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% g_C=16% (Government consumption-GDP ratio=16%) and  $\zeta=1.350$  (Bonds-money ratio=1.350)

% Parameter values
tau_k=0.276; % Capital income tax rate
tau_w=0.224; % Labor income tax rate
tau_c=0.08; % Consumption tax rate
g_I=0.035; % Ratio of government investment to real GDP
g_C=0.160; % Ratio of government consumption to real GDP
g_W=0; % Ratio of wasteful government expenditure to real GDP
beta=1.4; % Weight of leisure in utility
etha=0.12; % Weight of real balances in utility
phai=0.22; % Weight of government consumption in utility
share=0.6; % Labor share in output
effic=0.9; % TFP
rho=0.04; % Rate of time preference
gamma=-1.5; % Elasticity of intertemporal substitution in consumption
epsilon=0.6; % Geometric weight of average externalities derived from private capital
kusai=1.350; % Bonds-money ratio
t_ss=0.0137; % Lump-sum-tax-wealth ratio

nobs=200; % periods

% Steady state values of z and l
fun = @rootGT1;
x0 = [0.7, 0.7];

param=[tau_k tau_w tau_c g_I g_C g_W beta etha phai share effic rho gamma epsilon];

x = fsolve(fun, x0, optimset, param);

zss=x(1); % Steady state ratio of public capital to private capital, z*
lss=x(2); % Steady state value of leisure, l*

% Balanced growth equilibrium values

css=((1-tau_w)/(1+tau_c))*share*effic*(1-lss)^(share-1)*zss^(share*(1-epsilon))*lss/beta; % Consumption-capital ratio, c*
yss=effic*(1-lss)^share*zss^(share*(1-epsilon)); % Output-capital ratio, y*
(1-g_I-g_C-g_W)*yss; % (Consumption+Investment)/private capital
rss=(1-share)*effic*(1-lss)^share*zss^(share*(1-epsilon)); % Before-tax return on private capital, r*
(1-tau_k)*rss; % After-tax return on capital, (1-tau_k)r*
Pai=1-gamma-etha*gamma-phai*gamma; % value of  $\Pi$  in (12c)
Gss=((1-tau_k)*rss-rho)/Pai; % Balanced growth rate of real GDP
mss=((1+etha)*tau_c+etha)*css+(tau_k*(1-share)+tau_w*share-g_I-g_C-g_W)*yss-t_ss)/((t_ss+(1-1/Pai)*(1-tau_k)*rss+rho/Pai)*(1+kusai)); % Real
money balances-capital ratio, m*
mss/css; % Money-caonsumption ratio
theta_ss=(etha*(1+tau_c)*css/mss-((1-1/Pai)*(1-tau_k)*rss+rho/Pai)); % Money growth rate,  $\theta$ *
infl=etha*(1+tau_c)*css/mss-(1-tau_k)*rss; % Inflation rate,  $\pi$ *
Piketty=(Pai-1)*(1-tau_k)*rss/Pai-rho/Pai; % (1-tau_k)rss-Gss

% Local stability

theta_L=((1-share)*(1-gamma-phai*gamma)+phai*gamma)*lss/(1-lss)+(1-gamma-beta*gamma); %  $\Delta$  in (12c)

y_z=share*(1-epsilon)*yss/zss; % dy/dz
y_l=-share*yss/(1-lss); % dy/dl

c_z=(1-epsilon)*share*css/zss; % dc/dz
c_l=((1-share)*lss/(1-lss)+1)*css/lss; % dc/dl

r_z=share*(1-epsilon)*rss/zss; % dr/dz
r_l=-share*rss/(1-lss); % dr/dl

G_k=(1-g_I-g_C-g_W)*yss-css; % Growth rate of private capital
G_kp=g_I*effic*(1-lss)^share*zss^(share*(1-epsilon))-1; % Growth rate of public capital

G_kl=-share*(1-g_I-g_C-g_W)*yss/(1-lss)-((1-share)*lss/(1-lss)+1)*css/lss; % d(G_k)/dl
G_kz=share*(1-epsilon)*G_k/zss; % d(G_k)/dz

G_kpz=-(1-share*(1-epsilon))*G_kp/zss; % d(G_kp)/dz
G_kpl=-share*G_kp/(1-lss); % d(G_kp)/dl

% Elements of (15)

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a_31=mss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*y_l+(1-tau_k)*r_l*(1+kusai)*mss-(tau_c+etha*(1+tau_c))*c_l)/((1+kusai)*mss)-G_kl*mss: % a_31
a_32=mss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*y_z+(1-tau_k)*r_z*(1+kusai)*mss-(tau_c+etha*(1+tau_c))*c_z)/((1+kusai)*mss)-G_kz*mss: % a_32
a_33=-((g_l+g_c-(1-share)*tau_k-share*tau_w)*yss-(tau_c+etha*(1+tau_c))*css+t_ss)/((1+kusai)*mss); % a_33

a_21=(G_kpl-G_kl)*zss: % a_21
a_22=(G_kpz-G_kz)*zss: % a_22
a_23=0: % a_23

a_11=(etha*ganmma*a_31/mss-Pai*G_kl-share*(1-epsiron)*(1-gamma-phai*ganmma)*a_21/zss+(1-tau_k)*r_l*(lss/theta_L): % a_31
a_12=(etha*ganmma*a_32/mss-Pai*G_kz-share*(1-epsiron)*(1-gamma-phai*ganmma)*a_22/zss+(1-tau_k)*r_z*(lss/theta_L): % a_32
a_13=(etha*ganmma*a_33/mss)*(lss/theta_L): % a_33

