ONLINE APPENDIX FOR "WELFARE CONSEQUENCES OF GRADUAL DISINFLATION IN EMERGING ECONOMIES". NOT INTENDED FOR PUBLICATION[1]

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First Draft: July 2015

July 14, 2017

Abstract

This online appendix includes supplementary material on the data used in the project, the discussion of an alternative model with a representative firm that operates a constant returns-to-scale technology and households that face a transactions costs function that is homothetic in real money balances and consumption. The appendix also provides proofs to Lemmas 1 and 2 and describe the numerical solution algorithm for stationary and transition dynamics equilibria. Finally, I provide the results implied by a revenue neutral disinflation exercise which assumes that decline in seigniorage revenues during disinflation is matched with an increase in the distortionary labor income tax rate.

Keywords: Small open economy, incomplete markets, gradual disinflation.
JEL Classification: D31, E52, F41

[1]I am grateful to the editor Pok-sang Lam and two anonymous referees for their constructive insights. I also thank Enrique Mendoza, Pablo D'Erasmo, Carlos Végh, Yasin Mimir, Yasin Kursat Onder, seminar participants at the University of Maryland, College Park, Department of Economics, Central Bank of the Republic of Turkey, Canadian Economic Association Annual Conference 2010, the 15th LACEA Meetings 2010, the 16th CEF Conference 2010 and Midwest Macroeconomics Meetings 2010. The usual disclaimer applies.

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APPENDIX A: DATA

Deposits distributions. The data used to plot Figure 1 include demand and term deposit account bins that are differentiated by account sizes. Since the data on the distribution of cash are not available for emerging economies, I approximate money balances with demand deposits. For each bin, the share of account balances as well as *number* of accounts (as opposed to *depositors*) is reported. This creates a caveat because if an individual possesses multiple accounts that fall in different bins, then inequality in the distribution of these deposits would be understated. Furthermore, depending on the country-specific institutional arrangements, demand deposits might be dollarized or effectively pay an interest that is closely related to the inflation rate, missing the vulnerability of cash to inflation. Considering that the existing Gini coefficients are already too high, the first caveat is arguably not crucial. The second issue, on the other hand, is difficult to address, since data on the currency composition of deposits as well as cash in circulation are typically not available at the disaggregated level in emerging market economies.

The deposits data sources for listed countries are Autoridad de Supervision del Sistema Financiero (Bolivia), Bulgarian National Bank, Superintendencia de Bancos e Instituciones Financier- as (Chile), National Bank of Georgia, Bank of Lithuania, Central Reserve Bank of Peru, Bank of Thailand, and Banking Regulation and Supervision Agency (BRSA, Turkey). Table A1 presents summary statistics that lead to the plots in Figure 1 regarding the case of Turkey.

Structural disinflation. In order to determine the break points, I regress the time series of inflation for each country on a constant and perform the Chow test that incorporates a structural break date around the mid-1990s. For each country, I search alternative break dates and choose the ones that imply the highest F-statistic in the Chow test. If there is no evidence of a structural break, I just compute averages for 1975-1994 and 1995-2014. In order not to bias the structural change results, I omit data points that correspond to annual inflation rates of more than 200%. Among the 134 countries listed in Table A2, 105 pass the structural break test (at a 99% significance level). Countries that did not pass the test are marked with an asterisk.

APPENDIX B: A REPRESENTATIVE FIRM AND HOMOTHETIC TRANSACTIONS COSTS

The goal of this appendix is twofold. First, I demonstrate that in the absence of aggregate uncertainty, the portfolio of heterogeneous agents between bonds and capital is indeterminate in a small open economy with physical capital accumulation and a representative firm that operates a constant returns-to-scale (CRS) technology. As in standard models, I assume here that heterogeneous households rent capital to a representative firm, which utilizes the aggregate stock of capital and labor in the production process. I then provide an analytical and computational solution algorithm which uses a measure of *the sum of bonds and capital* in approximating for the stationary equilibrium in which aggregate bonds and capital holdings are determined without necessarily pinning down the *idiosyncratic* portfolio between these two assets. Indeed, the portfolio is undefined. Second, I show that when the assumption of non-homothetic transactions costs of consumption is relaxed, the model loses its ability of generating substantial inequality in the distribution of real balances. In the interest of brevity, this appendix will include a sketch of this alternative model and only emphasize the differences of it from the benchmark model in the main text. Furthermore, I only consider stationary equilibrium, since it is straightforward to extend this solution strategy to a transition dynamics equilibrium.

