

1 The econometric model

1.1 Model description

Observable variables:

y_t^1, \dots, y_t^N	real activity indicators
π_t	inflation
x_t	noisy proxy for trend inflation z_t
v_t	a regressor in the inflation equation

State variables:

g_t	output gap (common stationary component of y 's)
w_t^1, \dots, w_t^N	trends of the real activity indicators
$\delta_t^1, \dots, \delta_t^N$	drifts of the trends w
z_t	trend inflation
q_t	$= v_t$ state variable containing the regressor in the inflation equation
p_t	$= \pi_t$ state variable containing inflation

$$y_t^n = b^n(L)g_t + w_t^n + \varepsilon_t^n, \quad \text{for } n = 1, \dots, N, \quad (1a)$$

$$(\pi_t - z_t) = a_g(L)g_t + a_p(L)(\pi_{t-1} - z_{t-1}) + a_q(L)q_t + \varepsilon_t^\pi, \quad (1b)$$

$$x_t = c_0 + c_1 z_t + \varepsilon_t^x, \quad (1c)$$

$$v_t = q_t, \quad (1d)$$

where $\varepsilon_t^n, \varepsilon_t^\pi, \varepsilon_t^x$ are independent Gaussian errors, $b^n(L)$ and $a_\bullet(L)$ are polynomials in the lag operator L , and c_0 and c_1 are parameters.

The laws of motion for the state variables are

$$g_t = \phi(L)g_{t-1} + \eta_t^g, \quad (2a)$$

$$w_t^n = \delta_t^n + w_{t-1}^n + \eta_t^n, \quad \text{for } n = 1, \dots, N, \quad (2b)$$

$$\delta_t^n = \delta_{t-1}^n + \zeta_t^n, \quad \text{for } n = 1, \dots, N, \quad (2c)$$

$$z_t = d_z + f_z(L)z_{t-1} + \eta_t^z, \quad (2d)$$

$$q_t = d_q + f_q(L)q_{t-1} + \eta_t^q, \quad (2e)$$

where $\eta_t^g, \eta_t^n, \zeta_t^n, \eta_t^z, \eta_t^q$ are independent Gaussian errors, and ϕ_1, ϕ_2, d^n, d^z and f are coefficients.

1.2 The model in state-space form

Observation equation (for the case when the lag polynomials $a_g(L)$ and $B(L)$ are of order 3)

$$\begin{pmatrix} \pi_t \\ y_t \\ x_t \\ v_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ c_0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ B_1 & B_2 & B_3 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} g_{t+1} \\ g_t \\ g_{t-1} \\ w_t \\ \delta_t \\ z_t \\ q_t \\ p_t \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon_t^y \\ \varepsilon_t^x \\ 0 \end{pmatrix} \quad (3)$$

The shocks in the observation equations are mutually uncorrelated

$$\begin{pmatrix} \varepsilon_t^y \\ \varepsilon_t^x \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \text{diag}(\sigma_y^2) & 0 \\ 0 & \text{diag}(\sigma_x^2) \end{pmatrix} \right),$$

where σ_y^2 and σ_x^2 are vectors of length N_y and N_x respectively.

State equation

$$\begin{pmatrix} g_{t+2} \\ g_{t+1} \\ g_t \\ w_{t+1} \\ \delta_{t+1} \\ z_{t+1} \\ q_{t+1} \\ p_{t+1} \\ p_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ d_z \\ d_q \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & F_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_q & 0 & 0 \\ a_{g1} & a_{g2} & a_{g3} & 0 & 0 & a_z & a_q & a_{p1} & a_{p2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} g_{t+1} \\ g_t \\ g_{t-1} \\ w_t \\ \delta_t \\ z_t \\ q_t \\ p_t \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} \eta_t^g \\ 0 \\ 0 \\ \eta_t^w \\ \eta_t^\delta \\ \eta_t^z \\ \eta_t^q \\ \eta_t^p \\ 0 \end{pmatrix} \quad (4)$$

The shocks in the state equation are also mutually uncorrelated

$$\begin{pmatrix} \eta_t^g \\ \eta_t^w \\ \eta_t^\delta \\ \eta_t^z \\ \eta_t^q \\ \eta_t^p \end{pmatrix} \sim N(0, \Sigma).$$

Σ is a diagonal variance matrix of dimension N_Σ . Note that N_Σ is smaller than the number of state equations, since not all the state equations contain shocks.

The shocks in the observation equation are uncorrelated with the shocks in the state equation.

The model contains the following parameters.

Factor model for real activity (1a): $B = (B_1, B_2, B_3), \sigma_y^2$

Phillips curve (1b): $a_g(L), a_z(L), a_p(L), \sigma_\pi^2$

Other drivers of inflation (1c): c, σ_x^2

Other state equations (2a-2e): $\phi = (\phi_1, \phi_2)', d = (d'_{z0}, d'_z)', F, \Sigma$

1.3 Specifying the prior

The Matlab files with names beginning with ‘`mod...`’ specify a structure `prior` that contains all the specification choices. The structure has the following fields:

`prior.pspec` - Defines the Phillips curve.

`prior.pspec.ap_mean` - Prior mean of the coefficients of the lagged inflation gap. The length of `ap_mean` determines the number of lags.

`prior.pspec.ap_tightness` - Prior standard deviation.

`prior.pspec.ag_mean` - Prior mean of the coefficients of g_t . The length of `ag_mean` determines the number of lags.

`prior.pspec.ag_tightness` - Prior tightness. The ‘tightness’ times the ratio of standard deviations of inflation and gdp growth gives the prior standard deviation (see Appendix B).

`prior.xspec` - Defines the observation equation for x_t .

`prior.xspec.ME_b_mean` - Prior mean of the coefficients of the Measurement Equation (ME) of x_t . The coefficients are 1) constant term c_0 , and 2) the slope c_1 .

`prior.xspec.ME_b_std` - Prior standard deviations the same coefficients.

`prior.xspec.ME_sig2` - Prior mean of the standard deviation of the error term in this equation *as the fraction of the variance of the underlying observable* (see Appendix B).

`prior.xspec.LM_b_mean` - Prior mean of the coefficients of the Law of Motion (LM) of the state corresponding to x_t , i.e. of z_t . The coefficients are 1) constant term d_z , and 2) the autoregressive coefficients f_z . The length of `LM_b_mean` determines the number of lags.

`prior.xspec.LM_b_std` - Prior standard deviations the same coefficients.

`prior.xspec.LM_sig2` - Prior mean of the standard deviation of the error term in this equation *as the fraction of the variance of the underlying observable* (see Appendix B).

... etc. The remaining settings are either analogous or explained in the documentation of the Matlab file.