A=[a_11 a_12 a_13; a_21 a_22 a_23; a_31 a_32 a_33]; % Jacobian in (15)
e=eig(A); % Three eigenvalues in jacobian

% Saddle path
Meig=sort(e);
lambda=diag(Meig);
lambda(1,1): % Unique stable root

A_1=(lambda(1,1)-a_22)/a_21: % Coefficient of saddle path of leisure
A_2=a_32+a_31*A_1: % Coefficient of saddle path of money, Numerator
A_3=lambda(1,1)-a_33: % Coefficient of saddle path of money, Denominator

za=0.5759: % A value of z(0) equal to pre-shock value, Public capital-private capital ratio
z=[]: z=[z:0.5759];
la=lss+A_1*(za-zss): % A value of l(0), lesiure
l=[]: l=[l:la];
m=mss+(A_2/A_3)*(za-zss): % A value of m(0), real balances-capital ratio
m=[]: m=[m:ma];
ca=((1-tau_w)/(1+tau_c))*share*effic*(1-la)^(share-1)*za^(share*(1-epsiron))*la/beta: % A value of c(0), Initial consumption-capital ratio
c=[]: c=[c:ca];
ra=(1-share)*effic*(1-la)^share*za^(share*(1-epsiron)): % A value of r(0), Initial before-tax return on private capital
r=[]: r=[r:ra];
minfl_a=etha*(1+tau_c)*ca/ma-(1-tau_k)*ra: % A value of pi(0), Initial inflation rate
infl=[]: infl=[infl:minfl_a];

ya=effic*(1-la)^share*za^(share*(1-epsiron)): % A value of y(0), Initial ratio of GDP to private capital
y=[]: y=[y:ya];
mG_ka=(1-g_l-g_c-g_w)*ya-ca: % Growth rate of private capital in period 0
G_k=[]: G_k=[G_k:mG_ka];
mG_kpa=g_l*ya/za: % Growth rate of public capital in period 0
G_kp=[]: G_kp=[G_kp:mG_kpa];
mtheta_a=((g_l+g_c-g_w)-(1-share)*tau_k-share*tau_w)*ya-(tau_c-kusai*etha*(1+tau_c))*ca+(1+(1+kusai)*ma)*t_ss)/((1+kusai)*ma): % Money growth rate in period 0
theta=[]: theta=[theta:mtheta_a];
G_la=(la/theta_L)*(etha*ganmma*(theta-minfl_a-mG_ka)-Pai*mG_ka-share*(1-epsiron)*(1-gamma-phai*ganmma)*(mG_kpa-mG_ka)+(1-tau_k)*ra-rho): % Groth rate of leisure in period 0
G_l=[]: G_l=[G_l:G_la];
mG_ya=-share*(G_la/(1-la))+share*(1-epsiron)*mG_kpa+(1-share*(1-epsiron))*mG_ka: % Grwoth rate of GDP in period 0
G_y=[]: G_y=[G_y:mG_ya];
mPiketa=(1-tau_k)*ra-mG_ya: % (1-tau_k)r-G_y in period 0
Piket=[]: Piket=[Piket:mPiketa];
mG_wa=((1+kusai)*ma/(1+(1+kusai)*ma))*(theta-minfl_a)+(1/(1+(1+kusai)*ma))*mG_ka: % Grwoth rate of wealth in period 0
G_w=[]: G_w=[G_w:mG_wa];

% Inequality in a balnaced growth path
nyu=beta/(1+beta+etha): % nu in (17)
wss_tild=share*effic*(1-lss)^(share-1)*zss^(share*(1-epsiron)): % w*/K*, Real wage-private capital ratio in a balanced growth
w0_tild=share*effic*(1-la)^(share-1)*za^(share*(1-epsiron)): % w(0)/K(0) Real wage-private capital in period 0
wtild=[]: wtild=[wtild:w0_tild];

alpha_a=1+(1-tau_w)*(wss_tild/(1+(1+kusai)*mss))*(1-la/lss)/(-lambda(1,1)-(1-tau_w)*(wss_tild/(1+(1+kusai)*mss)))*(1-lss/nyu)): % A value of alpha(0) in (D-4);
alpha=[]: alpha=[alpha:alpha_a];

omega_a=1+(ma/(1+kusai)*ma)*(alpha_a-1+nyu/lss)/alpha_a: % A value of omega(0) in (D-6), Coefficient of (D-6)
omega=[]: omega=[omega:omega_a];

fey_a=(ra*(1+kusai)*ma)/(ra*(1+kusai)*ma+w0_tild*(1-la))*omega_a-(w0_tild*la/(ra*(1+kusai)*ma+w0_tild*(1-la)))*(1-nyu/lss)/alpha_a: % A value of phi(0) in (D-8a)
fey=[]: fey=[fey:fey_a];