B.1 *Characterization of optimality conditions*

The dynamic program of a household who supply capital and labor to a representative firm that operates under CRS and is in state (a, x) reads,

$$v(a, x; \varepsilon) = \max_{c,h,k',b',m'} \Big[u(c, h) + \beta E \left\{ v\left(a', x'; \varepsilon'\right) |x' \right\} \Big], \tag{B.1}$$

subject to

$$c\Big[1 + \chi\left(\kappa\right)\Big] + b' + m' + k' = (1 - \tau^h) wxh + a + \tau, \tag{B.2}$$

$$a' = \Big[1 + (1 - \tau^k) r_k\Big] k' + R^* b' + \frac{m'}{1 + \varepsilon} \tag{B.3}$$

and

$$c, m', k' > 0,\ 1 > h > 0,\ a', b' \geq \Omega \text{ with } \kappa = c/m' \text{ and } \chi\left(\kappa\right) = \phi \kappa^{\gamma}. \tag{B.4}$$

In this formulation, $\{r_k, w\}$ denote competitive factor prices of rental rate and wage rate. Note also that the definition of κ leads to homothetic transactions costs $c\chi(\kappa)$ in consumption and real balances. Solution to households' problem yields,

$$\psi h^{\eta - 1} = \frac{(1 - \tau^h) wx}{\Big[1 + \phi(1 + \gamma)\kappa^{\gamma}\Big]}, \tag{B.5}$$

$$\lambda = \Big[1 + (1 - \tau^k) r_k\Big] \Big[\beta E\Big\{\lambda' | x'\Big\} + \varphi\Big], \tag{B.6}$$

$$\lambda = R^* \Big[\beta E\Big\{\lambda' | x'\Big\} + \varphi\Big], \tag{B.7}$$

$$\lambda\Big[1 - \chi'(\kappa)\kappa^2\Big] = \left[\frac{1}{1 + \varepsilon}\right] \Big[\beta E\Big\{\lambda' | x'\Big\} + \varphi\Big], \tag{B.8}$$

the budget constraint (B.2), the evolution of total wealth (B.3), with $\kappa = c/m'$ and $\lambda = U_c(c, h) \Big/ \Big[1 + \phi(1 + \gamma)\kappa^{\gamma}\Big]$.

Notice that the absence of aggregate uncertainty ensures that I write the after-tax gross return earned over capital outside of the mathematical expectation taken over the stochastic idiosyncratic labor productivity process in equation (B.6), which stands for the Euler equation for capital. Therefore, I construct the portfolio indeterminacy result by combining this equation with the Euler equation for bonds (B.7) to reach

$$1 + (1 - \tau^k) r_k = R^*. \tag{B.9}$$

Any pair $\{k', b'\}$ consistent with equations (B.2) and (B.3) is an equilibrium portfolio, once this condition, which does not depend on the actual portfolio, holds.

Different from the benchmark model in the main text, I now assume that there is a single representative firm which operates under a CRS Cobb-Douglas technology and faces competitive factor prices. For given rental rate and wage rate, the firm uses the aggregate amount of capital K and labor in efficiency units H to produce the output $Y = K^\alpha H^{1-\alpha}$ of the economy. Capital is installed one period ahead of production and depreciates at the rate δ. Since aggregate TFP is fixed, output will be fixed in stationary equilibrium. Under these standard conditions, competitive factor prices of the firm satisfy the following optimality conditions;

$$\alpha \left(\frac{K}{H} \right)^{\alpha - 1} - \delta = r_k, \tag{B.10}$$

$$(1 - \alpha) \left(\frac{K}{H} \right)^{\alpha} = w. \tag{B.11}$$

Combining the two conditions and using the rate of return equivalence condition (B.9) suggest that the aggregate capital-output ratio should satisfy,

$$\frac{K}{H} = \left(\frac{\alpha}{1 - \alpha} \right) \frac{w}{\left(\frac{R^* - 1}{1 - \tau^k} + \delta \right)} \tag{B.12}$$

Notice that the aggregate capital-labor ratio obtained in the benchmark model coincides with (B.12) despite it assumes decreasing returns-to-scale (DRS) and household level production (see the proof of Lemma 1 below).

B.2 Transformed problem

The rate of return equivalence between capital and bonds can be exploited to simplify the dynamic programming problem of households by replacing these choice variables with their summation $q' = k' + b'$. After this transformation, the dynamic program of households reads,

$$v(a, x; \varepsilon) = \max_{c,h,q',m'} \Big[u(c, h) + \beta E \left\{ v \left(a', x'; \varepsilon' \right) |x' \right\} \Big], \tag{B.13}$$

subject to

$$c \Big[1 + \chi\left(\kappa\right) \Big] + q' + m' = (1 - \tau^h) w x h + a + \tau, \tag{B.14}$$

$$a' = R^* q' + \frac{m'}{1 + \varepsilon} \tag{B.15}$$

and

$$c, m' > 0,\ 1 > h > 0,\ a', q' \geq \Omega \text{ with } \kappa = c/m' \text{ and } \chi\left(\kappa\right) = \phi \kappa^{\gamma}. \tag{B.16}$$

Solution to households' problem yields,

$$\psi h^{\eta - 1} = \frac{(1 - \tau^h) w x}{\Big[1 + \phi(1 + \gamma)\kappa^{\gamma} \Big]}, \tag{B.17}$$

$$\lambda = R^* \left[\beta E \Big\{ \lambda' | x' \Big\} + \varphi \right], \tag{B.18}$$

$$\lambda \Big[1 - \chi'(\kappa)\kappa^2 \Big] = \left[\frac{1}{1 + \varepsilon} \right] \left[\beta E \Big\{ \lambda' | x' \Big\} + \varphi \right], \tag{B.19}$$

the budget constraint (B.14), the evolution of total wealth (B.15), with $\kappa = c/m'$ and $\lambda = U_c(c,h) \Big/ \left[1 + \phi(1+\gamma)\kappa^\gamma\right]$.

