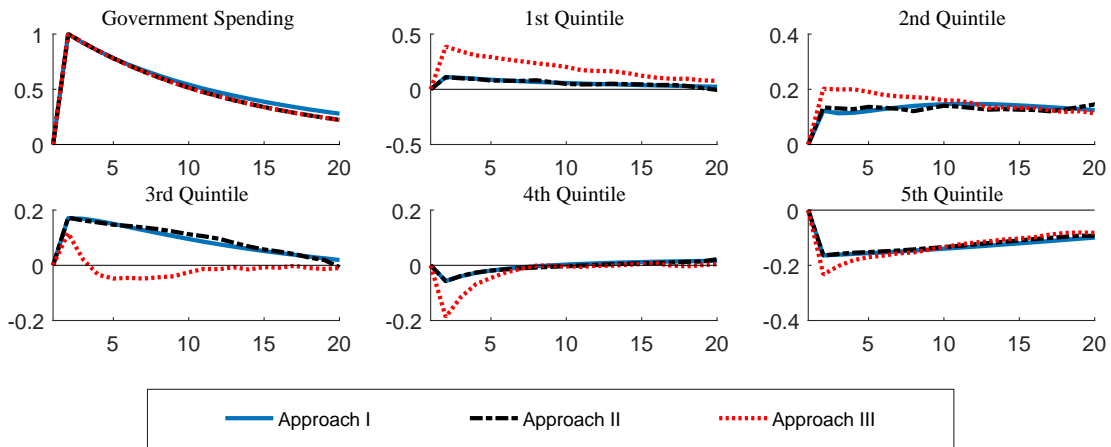

Figure A.5: Responses of Consumption across Income Quintiles

Note: Responses of average consumption across the income quintiles to a government spending shock using the three approaches. All variables are logged. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

References

Anderson, Emily, Atsushi Inoue, and Barbara Rossi. 2016. "Heterogeneous Consumers and Fiscal Policy Shocks." *Journal of Money, Credit and Banking*, 48(8): 1877–1888.

Attanasio, Orazio P., and Guglielmo Weber. 1995. "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey." *Journal of Political Economy*, 103(6): 1121–1157.

Chang, Yongsung, and Sun-Bin Kim. 2006. "From Individual to Aggregate Labor Supply: A Quantitative Analysis based on a Heterogeneous Agent Macroeconomy." *International Economic Review*, 47(1): 1–27.

De Giorgi, Giacomo, and Luca Gambetti. 2012. "The Effects of Government Spending on the Distribution of Consumption." Barcelona Graduate School of Economics Working Papers 645.

Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. "Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006." *Review of Economic Dynamics*, 13(1): 15 – 51. Special issue: Cross-Sectional Facts for Macroeconomists.

Krusell, Per, and Anthony A. Smith. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy, 106(5): 867-96.*

Ramey, Valerie A. 2011. "Identifying Government Spending Shocks: It's all in the Timing*." *The Quarterly Journal of Economics.*

Rios-Rull, Jose-Victor. 1997. "Computation of equilibria in heterogeneous agent models." Federal Reserve Bank of Minneapolis Staff Report 231.

Tauchen, George. 1986. "Finite state markov-chain approximations to univariate and vector autoregressions." *Economics Letters*, 20(2): 177 – 181.