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pusai_a=((1-tau_k)*ra*(1+kusai*ma)/((1-tau_k)*ra*(1+kusai*ma)+(1-tau_w)*w0_tild*(1-la))*omega_a-((1-tau_w)*w0_tild*la/((1-
tau_k)*ra*(1+kusai*ma)+(1-tau_w)*w0_tild*(1-la)))*(1-nyu/lss)/alpha_a:    % A value of  $\psi(0)$ , Coefficient of after-tax income inequality
pusai=[]: pusai=[pusai;pusai_a];

kapper=0.406;    % (bonds+money)/wealth in the benchmark parameter values

sigma_aa=1+((2*kapper-1)*(1-kapper)*kapper/((1-kapper)^2+kapper^2))*((1+kusai)*ma/0.6835-1);    % After-shock value of  $\sigma_a(0)$ 
sigma_ass=sigma_aa/alpha_a    % Balanced growth value of  $\sigma_a$ , i.e.,  $\sigma_a^*$ 
sigma_a=[]: sigma_a=[sigma_a:sigma_aa];

sigma_va=omega_a*sigma_aa/1.1634;    % Inequality of income-earning assets  $\sigma_v(0)$  relative to the benchmark value in
period 0
sigma_v=[]: sigma_v=[sigma_v:sigma_va];

sigma_ya=fey_a*sigma_aa/0.2837;    % Before-tax income inequality  $\sigma_y(0)$  relative to the benchmark value in period 0
sigma_y=[]: sigma_y=[sigma_y:sigma_ya];

sigma_yfa=pusai_a*sigma_aa/0.2544;    % After-tax income inequality  $\sigma_{ya}(0)$  relative to the benchmark value in period 0
sigma_yf=[]: sigma_yf=[sigma_yf:sigma_yfa];

% Transitional dynamics of average variables and standard deviations

nobs=200;

for t=1:nobs

    zt=zss+exp(lambda(1,1))*(za-zss);    % Dynamics of z(t)
    za=zt;
    z=[z:zt];

    lt=lss+exp(lambda(1,1))*(la-lss);    % Dynamics of l(t)
    la=lt;
    l=[l:lt];

    mt=mss+exp(lambda(1,1))*(ma-mss);    % Dynamics of m(t)
    ma=mt;
    m=[m:mt];

    ct=((1-tau_w)/(1+tau_c))*share*effic*(1-lt)^(share-1)*zt^(share*(1-epsiron))*lt/beta;    % Dynamics of c(t)
    c=[c:ct];

    rt=(1-share)*effic*(1-lt)^share*zt^(share*(1-epsiron));    % Dynamics of m(t)
    r=[r:rt];

    infl_t=etha*(1+tau_c)*ct/mt-(1-tau_k)*rt;    % Dynamics of inflation rate
    infl=[infl:infl_t];

    yt=effic*(1-lt)^share*zt^(share*(1-epsiron));    % Dynamics of GDP-capital ratio, y(t)
    y=[y:yt];

    G_kt=(1-g_l-g_c-g_w)*yt-ct;    % Growth rate of K(t)
    G_k=[G_k:G_kt];

    G_kpt=g_l*yt/zt;    % Growth rate of K_p(t)
    G_kp=[G_kp:G_kpt];

    theta_t=((g_l+g_c+g_w)-(1-share)*tau_k-share*tau_w)*yt-(tau_c-kusai*etha*(1+tau_c))*ct+(1+(1+kusai)*mt)*t_ss/((1+kusai)*mt);    %
Dynamics of  $\theta(t)$  money growth rate
    theta=[theta:theta_t];

    G_lt=(lt/theta_L)*(etha*gamma*(theta_t-infl_t-G_kt)-Pai*G_kt-share*(1-epsiron)*(1-gamma-phai*gamma)*(G_kpt-G_kt)+(1-tau_k)*rt-rho);    %
dl^dot/dt
    G_l=[G_l:G_lt];

    G_yt=-share*(G_lt/(1-lt))+share*(1-epsiron)*G_kpt+(1-share*(1-epsiron))*G_kt;    % Growth rate of real GDP
    G_y=[G_y:G_yt];

    Piket_t=(1-tau_k)*rt-G_yt;    % Dynamics of  $(1-\tau_k)r-G_y$ 
    Piket=[Piket:Piket_t];

    G_wt=((1+kusai)*mt/(1+(1+kusai)*mt))*(theta_t-infl_t)+(1/(1+(1+kusai)*mt))*G_kt;    % Growth rate of wealth A(t)
    G_w=[G_w:G_wt];

    alpha_t=1+(1-tau_w)*(wss_tild/(1+(1+kusai)*mss))*(1-lt/lss)/(-lambda(1,1)-(1-tau_w)*(wss_tild/(1+(1+kusai)*mss))*(1-lss/nyu));    % Dynamics