Now I proceed to showing that real balances are proportional to consumption under homothetic transactions costs, which makes the distribution of the two identical. Combining Euler equations for total interest-bearing assets (B.18) and real balances (B.19), I obtain

$$\left(\frac{1}{1+\varepsilon}\right)\left(\frac{1}{1-\chi'(\kappa)\kappa^2}\right) = R^*, \tag{B.20}$$

which can also be rewritten as

$$\chi'(\kappa)\kappa^2 = \frac{i}{1+i}, \tag{B.21}$$

using the definition of nominal interest rate $1+i = (1+\varepsilon)R^*$ under the absence of aggregate uncertainty. Given that $\chi(\kappa) = \phi\kappa^\gamma$ and $\kappa = c/m'$, equation (B.21) implies a solution for the consumption velocity of money $c/m' = \left[\frac{1}{\gamma\phi}\left(\frac{i}{1+i}\right)\right]^{\frac{1}{1+\gamma}}$, which does not depend on any idiosyncratic variable. The proportionality between these two variables resembles the implication of the setup studied by Sunel (2013) in which the cross-sectional distributions of real balances and consumption coincide. Obviously, this result is useful for tractability reasons since it makes households' budget constraint linear in consumption. It also implies that since marginal utility of consumption declines as wealth increases, the poor would *unambiguously* hold a financial portfolio that is biased towards real balances. However, it comes at the expense of the model's inability in delivering substantial inequality in the distribution of real balances.

B.3 Solution strategy

I now make use of the analytical properties elaborated above in suggesting a numerical solution strategy to the model with a CRS technology, representative firm.

1. Guess the pair of endogeneous equilibrium objects consisting of real wages and lump-sum transfers $\{w, \tau\}$.

2. Since real interest rate R^*, inflation rate ε and labor-income tax rate τ^h are given and fixed in stationary equilibrium, households' dynamic program enlisted in Section B.2 can be solved for a given double $\{w, \tau\}$. This yields the decision rules of a generic household $\{c(a,x), h(a,x), q'(a,x), m'(a,x)\}$ and the stationary distribution across agents $\Gamma(a,x)$, which is a fixed-point to the following condition;

$$\Gamma(a', x') = \sum_{\left\{a : a' = R^* q' + \frac{m'}{1+\varepsilon}\right\}} \sum_x \Gamma(a,x) p_{x'|x}. \tag{B.22}$$

3. Back out the rental rate of capital by using the rate of return equivalence condition. That is, $r_k = (R^* - 1)/(1 - \tau^k)$. Once the rental rate is obtained, the capital-labor ratio of the representative firm can be solved for by using the competitive capital demand of the firm listed in equation (B.10). This implies that

$$\frac{K}{H} = \left[\frac{r_k + \delta}{\alpha}\right]^{\frac{1}{\alpha - 1}}. \tag{B.23}$$

4. Using the decision rules of households and the stationary distribution, aggregate labor hours in efficiency units can be solved for by $\sum_{a,x} \Gamma(a,x) x h(a,x) = H$. This object can be plugged in equation (B.23) to obtain aggregate capital stock K. Once I have the aggregate measures for both factors of production, I can compute the output of the economy Y by using the Cobb-Douglas production function.

5. The decision rules and stationary distribution allows for aggregating over households' idiosyncratic choice for the sum of capital and bond holdings. That is, $\sum_{a,x} \Gamma(a,x) q'(a,x) = Q' = Q$ in the stationary equilibrium. Since $q = k + b$ and aggregate capital stock K is solved for, aggregate bonds can be backed out by $B = Q - K$.

This establishes the property of this alternative model that without solving for the exact idiosyncratic portfolio between capital and bonds, one can obtain their aggregate equilibrium levels separately.

6. The last step of the solution strategy would only involve iterating on $\{w, \tau\}$ until two conditions hold. First, the real wage has to be consistent with equilibrium capital-output ratio. That is,

$$\frac{K}{H} = \left[\frac{w}{1-\alpha}\right]^{\frac{1}{\alpha}}. \tag{B.24}$$

Second, the government budget constraint has to be in balance. That is,

$$G + \tau = \tau^k r_k K + \tau^h w H + \left(\frac{\varepsilon}{1+\varepsilon}\right) M^s \tag{B.25}$$

where, equilibrium in money market necessitates that the aggregate money supply can be obtained by aggregating over the policy for real balances with $\sum_{a,x} \Gamma(a,x) m'(a,x) = M^s$.

