Online Appendix

Housing boom-bust cycles and asymmetric macroprudential policy

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Appendix A First order conditions

A.1 Patient households

The first-order conditions patient households are:

$$\frac{1-\varrho}{C_{s,t}-\varrho C_{s,t-1}} = \beta_s\,\mathbb{E}_t\left\{\left(\frac{1-\varrho}{C_{s,t+1}-\varrho C_{s,t}}\right)\frac{R_t}{\pi_{t+1}}\right\} \tag{1}$$

$$q_t\left(\frac{1-\varrho}{C_{s,t}-\varrho C_{s,t-1}}\right) = \frac{\mathrm{j}_t}{H_{s,t}} + \beta_s\,\mathbb{E}_t\left\{q_{t+1}\left(\frac{1-\varrho}{C_{s,t+1}-\varrho C_{s,t}}\right)\right\} \tag{2}$$

$$w_{s,t}\left(\frac{1-\varrho}{C_{s,t}-\varrho C_{s,t-1}}\right) = \tau N_{s,t}^{\varphi} \tag{3}$$

Equation (1) is the typical Euler equation over lending and equation (2) is the Euler equation specifying demand for housing. Savers aim to smoothen consumption by matching the return on saving to the cost of foregone consumption. Given that housing is a durable good, it not only increases utility in the current period but it also increases the amount of resources available in the next period, through its resale value. Equation (3) defines labour supply by equating marginal utilities over consumption and leisure.

A.2 Impatient households

First-order conditions for impatient households are:

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$$\frac{1-\varrho}{C_{b,t}-\varrho C_{b,t-1}} = \beta_b \,\mathbb{E}_t \left\{ \left(\frac{1-\varrho}{C_{b,t+1}-\varrho C_{b,t}} \right) \frac{R_t}{\pi_{t+1}} \right\} + R_t \mu_t \tag{4}$$

$$q_t \left(\frac{1-\varrho}{C_{b,t}-\varrho C_{b,t-1}} \right) = \frac{\mathsf{j}_t}{H_{b,t}} + \beta_b \,\mathbb{E}_t \left\{ q_{t+1} \left(\frac{1-\varrho}{C_{b,t+1}-\varrho C_{b,t}} \right) \right\}$$
$$+ \mu_t \,\mathbb{E}_t \left\{ \mathsf{m}_t q_{t+1} \pi_{t+1} \right\} \tag{5}$$

$$w_{b,t} \left(\frac{1-\varrho}{C_{b,t}-\varrho C_{b,t-1}} \right) = \tau N_{b,t}^{\varphi} \tag{6}$$

where $\mu_t > 0$ is the Lagrange multiplier on the borrowing constraint. Equations (4)-(6) are the Euler equations over borrowing and housing demand respectively and the intratemporal labour supply equation. It can be seen that the borrowing constraint introduces a wedge between the marginal benefit and marginal cost of decisions. Borrowers are constrained by their borrowing limit and are therefore not able to fully smoothen consumption, making them unable to adjust fully in the wake of shocks. This implies that they have a higher marginal propensity to consume out of current income than savers. Note that shocks to housing preferences j_t generate an immediate response in housing demand and house prices, for both household types.[1]

A.3 Intermediate goods firms

The first order conditions characterising optimal labour demand are:

$$n_{s,j,t} = \alpha \frac{MC_{j,t} y_{j,t}}{w_{s,t}} \tag{7}$$

$$n_{b,j,t} = (1-\alpha) \frac{MC_{j,t} y_{j,t}}{w_{b,t}} \tag{8}$$

Using these optimality conditions, we can define marginal costs as

$$MC_t = \left(\frac{w_{s,t}}{\alpha} \right)^{\alpha} \left(\frac{w_{b,t}}{1-\alpha} \right)^{1-\alpha} \tag{9}$$

As marginal costs of production do not depend on characteristics of any firm j, and since technology is symmetric across all firms, I drop the subscript j in (7) and (8) to ease notation.

[1] Solving for H_s in (2), we get:

$$H_{s,t} = \mathsf{j}_t \left(\frac{\widehat{C}_{s,t}}{q_t} - \beta_s \,\mathbb{E}_t \left\{ \frac{\widehat{C}_{s,t+1}}{q_{t+1}} \right\} \right)$$

where $\widehat{C}_{s,t} = \frac{C_{s,t}-\varrho C_{s,t-1}}{1-\varrho}$. Housing demand $H_{s,t}$ increases as the preference term j_t rises. The same holds for borrowers, although their housing demand function also includes the shadow price of the borrowing constraint. In that case an increase in the LTV ratio m_t or an increase in inflation π_t also increases housing demand by borrowers, as both of these variables relax the borrowing constraint. The former directly, by increasing outright the borrowing limit, and the latter by reducing the real burden of debt.