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of  $\alpha(t)$ :
    alpha=[alpha:alpha_t];

    sigma_at=alpha_t*sigma_ass; % Dynamics of  $\sigma_a(t)$ 

    omega_t=1+(mt/(1+kusai*mt))*(alpha_t-1+nyu/lss)/alpha_t; % Dynamics of  $\omega(t)$ 
    omega=[omega:omega_t];

    sigma_vt=omega_t*sigma_at/1.1634; % Dynamics of  $\sigma_v(t)$ 

    wtild_t=share*effic*(1-lt)^(share-1)*zt^(share*(1-epsiron)); % Dynamics of  $wtild(t)$ 
    wtild=[wtild:wtild_t];

    fey_t=(rt*(1+kusai*mt)/(rt*(1+kusai*mt)+wtild_t*(1-lt))*omega_t-(wtild_t*lt/(rt*(1+kusai*mt)+wtild_t*(1-lt)))*(1-nyu/lss)/alpha_t; %
Dynamics of  $\phi(t)$ 
    sigma_yt=fey_t*sigma_at/0.2837;

    pusai_t=((1-tau_k)*rt*(1+kusai*mt)/((1-tau_k)*rt*(1+kusai*mt)+(1-tau_w)*wtild_t*(1-lt))*omega_t-((1-tau_w)*wtild_t*lt/((1-
tau_k)*rt*(1+kusai*mt)+(1-tau_w)*wtild_t*(1-lt)))*(1-nyu/lss)/alpha_t; % Dynamics of  $\psi(t)$ 
    sigma_yft=pusai_t*sigma_at/0.2544;

    fey=[fey:fey_t];
    pusai=[pusai:pusai_t];

    sigma_a=[sigma_a:sigma_at];
    sigma_v=[sigma_v:sigma_vt];
    sigma_y=[sigma_y:sigma_yt];
    sigma_yf=[sigma_yf:sigma_yft];

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end
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```
% g_C=16% (Government consumption-GDP ratio=16%) and  $\xi=1.10$  (Bonds-money ratio=1.10)
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% Parameter values
tau_k=0.276; % Capital income tax rate
tau_w=0.224; % Labor income tax rate
tau_c=0.08; % Consumption tax rate
g_I=0.035; % Ratio of government investment to real GDP
g_C=0.160; % Ratio of government consumption to real GDP
g_W=0; % Ratio of wastful government expenditure to real GDP
beta=1.4; % Weight of leisure in utility
etha=0.12; % Weight of real mney balances in utility
phai=0.22; % Weight of government consumption in utility
share=0.6; % Labor share in output
effic=0.9; % TFP
rho=0.04; % Rate of time preference
gamma=-1.5; % Elasticity of intertemporal substitution in consumption
epsiron=0.6; % Geometric weight of average externalities derived from private capital
kusai=1.10; % Bonds-money ratio
t_ss=0.0137; % Lump-sum-tax-wealth ratio

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```
nobs=200; % periods
```

```
% Steady state values of z and l
```

```

fun = @rootGT1;
x0 = [0.7, 0.7];

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param=[tau_k tau_w tau_c g_I g_C g_W beta etha phai share effic rho gamma epsiron];
```

```
x = fsolve(fun, x0, optimset, param);
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nzss=x(1); % Steady state ratio of public capital to private capital, z*
```