APPENDIX C: PROOFS TO LEMMAS 1 AND 2

Lemma 1. Euler equations for capital (11) and bonds (12) imply that $\Big[(1-\tau^k)(F_k(k_{t+1}, l_{t+1}) - \delta) + 1\Big] = R^*$. Since the production function $F(k_t, l_t) = \Big[k_t^{\alpha} l_t^{1-\alpha}\Big]^{\theta}$ with $0 < \alpha, \theta < 1$ exhibits DRS, it is straightforward to show that the rate of return equivalence between bonds and capital together with equation (10) yields,

$$\frac{k_{t+1}(a_t, x_t)}{l_{t+1}(a_t, x_t)} = q_{t+1}(w_{t+1}; R^*) = \left(\frac{\alpha}{1-\alpha}\right) . \frac{w_{t+1}}{\left(\frac{R^*-1}{1-\tau^k} + \delta\right)} \quad \forall\ (a_t, x_t)\ \ with, \tag{C.1}$$

$$l_{t+1}(a_t, x_t) = \left[\frac{w_{t+1}}{(1-\alpha)\theta q_{t+1}^{\alpha\theta}}\right]^{\frac{1}{\theta-1}} \quad \forall\ (a_t, x_t). \tag{C.2}$$

Q.E.D.

Proof of Lemma 2. Euler equations for bonds (12) and real balances (13) can be combined to obtain

$$\left(\frac{1}{1+\varepsilon_{t+1}}\right)\left(\frac{1}{1-\varrho m_{t+1}^{\varrho-1}\chi'(\kappa_t)\kappa_t^2}\right) = R^*, \tag{C.3}$$

which can also be rewritten as

$$\varrho m_{t+1}^{\varrho-1}\chi'(\kappa_t)\kappa_t^2 = \frac{i_t}{1+i_t}\,, \tag{C.4}$$

using the definition of nominal interest rate $1+i_t = (1+\varepsilon_{t+1})R^*$ under the absence of aggregate uncertainty. Given that $\chi(\kappa_t) = \phi\kappa_t^\gamma$ and $\kappa_t = c_t/m_{t+1}^\varrho$, equation (C.4) implies a solution for the consumption velocity of money, that is, $c_t/m_{t+1} = c_t^{\frac{\gamma(\varrho-1)}{1+\gamma\varrho}}\left[\frac{1}{\varrho\gamma\phi}\left(\frac{i}{1+i}\right)\right]^{\frac{1}{1+\gamma\varrho}}$. It is typical in Bewley-Huggett-Aiyagari class of models that policy function for consumption is increasing in total wealth and idiosyncratic labor productivity. Therefore, since $0<\varrho<1$ and $\gamma,\phi,i>0$ and i is fixed for given inflation and real interest rates, consumption velocity of money decreases with total wealth and idiosyncratic labor productivity. This finding is also confirmed numerically as shown in Figure 4.

Q.E.D.

APPENDIX D: NUMERICAL SOLUTION ALGORITHM FOR THE BENCHMARK MODEL

D.1 Stationary recursive equilibrium

I solve the household optimization problem formulated in Section 4.1 by value function iteration on a discretized state space defined over total wealth and idiosyncratic labor productivity pairs. By Lemma 1, decision rules for capital and labor demand can be solved outside of the value function iteration routine. Lemma 2 on the other hand, suggests a non-linear relationship between consumption and money policies and is useful to replace money choice in the budget constraint to arrive a univariate non-linear equation in consumption for given capital, labor supply and bond choices. Consequently, I only use a grid for bonds choices while iterating on the value and use the Newton method for univariate non-linear equations to obtain implied consumption policies. The grids that I use for wealth (in $[\Omega,\ 24.8]$), productivity (in $\{4.74, 0.848, 0.17\}$), and bonds (in [0, 15]) have 400, 3, and 2000 nodes, respectively.

The solution algorithm that I implement to compute the stationary recursive equilibrium involves the following steps:

1. Fix two pairs of real wage rate and lump-sum transfers, $w^1 > w^2$ and $\tau^1 > \tau^2$, for which the solution of the model implies an excess supply of (demand for) labor at w^1 (w^2) and a deficit (surplus) in the government budget constraint at τ^1 (τ^2). Set $w = w^1$ and $\tau = \tau^1$.

2. Use Lemma 1 to compute homogeneous policy functions for capital and labor demand k', l' as functions of the real wage rate w and the real interest rate R^*. Given k' and l', compute idiosyncratic output, $F(k', l')$ and the after-tax gross return to capital, $k' + (1 - \tau^k) [F(k', l') - \delta k' - w l']$.