A.4 Aggregate output

Following Yun (1996) and Christiano, Trabandt, and Walentin (2011), let Y_t^* be the unweighed sum of output from intermediate goods firms. Since all firms use labour in the same proportions, this can be written as

$$Y_t^* = \int_0^1 y_{j,t}\, dj = \int_0^1 n_{s,j,t}^{\alpha} n_{b,j,t}^{1-\alpha}\, dj = A_t N_{s,t}^{\alpha} N_{b,t}^{1-\alpha}$$

Alternatively, summing over the demand across all intermediate firms and equating Y_t^*:

$$\begin{aligned} Y_t^* &= \int_0^1 y_{j,t}\, dj = Y_t \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\sigma} dj \\ \Rightarrow \quad Y_t &= \frac{Y_t^*}{s_t} = \frac{1}{s_t} N_{s,t}^{\alpha} N_{b,t}^{1-\alpha} \end{aligned} \tag{10}$$

where $s_t = \int_0^1 \left(\frac{p_{j,t}}{P_t}\right)^{-\sigma} dj > 1$ is the measure of output cost of price dispersion, which reduces aggregate output compared with an economy with flexible prices (Yun 1996).[2] This measure can be written recursively as:

$$\begin{aligned} s_t &= (1-\omega)\left(\frac{p_t^*}{P_t}\right)^{-\sigma} + \omega \pi_t^{\sigma} s_{t-1} \\ &= (1-\omega)\left(\frac{1-\omega\pi_t^{\sigma-1}}{1-\omega}\right)^{\frac{\sigma}{\sigma-1}} + \omega \pi_t^{\sigma} s_{t-1}. \end{aligned} \tag{11}$$

Appendix B Derivation of price setting behaviour

The maximisation problem faced by price setting firms is:

$$\max_{p_{j,t}} \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \omega^i \Lambda_{i,t+i} Y_{t+i} \left[\left(\frac{p_{j,t}}{P_{t+i}}\right)^{1-\sigma} - MC_{t+i} \left(\frac{p_{j,t}}{P_{t+i}}\right)^{-\sigma} \right] \right\}$$

Maximising with respect to $p_{j,t}$, and multiplying out all constants with respect to the sum:

$$\begin{aligned} &\mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \omega^i \Lambda_{i,t+i} Y_{t+i} (1-\sigma) \left(\frac{p_{j,t}}{P_{t+i}}\right)^{-\sigma} \left[\left(\frac{p_{j,t}}{P_{t+i}}\right) - \frac{\sigma}{\sigma-1} MC_{t+i} \right] \right\} = 0 \\ &\Rightarrow \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \omega^i \Lambda_{i,t+i} Y_{t+i} P_{t+i}^{\sigma} \left[\left(\frac{p_{j,t}}{P_{t+i}}\right) - \frac{\sigma}{\sigma-1} MC_{t+i} \right] \right\} = 0 \end{aligned}$$

Using the definition for the stochastic discount factor, and noting that $\widetilde{C}_{s,t}$ is constant with respect to the problem, we get:

$$\mathbb{E}_t \left\{ \sum_{i=0}^{\infty} (\omega\beta)^i \frac{Y_{t+i}}{\widetilde{C}_{s,t+i}} P_{t+i}^{\sigma} \left[\left(\frac{p_{j,t}}{P_{t+i}}\right) - \frac{\sigma}{\sigma-1} MC_{t+i} \right] \right\} = 0$$

[2] Note that this variable drops out from any linear approximations of the model around a point, as the variance of prices has only second-order effects on output.

The price p_t^* which solves this can be written as:

$$p_t^* = \left(\frac{\sigma}{\sigma-1}\right)\frac{\mathbb{E}_\mathrm{t}\left\{\sum_{i=0}^{\infty}(\omega\beta_s)^i\frac{Y_{t+i}}{\widetilde{C}_{s,t+i}}MC_{t+i}P_{t+i}^{\sigma}\right\}}{\mathbb{E}_\mathrm{t}\left\{\sum_{i=0}^{\infty}(\omega\beta_s)^i\frac{Y_{t+i}}{\widetilde{C}_{s,t+i}}P_{t+i}^{\sigma-1}\right\}} \tag{12}$$

where the subscript j is dropped since all firms have the same technology and face the same demand curve, and hence will optimise in the same way. Multiplying both sides by P_t^{-1} we get relative prices.[3]

$$\frac{p_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\frac{\mathbb{E}_\mathrm{t}\left\{\sum_{i=0}^{\infty}(\omega\beta_s)^i\frac{Y_{t+i}}{\widetilde{C}_{s,t+i}}MC_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\sigma}\right\}}{\mathbb{E}_\mathrm{t}\left\{\sum_{i=0}^{\infty}(\omega\beta_s)^i\frac{Y_{t+i}}{\widetilde{C}_{s,t+i}}\left(\frac{P_{t+i}}{P_t}\right)^{\sigma-1}\right\}} \tag{13}$$