```
nlss=x(2); % Steady state value of leisure, l*
```

```
% Balanced growth equilibrium values
```

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ncss=((1-tau_w)/(1+tau_c))*share*effic*(1-nlss)^(share-1)*nzss^(share*(1-epsiron))*nlss/beta; % Consumption-capital ratio, c*
nyss=effic*(1-nlss)^share*nzss^(share*(1-epsiron)); % Output-capital ratio, y*

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(1-g_l-g_c-g_w)*nyss; % Consumption+investment/capital
nrss=(1-share)*effic*(1-nlss)^(share*nzss^(share*(1-epsilon))); % Before-tax return on private capital, r*
(1-tau_k)*nrss; % After-tax return on private capital, (1-tau_k)r*
nPaI=1-gamma-etha*gamma-phai*gamma; %  $\Pi$  in (12c)
nGss=((1-tau_k)*nrss-rho)/nPaI; % Balanced growth rate of real GDP
nmss=((1+etha)*tau_c+etha)*ncss+(tau_k*(1-share)+tau_w*share-g_l-g_c-g_w)*nyss-t_ss/((t_ss+(1-1/nPaI)*(1-tau_k)*nrss+rho/nPaI)*(1+kusai)); % Real
money balances-capital ratio, m*
nmss/ncss; % Money-consumption ratio
ntheta_ss=(etha*(1+tau_c)*ncss/nmss-((1-1/nPaI)*(1-tau_k)*nrss+rho/nPaI)); % Money growth rate
ninfl=etha*(1+tau_c)*ncss/nmss-(1-tau_k)*nrss; % Inflation rate
nPiketty=(nPaI-1)*(1-tau_k)*nrss/nPaI-rho/nPaI; % (1-tau_k)rss-Gss

% Local stability
ntheta_L=((1-share)*(1-gamma-phai*gamma)+phai*gamma)*nlss/(1-nlss)+(1-gamma-beta*gamma); %  $\Delta$  in (12c)

ny_z=share*(1-epsilon)*nyss/nzss; % dy/dz
ny_l=-share*nyss/(1-nlss); % dy/dl

nc_z=(1-epsilon)*share*ncss/nzss; % dc/dz
nc_l=((1-share)*nlss/(1-nlss)+1)*ncss/nlss; % dc/dl

nr_z=share*(1-epsilon)*nrss/nzss; % dr/dz
nr_l=-share*nrss/(1-nlss); % dr/dl

nG_k=(1-g_l-g_c-g_w)*nyss-ncss; % Growth rate of private capital
nG_kp=g_l*effic*(1-nlss)^(share*nzss^(share*(1-epsilon)-1)); % Growth rate of public capital

nG_kl=-share*(1-g_l-g_c-g_w)*nyss/(1-nlss)-((1-share)*nlss/(1-nlss)+1)*ncss/nlss; % d(G_k)/dl
nG_kz=share*(1-epsilon)*nG_k/nzss; % d(G_k)/dz

nG_kpz=-((1-share*(1-epsilon))*nG_kp/nzss); % d(G_kp)/dz
nG_kpl=-share*nG_kp/(1-nlss); % d(G_kp)/dl

% Elements of (15)
na_31=nmss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*ny_l+(1-tau_k)*nr_l*(1+kusai)*nmss-(tau_c+etha*(1+tau_c))*nc_l)/((1+kusai)*nmss)-nG_kl*nmss; %
a_31
na_32=nmss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*ny_z+(1-tau_k)*nr_z*(1+kusai)*nmss-(tau_c+etha*(1+tau_c))*nc_z)/((1+kusai)*nmss)-nG_kz*nmss; %
a_32
na_33=-((g_l+g_c-(1-share)*tau_k-share*tau_w)*nyss-(tau_c+etha*(1+tau_c))*ncss+t_ss)/((1+kusai)*nmss); %
a_33

na_21=(nG_kpl-nG_kl)*nzss; % a_21
na_22=(nG_kpz-nG_kz)*nzss; % a_22
na_23=0; % a_23

na_11=(etha*gamma*na_31/nmss-nPaI*nG_kl-share*(1-epsilon)*(1-gamma-phai*gamma)*na_21/nzss+(1-tau_k)*nr_l)*(nlss/ntheta_L); % a_11
na_12=(etha*gamma*na_32/nmss-nPaI*nG_kz-share*(1-epsilon)*(1-gamma-phai*gamma)*na_22/nzss+(1-tau_k)*nr_z)*(nlss/ntheta_L); % a_12
na_13=(etha*gamma*na_33/nmss)*(nlss/ntheta_L); % a_13

nA=[na_11 na_12 na_13; na_21 na_22 na_23; na_31 na_32 na_33]; % Jacobian in (15)
ne=eig(nA); % Three eigenvalues in jacobian

% Saddle path
nMeig=sort(ne);
nlambda=diag(nMeig);
nlambda(1,1); % Unique stable root

nA_1=(nlambda(1,1)-na_22)/na_21; % Coefficient of saddle path of leisure
nA_2=na_32+na_31*nA_1; % Coefficient of saddle path of money, Numerator
nA_3=nlambda(1,1)-na_33; % Coefficient of saddle path of money, Denominator

nza=0.5759; % A value of z(0) equal to pre-shock value, Public capital-private capital ratio
nz=[]; nz=[nz;0.5759];
nla=nlss+nA_1*(nza-nzss); % A value of l(0), lesiure
nl=[]; nl=[nl;nla];
nma=nmss+(nA_2/nA_3)*(nza-nzss); % A value of m(0), real balances-capital ratio
nm=[]; nm=[nm;nma];
nca=((1-tau_w)/(1+tau_c))*share*effic*(1-nla)^(share-1)*nza^(share*(1-epsilon))*nla/beta; % A value of c(0), Initial consumption-capital ratio
nc=[]; nc=[nc;nca];