3. Start the value function iteration routine: For each bonds choice over the grid, use policies for capital and labor demands to obtain candidate policies for labor supply and consumption by using the Newton algorithm in solving the non-linear budget constraint. The policy for real balances is implied by Lemma 2. For each triple of real balances, bonds and capital choice, keep track of the law of motion of total wealth and linearly interpolate the next iteration's value by using this law of motion. Re-iterate on the value function until the difference between the obtained maximized value and the one in the previous iteration is less then 10^{-6}.

4. Once the decision rules are computed, solve for the stationary distribution over wealth and productivity $\Gamma(a, x)$ by employing standard methods. Aggregate over heterogeneous agents by using the obtained distribution, and compute the public surplus $PS = \tau^k \Big(Y - \delta K - wL \Big) + \tau^h wH + \left(\frac{\varepsilon}{1+\varepsilon}\right) M^s - G - \tau$ and the excess supply of labor $H - L$.

5. Depending on the sign of the implied public surplus and excess supply of labor, update the boundaries $[w^1, w^2]$ and $[\tau^1, \tau^2]$ akin to a bisection algorithm. Finalize the procedure if $\max \Big\{ |PS|, |H - L| \Big\} < 10^{-4}$. Otherwise, update w, τ by using the new boundaries and go back to step 2.

D.2 *Transition dynamics equilibrium*

The numerical solution algorithm of the transition dynamics exercise involves the following steps:

1. Solve for the stationary equilibria that correspond to high (14%) and low (2%) inflation rates by following the algorithm presented in Appendix D.1. In the following, variables with superscript 0 (1) represent the permanent value of that variable in the high (low) inflation regime. Store $v_0(a, x) = v^0(a, x), \Gamma_0(a, x) = \Gamma^0(a, x), K_0 = K^0, L_0 = L^0, Y_0 = Y^0, B_0 = B^0, M_0 = M^0$ as initial conditions and $v^1(a, x), \Gamma^1(a, x), \tau^1, w^1$ as terminal conditions.

2. Feed the calibrated time profile for inflation rates,

$$\varepsilon_0 = 14\% \tag{D.1}$$

$$\{\varepsilon_t\}_{t=1}^{T_1} = \{\varepsilon_t^1\}_{t=1}^{T_1}$$

where $\{\varepsilon_t^1\}_{t=1}^{T_1}$ is the finite sequence of inflation rates that satisfies, $\varepsilon_0 > \varepsilon_1^1 > \varepsilon_2^1 ... > \varepsilon_T^1$ and $\{\varepsilon_t^1\}_{t=T+1}^{T_1} = 2\%$ for finite T and T_1. I set $T = 27$ and $T_1 = 570$ in the benchmark experiment.

3. Set $\tau_{T_1} = \tau^1$ and $w_{T_1} = w^1$. Guess a sequence of lump-sum transfers and real wages $\{\tau_t', w_t'\}_{t=0}^{T_1-1}$. Set $v_{T_1}(a,x) = v^1(a,x)$. Solve for the sequence of functions $\{v_t, k_{t+1}, l_{t+1}, b_{t+1}, m_{t+1}, c_t, h_t\}_{t=0}^{T_1-1}$ by backward recursion. The solution takes as given the guessed sequences for transfers and real wages, as well as the terminal value function.

4. Compute the sequence of distributions over total wealth and labor productivity, $\left\{\Gamma_t(a_t, x_t)\right\}_{t=1}^{T_1-1}$ by using the Markov transition probabilities of the idiosyncratic productivity process and the policy functions for total wealth $a_{t+1} = (1-\tau^k)\left[F(k_{t+1}, l_{t+1}) - \delta k_{t+1} - w_{t+1} l_{t+1}\right] + k_{t+1} + R^* b_{t+1} + \frac{m_{t+1}}{1+\varepsilon_{t+1}}$.

$$\Gamma_{t+1} = \sum_{\left\{a: a_{t+1} = (1-\tau^k)\left[y_{t+1} - \delta k_{t+1} - w_{t+1} l_{t+1}\right] + k_{t+1} + R^* b_{t+1} + \frac{m_{t+1}}{1+\varepsilon_{t+1}}\right\}} \sum_x \Gamma_t p_{x_{t+1}|x_t},$$

with $y_{t+1} = F(k_{t+1}, l_{t+1})$ and $\Gamma_t = \Gamma_t(a_t, x_t)\ \ \forall t$.

5. Use the obtained decision rules and distributions to do aggregation as described in Section 4.1. Compute $\{K_{t+1}, L_{t+1}, Y_t, B_{t+1}, M_{t+1}, C_t, H_t, TC_t\}_{t=0}^{T_1-1}$.