Following Christiano, Trabandt, and Walentin (2011) and **ascari2014macroeconomics**, it is useful to represent the New Keyesian Phillips curve as:

$$\frac{p_t^*}{P_t} = \left(\frac{\sigma}{\sigma-1}\right)\frac{\mathbb{E}_\mathrm{t}\left\{\sum_{i=0}^{\infty}(\omega\beta_s)^i\frac{Y_{t+i}}{\widetilde{C}_{s,t+i}}MC_{t+i}\Theta_{t,t+i}^{\sigma}\right\}}{\mathbb{E}_\mathrm{t}\left\{\sum_{i=0}^{\infty}(\omega\beta_s)^i\frac{Y_{t+i}}{\widetilde{C}_{s,t+i}}\Theta_{t,t+i}^{\sigma-1}\right\}} = \left(\frac{\sigma}{\sigma-1}\right)\frac{\Upsilon_t}{\Phi_t} \tag{14}$$

where $\Theta_{t,t+i}$ represents cumulative gross inflation between two periods:

$$\Theta_{t,t+i} = \begin{cases} 1 & \text{if } j = 0 \\ \frac{P_{t+1}}{P_t}\times\cdots\times\frac{P_{t+i}}{P_{t+i-1}} & \text{if } j \geq 1 \end{cases} \tag{15}$$

The numerator Υ_t and denominator Φ_t can be written in recursive form:

$$\Upsilon_t = \frac{Y_t}{\widetilde{C}_{s,t}}MC_t + \omega\beta_s\,\mathbb{E}_\mathrm{t}\left\{\pi_{t+1}^{\sigma}\Upsilon_{t+1}\right\} \tag{16}$$

$$\Phi_t = \frac{Y_t}{\widetilde{C}_{s,t}} + \omega\beta_s\,\mathbb{E}_\mathrm{t}\left\{\pi_{t+1}^{\sigma-1}\Phi_{t+1}\right\} \tag{17}$$

Since the probability of adjusting prices is independent of a firm's history, from the law of large numbers the aggregate price[4] is a weighted average of optimised prices and previous period prices:

$$P_t^{1-\sigma} = (1-\omega)(p_t^*)^{1-\sigma} + \omega P_{t-1}^{1-\sigma} \tag{18}$$

which can be used to solve for relative prices as a function of inflation:

$$\frac{p_t^*}{P_t} = \left(\frac{1-\omega}{1-\omega\pi_t^{\sigma-1}}\right)^{\frac{1}{\sigma-1}} \tag{19}$$

[3]Use of the fact that $P_t^{-1} \equiv P_t^{(-1+\sigma-\sigma)}$ has been made.

[4]The aggregate price is a CES aggregate of prices over the continuum of firms:

$$P_t = \left(\int_0^1 p_{j,t}^{1-\sigma}\,dj\right)^{\frac{1}{1-\sigma}}$$

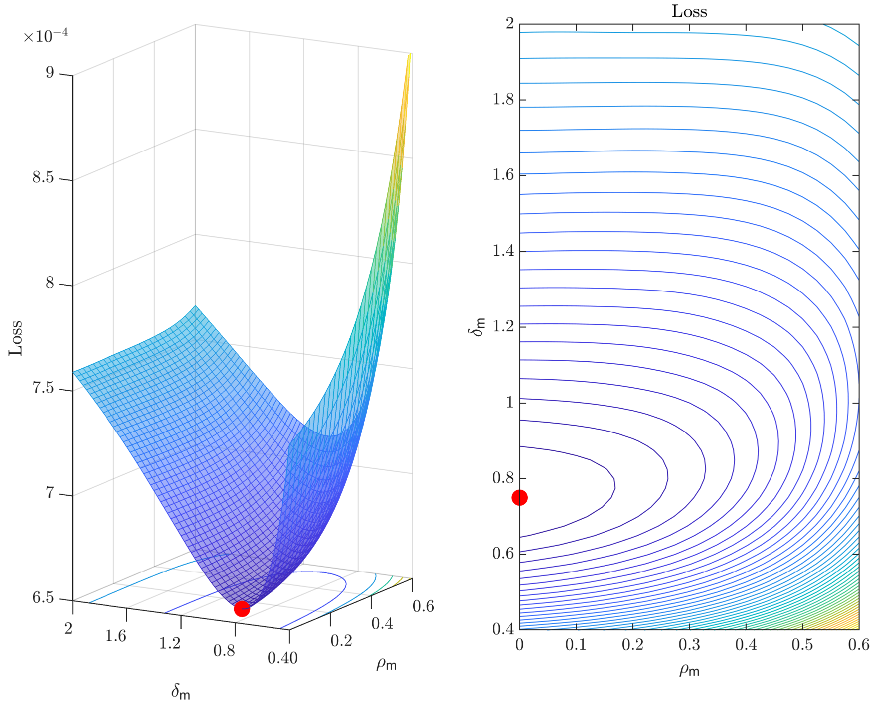

Figure 1: The mapping of symmetric LTV rule parameters and the loss function

Notes: The figure shows the macroprudential policymaker's loss as a function of the two arguments in the minimisation problem, the policy response parameter δ_{m} and the persistence parameter ρ_{m}, as surface (left) and countour (right) plots. The dots denote the minimum.

This can be used to elimate p^*, and the optimal pricing equation can therefore be written as:

$$\Upsilon_t = \left(\frac{\sigma - 1}{\sigma}\right)\left(\frac{1-\omega}{1-\omega\pi_t^{\sigma-1}}\right)^{\frac{1}{\sigma-1}} \Phi_t \tag{20}$$

Equations (16), (17) and (20) jointly determine price dynamics.

Appendix C The optimal symmetric LTV rule

I calibrate the symmetric macroprudential policy rule by finding the parameters that minimise the policymaker's loss, which in turn is a function of the volatility of the credit to output ratio and the LTV ratio. I find the optimal values $\{\delta_{\mathsf{m}}^*, \rho_{\mathsf{m}}^*\}$ by repeatedly solving the model using the solution method of Guerrieri and Iacoviello (2015) and conducting stochastic simulations over a fine grid for both of these parameters. The grid is constrained for positive values of both δ_{m} and ρ_{m}, with domain $\mathcal{G}(\delta_{\mathsf{m}}, \rho_{\mathsf{m}}) : \mathbb{R}^2 = [0.0, 2.0] \times [0.0, 0.9]$. The results of this minimization problem, shown in Figure 1, are $\delta_{\mathsf{m}}^* = 0.75$ and $\rho_{\mathsf{m}}^* = 0$, which is a global minimum in $\mathcal{G}(\delta_{\mathsf{m}}, \rho_{\mathsf{m}})$. The plot is over a sub-domain of ρ_{m} to make it more readable. Figure 2 shows the evolution of the sub-components of the loss function (variances of credit-to-output and LTV ratios) as slices around the global minimum.

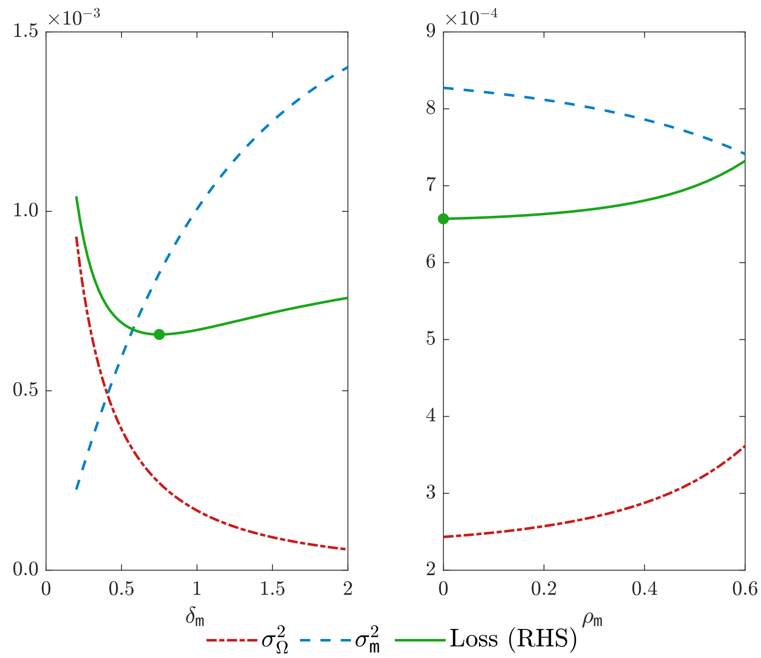

Figure 2: The components of the loss function at the minimum

Notes: The plots show the variance of the arguments in the loss function as slices at the minimum. The dots denote the minimum showed in Figure 1.

References

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