nra=(1-share)*effic*(1-nla)^(share*nza^(share*(1-epsilon))); % A value of r(0), Initial before-tax return on private
capital
nr=[]; nr=[nr;nra];
ninfl_a=etha*(1+tau_c)*nca/nma-(1-tau_k)*nra; % A value of  $\pi$ (0), Initial inflation rate
ninfl=[]; ninfl=[ninfl;ninfl_a];

nya=effic*(1-nla)^(share*nza^(share*(1-epsilon))); % A value of y(0), Initial ratio of GDP to private

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capital
ny=[]; ny=[ny:nya];
nG_ka=(1-g_l-g_C-g_W)*nya-nca; % Growth rate of private capital in period 0
nG_k=[]; nG_k=[nG_k:nG_ka];
nG_kpa=g_l*nya/nza; % Growth rate of public capital in period 0
nG_kp=[]; nG_kp=[nG_kp:nG_kpa];
ntheta_a=((g_l+g_C+g_W-(1-share)*tau_k-share*tau_w)*nya-(tau_c-kusai*etha*(1+tau_c))*nca+(1+(1+kusai)*nma)*t_ss)/((1+kusai)*nma); %
Money growth rate in period 0
ntheta=[]; ntheta=[ntheta:ntheta_a];
nG_la=(nla/ntheta_l)*(etha*gamma*(ntheta-ninfl_a-nG_ka)-nPai*nG_ka-share*(1-epsiron)*(1-gamma-phai*gamma)*(nG_kpa-nG_ka)+(1-tau_k)*nra-rho); %
Grfowth rate of leisure in period 0
nG_l=[]; nG_l=[nG_l:nG_la];
nG_ya=-share*(nG_la/(1-nla))+share*(1-epsiron)*nG_kpa+(1-share*(1-epsiron))*nG_ka; % Grwoth rate of real GDP in period 0
nG_y=[]; nG_y=[nG_y:nG_ya];
nPiketa=(1-tau_k)*nra-nG_ya; % (1-tau_k)r-G_y in period 0
nPiket=[]; nPiket=[nPiket:nPiketa];
nG_wa=((1+kusai)*nma/(1+(1+kusai)*nma))*(ntheta-ninfl_a)+(1/(1+(1+kusai)*nma))*nG_ka; % Grwoth rate of wealth in period 0
nG_w=[]; nG_w=[nG_w:nG_wa];

% Inequality in a balnaced growth path
nyu=beta/(1+beta+etha); % v in (17)
wss_tild=share*effio*(1-nlss)^(share-1)*nzss^(share*(1-epsiron)); % w*/K*, Real wage-private capital ratio in a balanced growth
nw0_tild=share*effio*(1-nla)^(share-1)*nza^(share*(1-epsiron)); % w(0)/K(0), real wage-private capital ratio in period 0
nwtild=[]; nwtild=[nwtild:nw0_tild];

nalpha_a=1+(1-tau_w)*(wss_tild/(1+(1+kusai)*nmss))*(1-nla/nlss)/(-nlambd(1,1)-(1-tau_w)*(wss_tild/(1+(1+kusai)*nmss))*(1-nlss/nyu)); % A value of
alpha(0) in (D-4);
nalpha=[]; nalpha=[nalpha:nalpha_a];

nomega_a=1+(nma/(1+kusai)*nma)*(nalpha_a-1+nyu/nlss)/nalpha_a; % A value of omega(0) in (D-6), Coefficient of (D-6)
nomega=[]; nomega=[nomega:nomega_a];

nfey_a=(nra*(1+kusai)*nma)/(nra*(1+kusai)*nma+nw0_tild*(1-nla))*nomega_a-(nw0_tild*nla/(nra*(1+kusai)*nma+nw0_tild*(1-nla)))*(1-
nyu/nlss)/nalpha_a; % A value of phi(0) in (D-8a)
nfey=[]; nfey=[nfey:nfey_a];

npusai_a=((1-tau_k)*nra*(1+kusai)*nma)/((1-tau_k)*nra*(1+kusai)*nma+(1-tau_w)*nw0_tild*(1-nla))*nomega_a-((1-tau_w)*nw0_tild*nla/((1-
tau_k)*nra*(1+kusai)*nma)+(1-tau_w)*nw0_tild*(1-nla))*(1-nyu/nlss)/nalpha_a; % A value of psi(0), Coefficient of after-tax income inequality
npusai=[]; npusai=[npusai:npusai_a];

nkapper=0.406; % (bonds+money)/wealth in the benchmark parameter values

nsigma_aa=1+((2*nkapper-1)*(1-nkapper)*nkapper/((1-nkapper)^2+nkapper^2))*((1+kusai)*nma/0.6835-1)); % After-shock value of sigma_a(0) in period
0
nsigma_ass=nsigma_aa/nalpha_a % Balanced growth value of sigma_a, i.e., sigma_a*
nsigma_a=[]; nsigma_a=[nsigma_a:nsigma_aa];

nsigma_va=nomega_a*nsigma_aa/1.1634; % Inequality of income-earning assets sigma_v(0) relative to the benchmark value in
period 0
nsigma_v=[]; nsigma_v=[nsigma_v:nsigma_va];

nsigma_ya=nfey_a*nsigma_aa/0.2837; % Before-tax income inequality sigma_y(0) relative to the benchmark value in period
0
nsigma_y=[]; nsigma_y=[nsigma_y:nsigma_ya];

nsigma_yfa=npusai_a*nsigma_aa/0.2544; % After-tax income inequality sigma_yf(0) relative to the benchmark value in period
0
nsigma_yf=[]; nsigma_yf=[nsigma_yf:nsigma_yfa];

% Transitional dynamics of average variables and standard deviations
nobs=200;

for t=1:nobs

    nzt=nzss+exp(nlambd(1,1))*(nza-nzss); % Dynamics of z(t)
    nza=nzt;
    nz=[nz:nzt];

    nlt=nlss+exp(nlambd(1,1))*(nla-nlss); % Dynamics of l(t)
    nla=nlt;
    nl=[nl:nlt];