6. Compute the sequences of public surpluses, $\left\{PS_t\right\}_{t=0}^{T_1-1} = \left\{\tau^k\left(Y_t - \delta K_t - w_t' L_t\right) + \tau^h w_t' H_t + M_{t+1} - \frac{M_t}{1+\varepsilon_t} - G - \tau_t'\right\}_{t=0}^{T_1-1}$ and excess demands in the labor market, $\left\{ED_t^H\right\}_{t=0}^{T_1-1} = \left\{L_t - H_t\right\}_{t=0}^{T_1-1}$. Update the guesses for transition dynamics equilibrium sequences of transfers and real wages;

$$\{\tau_t''\}_{t=0}^{T_1-1} = \left\{\tau_t' + \chi PS_t\right\}_{t=0}^{T_1-1} \tag{D.2}$$

$$\{w_t''\}_{t=0}^{T_1-1} = \left\{w_t' + \chi ED_t^H\right\}_{t=0}^{T_1-1} \tag{D.3}$$

for $0 < \chi < 1$. In the baseline case, $\chi = 0.05$.

7. For a small tolerance value ζ, proceed to the next step if $\max\Big\{\max\Big\{|\tau'_{T_1-1} - \tau^1|, |w'_{T_1-1} - w^1|\Big\}, sup||\Gamma_{T_1-1} - \Gamma^1||\Big\} < \zeta$. Otherwise, increase T_1 and go back to step 2.

8. If $\max\Big\{\max\Big\{|\tau''_t - \tau'_t|, |PS_t|\Big\}_{t=0}^{T_1-1}, \max\Big\{|w''_t - w'_t|, |ED^H_t|\Big\}_{t=0}^{T_1-1}\Big\} < \zeta$, the transition equilibrium is solved for. Otherwise, set $\Big\{\tau'_t, w'_t\Big\}_{t=0}^{T_1-1} = \Big\{\tau''_t, w'_t\Big\}_{t=0}^{T_1-1}$ and go back to step 3. In the benchmark economy, I set $\zeta = 0.0015$.

The numerical solution algorithm for the declining interest rates and revenue neutral disinflation economies involve similar steps to those of the baseline economy. In the former, I replace the constant real interest rate R^* with an exogeneous sequence $\Big\{R^*_t\Big\}_{t=0}^{T_1-1}$, which displays positive correlation with the sequence of inflation rates. In the latter, I fix $\Big\{\tau_t\Big\}_{t=0}^{T_1-1} = \tau^0$ and endogenize labor tax rates $\Big\{\tau^h_t\Big\}_{t=0}^{T_1-1}$, and solve for transition dynamics equilibrium sequence for taxes (instead of lump-sum transfers) in the spirit of the algorithm described above.

APPENDIX E: DISINFLATION UNDER REVENUE NEUTRALITY

Table E1 reports the steady state implications of revenue neutral disinflation. Transfers-to-GDP ratio is roughly constant and labor income tax rate increases to 21.5% when inflation is permanently reduced to 2%. The rise in distortionary labor income tax rate does not warrant significant departures in the aggregates of this economy from the baseline case. Distributional and welfare consequences of disinflation under revenue neutrality also resemble those delivered by the benchmark model (see Tables E2 and E3). The only difference is that the rise in labor income tax rates redistributes part of welfare gains from the high-productivity and wealthy households to low-productivity and wealth poor households. This causes the additional distortion introduced to the consumption-leisure margin to exactly offset the cost of reducing insurance provided to the poor in the benchmark experiment.

Figure E1 shows the transition dynamics of selected macroeconomic variables when labor tax rates respond to disinflation. The main adjusting variable, labor income tax rate follows a convex increasing trajectory toward its terminal stationary level during disinflation. This is because the government responds by increasing the labor income tax rate to make up for the missing inflation tax revenues. Recall that the sharp decline in transfers in the baseline economy towards the end of the disinflation episode creates a decline in consumption and a reversal in trade deficit. Since this dynamic, which predominantly affects the consumption of the poor is absent under the revenue neutral exercise, consumption and net exports follow a smoother trajectory than the benchmark experiment. Other macroeconomic variables evolve along disinflation as they do under the baseline experiment.

LITERATURE CITED

Sunel, Enes. (2013) “Distributional and Welfare Consequences of Disinflation in Emerging Economies.” Central Bank of the Republic of Turkey Working Paper Series No. 13.

Table A1: Summary statistics on the distribution of deposits in Turkey

Turkey (2002-2008)	Account sizes up to 10K[a]	10K-50K	50K-250K	250K-1,000K	1,000K and up
Demand deposits					
Share of balances	21.11[b]	17.13	19.32	13.02	29.41
Share of account numbers	97.32	1.97	0.59	0.08	0.02
Term deposits					
Share of balances	9.62	18.45	22.21	13.02	36.28
Share of account numbers	70.44	21.16	7.12	0.96	0.30
Share of term deposits within the account bin	64.03	82.72	83.54	81.29	84.13

[a]In Turkish liras. Source: Banking Regulation and Supervision Agency.
[b]In percentage terms, the average over the period.

Table A2: Disinflation as a worldwide phenomenon

Country	High	Low	Country	High	Low
Brazil	135[a] (60-94)[b]	10 (95-14)	Gambia*	14 (75-94)	5 (95-14)
Argentina	115 (75-94)	7 (95-14)	Myanmar*	14 (75-94)	20 (95-14)
Uganda	106 (60-88)	12 (89-14)	Egypt	14 (72-95)	7 (96-14)
Zambia	104 (60-93)	20 (94-14)	Guatemala	14 (73-90)	8 (91-14)
Indonesia	96 (60-69)	12 (70-14)	Cote D.	14 (72-79)	5 (80-14)
Israel	91 (77-86)	7 (87-14)	Swaziland	14 (73-94)	8 (95-14)
Sierra Leo.	75 (81-91)	16 (92-14)	Algeria	14 (75-94)	6 (95-14)
Peru	71 (74-91)	7 (92-14)	Honduras	14 (79-97)	8 (98-14)
Congo, Dem.	69 (75-97)	14 (98-14)	Spain	13 (71-87)	3 (88-14)
Ghana	66 (74-83)	22 (84-14)	Gabon	13 (73-81)	3 (82-14)
Uruguay	63 (75-94)	12 (95-14)	Samoa	13 (71-86)	5 (87-14)
Turkey	60 (77-02)	10 (03-14)	South Af.	12 (71-95)	6 (96-14)
Sudan	60 (78-96)	14 (97-14)	New Zealand	12 (70-86)	3 (87-14)
Mexico	53 (74-88)	11 (89-14)	Trinidad & T.	12 (72-93)	6 (94-14)
Guinea-B.	51 (60-96)	5 (97-14)	Barbados	12 (60-83)	4 (84-14)
Venezuela	51 (87-97)	24 (98-14)	Ireland	12 (67-86)	2 (87-14)
Emerg.&Dev.	49 (79-95)	9 (96-14)	Haiti*	12 (75-94)	14 (95-14)
Mozambique	46 (60-94)	13 (95-14)	Papua N.G.	12 (60-03)	4 (04-14)
Ecuador	42 (82-00)	7 (01-14)	Botswana	12 (60-93)	8 (94-14)
Nigeria	41 (87-95)	12 (96-14)	Sri Lanka*	11 (75-94)	9 (95-14)
Suriname	38 (86-00)	12 (01-14)	St. Lucia	11 (60-80)	3 (81-14)
Poland	37 (81-96)	5 (97-14)	Solomon I.*	11 (75-94)	8 (95-14)
Iceland	36 (71-88)	6 (89-14)	Thailand	11 (72-82)	3 (83-14)
Bolivia	35 (73-83)	8 (84-14)	Dominica	11 (60-81)	3 (82-14)
Chile	29 (74-90)	6 (91-14)	Tonga	11 (60-92)	5 (93-14)
Dom. Rep.	28 (83-90)	11 (91-14)	Pakistan	10 (73-97)	8 (98-14)
Tanzania	25 (74-95)	17 (96-14)	Neth. Ant.	10 (72-81)	3 (82-14)
Colombia	24 (72-94)	8 (95-14)	Burundi*	10 (75-94)	12 (95-14)
Jamaica	23 (73-96)	10 (97-14)	UK	10 (70-91)	3 (92-14)
Lao P.D.R.*	22 (75-94)	21 (95-14)	Bhutan	10 (60-98)	6 (99-14)
Nicaragua	22 (60-93)	9 (94-14)	Ethiopia*	10 (75-94)	10 (95-14)
Costa Rica	21 (72-82)	13 (83-14)	Nepal*	10 (75-94)	7 (95-14)
Congo, Rep.	21 (94-98)	3 (99-14)	Senegal	9 (60-85)	2 (86-14)

[a] In percentage terms. Source: International Financial Statistics, IMF.
[b] The numbers in parentheses denote years for which the average inflation is computed.