```

```

nmt=nmss+exp(nlambd(1,1))*(nma-nmss); % Dynamics of m(t)
nma=nmt;
nm=[nm:nmt];

nct=((1-tau_w)/(1+tau_c))*share*effic*(1-nlt)^(share-1)*nzt^(share*(1-epsiron))*nlt/beta; % Dynamics of c(t)
nc=[nc:nct];

nrt=(1-share)*effic*(1-nlt)^share*nzt^(share*(1-epsiron)); % Dynamics of r(t)
nr=[nr:nrt];

ninfl_t=etha*(1+tau_c)*nct/nmt-(1-tau_k)*nrt; % Dynamics of inflation rate  $\pi(t)$ 
ninfl=[ninfl:ninfl_t];

nyt=effic*(1-nlt)^share*nzt^(share*(1-epsiron)); % Dynamics of y(t)
ny=[ny:nyt];

nG_kt=(1-g_I-g_C-g_W)*nyt-nct; % Growth rate of K(t)
nG_k=[nG_k:nG_kt];

nG_kpt=g_I*nyt/nzt; % Growth rate of K_p(t)
nG_kp=[nG_kp:nG_kpt];

ntheta_t=((g_I+g_C+g_W-(1-share)*tau_k-share*tau_w)*nyt-(tau_c-kusai*etha*(1+tau_c))*nct+(1+(1+kusai)*nmt)*t_ss)/((1+kusai)*nmt); %
Dynamics of money growth rate
ntheta=[ntheta:ntheta_t];

nG_lt=(nlt/ntheta_L)*(etha*gamma*(ntheta_t-ninfl_t-nG_kt)-nPai*nG_kt-share*(1-epsiron)*(1-gamma-phi*gamma)*(nG_kpt-nG_kt)+(1-
tau_k)*nrt-rho); %  $d\dot{l}/dt$ 
nG_l=[nG_l:nG_lt];

nG_yt=-share*(nG_lt/(1-nlt))+share*(1-epsiron)*nG_kpt+(1-share*(1-epsiron))*nG_kt; % Growth rate of real GDP
nG_y=[nG_y:nG_yt];

nPiket_t=(1-tau_k)*nrt-nG_yt; %  $(1-\tau_k)r-G_y$ 
nPiket=[nPiket:nPiket_t];

nG_wt=((1+kusai)*nmt/(1+(1+kusai)*nmt))*(ntheta_t-ninfl_t)+(1/(1+(1+kusai)*nmt))*nG_kt; % Growth rate of wealth A(t)
nG_w=[nG_w:nG_wt];

nalpha_t=1+(1-tau_w)*(wss_tild/(1+(1+kusai)*nmss))*(1-nlt/nlss)/(-nlambd(1,1)-(1-tau_w)*(wss_tild/(1+(1+kusai)*nmss))*(1-nlss/nyu)); %
Dynamics of  $\alpha(t)$ 
nalpha=[nalpha:nalpha_t];

nsigma_at=nalpha_t*nsigma_ass; % Dynamics of  $\sigma_a(t)$ 

nomega_t=1+(nmt/(1+kusai*nmt))*(nalpha_t-1+nyu/nlss)/nalpha_t; % Dynamics of  $\omega(t)$ 
nomega=[nomega:nomega_t];

nsigma_vt=nomega_t*nsigma_at/1.1634; % Dynamics of  $\sigma_v(t)$ 

nwtild_t=share*effic*(1-nlt)^(share-1)*nzt^(share*(1-epsiron));

nfey_t=(nrt*(1+kusai*nmt)/(nrt*(1+kusai*nmt)+nwtild_t*(1-nlt)))*nomega_t-(nwtild_t*nlt/(nrt*(1+kusai*nmt)+nwtild_t*(1-nlt)))*(1-
nyu/nlss)/nalpha_t; % Dynamics of  $\phi(t)$ 
nsigma_yt=nfey_t*nsigma_at/0.2837;

npusai_t=((1-tau_k)*nrt*(1+kusai*nmt)/((1-tau_k)*nrt*(1+kusai*nmt)+(1-tau_w)*nwtild_t*(1-nlt)))*nomega_t-((1-tau_w)*nwtild_t*nlt/((1-
tau_k)*nrt*(1+kusai*nmt)+(1-tau_w)*nwtild_t*(1-nlt)))*(1-nyu/nlss)/nalpha_t; % Dynamics of  $\psi(t)$ 
nsigma_yft=npusai_t*nsigma_at/0.2544;

nwtild=[nwtild:nwtild_t];
nfey=[nfey:nfey_t];
npusai=[npusai:npusai_t];

nsigma_a=[nsigma_a:nsigma_at];
nsigma_v=[nsigma_v:nsigma_vt];
nsigma_y=[nsigma_y:nsigma_yt];
nsigma_yf=[nsigma_yf:nsigma_yft];