Table A2 continued

Country	High	Low	Country	High	Low
Saudi Arab.	20[a] (72-77)[b]	1 (78-14)	Australia	9 (71-90)	3 (91-14)
Hungary	20 (86-98)	5 (99-14)	Denmark	9 (71-85)	2 (86-14)
Iran, I.R.	18 (71-95)	19 (96-14)	Rwanda*	9 (75-94)	7 (95-14)
Malawi*	18 (74-94)	22 (95-14)	Niger	9 (60-82)	2 (83-14)
Paraguay	18 (72-95)	7 (96-14)	Cameroon*	9 (75-94)	3 (95-14)
Portugal	18 (71-91)	3 (92-14)	Fiji	9 (60-87)	4 (88-14)
Madagascar	17 (74-96)	9 (97-14)	Morocco	9 (71-86)	3 (87-14)
Syrian A.R.	17 (73-94)	6 (95-14)	France	9 (68-85)	2 (86-14)
Maldives	17 (60-93)	4 (94-14)	Libya*	8 (75-94)	2 (95-14)
Philippines	17 (70-85)	6 (86-14)	Sweden	8 (70-91)	1 (92-14)
Mauritius	17 (72-80)	7 (81-14)	Vanuatu	8 (60-88)	3 (89-14)
Seychelles	16 (60-80)	5 (81-14)	Jordan*	8 (75-94)	4 (95-14)
World	16 (75-94)	6 (95-14)	China, H.K.	8 (60-97)	1 (98-14)
Grenada	16 (60-83)	3 (84-14)	Norway	8 (70-91)	2 (92-14)
Zimbabwe*	16 (74-94)	53 (95-14)	India*	8 (75-94)	7 (95-14)
Kenya	16 (73-93)	10 (95-14)	China, M.*	8 (75-94)	3 (95-14)
Greece	16 (71-97)	3 (98-14)	Panama	8 (72-80)	2 (81-14)
Italy	15 (73-85)	3 (86-14)	Togo*	8 (75-94)	4 (95-14)
Korea	15 (60-82)	4 (83-14)	Finland	8 (60-90)	2 (91-14)
Bahrain,K.	15 (73-80)	2 (81-14)	Cyprus	7 (71-85)	3 (86-14)
El Salvador	15 (71-95)	3 (96-14)	Advanced	7 (60-90)	2 (91-14)
Mauritania*	7 (75-94)	6 (95-14)	Bangladesh*	6 (75-94)	7 (95-14)
Japan	7 (60-81)	1 (82-14)	Malaysia	6 (70-82)	3 (83-14)
Malta	7 (71-82)	2 (83-14)	Belgium	5 (60-85)	2 (86-14)
Singapore	7 (71-82)	2 (83-14)	Chad*	5 (75-94)	4 (95-14)
U.S.	7 (70-85)	3 (86-14)	Austria	4 (60-92)	2 (93-14)
Burkina F.*	7 (75-94)	3 (95-14)	Aruba*	4 (75-94)	3 (95-14)
Canada	7 (71-91)	2 (92-14)	St. Kitts &N.*	4 (75-94)	4 (95-14)
St. Vincent	7 (60-91)	3 (92-14)	Switzerland	4 (60-93)	1 (94-14)
Luxembourg	7 (71-85)	2 (86-14)	Cent Af.*	4 (75-94)	3 (95-14)
Cape Verde	7 (60-97)	2 (98-14)	Belize*	4 (75-94)	2 (95-14)
Tunisia	7 (60-94)	4 (95-14)	Kuwait*	3 (75-94)	3 (95-14)
Bahamas	6 (60-92)	2 (93-09)	Qatar*	3 (75-94)	4 (95-14)
Netherlands	6 (60-84)	2 (85-14)	Equatorial G.*	1 (75-94)	6 (95-14)

[a] In percentage terms.
[b] The numbers in parentheses denote years for which the average inflation is computed.

Table E1: Steady state implications of disinflation (14% $\rightarrow$ 2%)

Revenue neutral disinflation

Aggregates	$\varepsilon = 14\%$, $\tau^h = 21\%$	$\varepsilon = 2\%$, $\tau^h = 21.5\%$
Y	0.786	0.788
H	0.329	0.330
K	4.220	4.235
w	1.480	1.477
C/M	15.538	6.116
τ/Y	4.15%	4.14%
C/Y	69.52%	70.63%
NX/Y	-4.31%	-4.92%
TC/Y	1.20%	0.79%
I/Y	20.15%	20.15%
$\frac{\varepsilon}{1+\varepsilon} M/Y$	0.55%	0.23%
$B/(B+M)$	98.39%	96.42%

Table E2: Distributional consequences of disinflation (14% → 2%)

Revenue neutral disinflation		
Gini coefficients[a]	$\varepsilon = 14\%$, $\tau^h = 21\%$	$\varepsilon = 2\%$, $\tau^h = 21.5\%$
Financial wealth	79	76
Bonds	79	77
Real balances	57	58
Consumption	23	24
Income	24	24

[a]Computed by using model generated Lorenz curves. In percentages.

Table E3: Welfare consequences of disinflation (14% → 2%)

	(1)	(2)		(3)	(4)
Revenue neutral disinflation	*Aggregating distribution*				
Welfare Gains	$\Gamma^0(a,x)$	$\Gamma^1(a,x)$		$\Gamma^0(a,x)$	$\Gamma^1(a,x)$
Aggregate, *EG*	0.37	0.37		-	-
Aggregate, *U*	0.39	0.39		-	-
Wealth			*Productivity*		
Bottom 20%	0.35	0.35	*L*	0.29	0.29
Median	0.43	0.43	*M*	0.36	0.36
Top 1%	1.11	1.05	*H*	1.23	1.15

[a]Welfare gains are computed as percentage changes that compensate variation in consumption.
[b]Average welfare gains of percentiles are ordered according to total wealth and labor productivities, respectively.
[c]Panels 2, 3 and 4 compare weighted welfare gains from transition for initial and terminal wealth distributions as aggregators.

Figure E1: Transition dynamics of disinflation under revenue neutrality