```

end

```

% g_I=4.5% (Government investment-GDP ratio=4.5%) and  $\xi=1.35$  (Bonds-money ratio=1.35)

% Parameter values
tau_k=0.276: % Capital income tax rate
tau_w=0.224: % Labor income tax rate
tau_c=0.08: % Consumption tax rate
g_I=0.045: % Ratio of government investment to real GDP
g_C=0.150: % Ratio of government consumption to real GDP
g_W=0: % Ratio of wastful government expenditure to real GDP
beta=1.4: % Weight of leisure in utility
etha=0.12: % Weight of real money balances in utility
phai=0.22: % Weight of government consumption in utility
share=0.6: % Labor share in output
effic=0.9: % TFP
rho=0.04: % Rate of time preference
gamma=-1.5: % Elasticity of intertemporal substitution in consumption
epsilon=0.6: % Geometric weight of average externalities derived from private capital
kusai=1.35: % Bonds-money ratio
t_ss=0.0137: % Lump-sum-tax-wealth ratio

nobs=200: % periods

% Steady state values
fun = @rootGT1;
x0 = [0.7, 0.7];

param=[tau_k tau_w tau_c g_I g_C g_W beta etha phai share effic rho gamma epsilon];

x = fsolve(fun, x0, optimset, param);

Izss=x(1): % Steady state ratio of public capital to private capital, z*
Ilss=x(2): % Steady state value of leisure, l*

% Balanced growth equilibrium values
Icss=((1-tau_w)/(1+tau_c))*share*effic*(1-Ilss)^(share-1)*Izss^(share*(1-epsilon))*Ilss/beta: % Consumption-capital ratio, c*
lyss=effic*(1-Ilss)^share*Izss^(share*(1-epsilon)); % Output-capital ratio, y*
(1-g_I-g_C-g_W)*lyss: % (Consumption+investment)/capital
Irss=(1-share)*effic*(1-Ilss)^share*Izss^(share*(1-epsilon)); % Before-tax return on private capital, r*
(1-tau_k)*Irss: % After-tax return on private capital, (1-tau_k)r*
IPai=1-gamma-etha*gamma-phai*gamma: %  $\Pi$  in (12c)
IGss=((1-tau_k)*Irss-rho)/IPai: % Balanced growth rate
Imss=((1+etha)*tau_c+etha)*Icss+(tau_k*(1-share)+tau_w*share-g_I-g_C-g_W)*lyss-t_ss)/((t_ss+(1-1/IPai)*(1-tau_k)*Irss+rho/IPai)*(1+kusai)): % Real
money balances-capital ratio, m*
Imss/Icss: % Money-consumption ratio
ltheta_ss=(etha*(1+tau_c)*Icss/Imss-((1-1/IPai)*(1-tau_k)*Irss+rho/IPai)): % Money growth rate
linfl=etha*(1+tau_c)*Icss/Imss-(1-tau_k)*Irss: % Inflation rate
IPiketty=(IPai-1)*(1-tau_k)*Irss/IPai-rho/IPai: %  $(1-\tau_k)r_{ss}-g_{ss}$ 

% Local stability
ltheta_L=((1-share)*(1-gamma-phai*gamma)+phai*gamma)*Ilss/(1-Ilss)+(1-gamma-beta*gamma): %  $\Delta$  in (12c)

ly_z=share*(1-epsilon)*lyss/Izss: % dy/dz
ly_l=-share*lyss/(1-Ilss): % dy/dl

lc_z=(1-epsilon)*share*Icss/Izss: % dc/dz
lc_l=((1-share)*Ilss/(1-Ilss)+1)*Icss/Ilss: % dc/dl

lr_z=share*(1-epsilon)*Irss/Izss: % dr/dz
lr_l=-share*Irss/(1-Ilss): % dr/dl

IG_k=(1-g_I-g_C-g_W)*lyss-Icss: % Growth rate of private capital, G_k
IG_kp=g_I*effic*(1-Ilss)^share*Izss^(share*(1-epsilon)-1): % Growth rate of public capital, G_kp

IG_kl=-share*(1-g_I-g_C-g_W)*lyss/(1-Ilss)-((1-share)*Ilss/(1-Ilss)+1)*Icss/Ilss: % d(G_k)/dl
IG_kz=share*(1-epsilon)*IG_k/Izss: % d(G_k)/dz

IG_kpz=-(1-share*(1-epsilon))*IG_kp/Izss: % d(G_kp)/dz
IG_kpl=-share*IG_kp/(1-Ilss): % d(G_kp)/dl

```



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% Elements of (15)
Ia_31=Imss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*ly_l+(1-tau_k)*lr_l*(1+kusai)*Imss-(tau_c+etha*(1+tau_c))*lc_l)/((1+kusai)*Imss)-IG_kl*Imss; %
a_31
Ia_32=Imss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*ly_z+(1-tau_k)*lr_z*(1+kusai)*Imss-(tau_c+etha*(1+tau_c))*lc_z)/((1+kusai)*Imss)-IG_kz*Imss; %
a_32
Ia_33=-((g_l+g_c-(1-share)*tau_k-share*tau_w)*lyss-(tau_c+etha*(1+tau_c))*lcss+t_ss)/((1+kusai)*Imss); %
a_33

Ia_21=(IG_kpl-IG_kl)*Izss; % a_21
Ia_22=(IG_kpz-IG_kz)*Izss; % a_22
Ia_23=0; % a_23

Ia_11=(etha*gamma*Ia_31/Imss-IPai*IG_kl-share*(1-epsiron)*(1-gamma-phai*gamma)*Ia_21/Izss+(1-tau_k)*lr_l)*(I/ss/Itheta_L); % a_11
Ia_12=(etha*gamma*Ia_32/Imss-IPai*IG_kz-share*(1-epsiron)*(1-gamma-phai*gamma)*Ia_22/Izss+(1-tau_k)*lr_z)*(I/ss/Itheta_L); % a_12
Ia_13=(etha*gamma*Ia_33/Imss)*(I/ss/Itheta_L); % a_13

IA=[Ia_11 Ia_12 Ia_13; Ia_21 Ia_22 Ia_23; Ia_31 Ia_32 Ia_33]; % Jacobian in (15)
Ie=eig(IA); % Three eigenvalues in the Jacobian

% Saddle path
I梅ig=sort(Ie);
Ilambda=diag(I梅ig);
Ilambda(1,1); % Unique stable root

IA_1=(Ilambda(1,1)-Ia_22)/Ia_21; % Coefficient of saddle path of leisure
IA_2=Ia_32+Ia_31*IA_1; % Coefficient of saddle path of money, Numerator
IA_3=Ilambda(1,1)-Ia_33; % Coefficient of saddle path of money, Denominator

Iza=0.5759; % A value of z(0) equal to pre-shock value, Public capital-private capital ratio
Iz=[]; Iz=[Iz;0.5759];
Ila=I/ss+IA_1*(Iza-Izss); % A value of l(0)
Il=[]; Il=[Il;Ila];
Ima=Imss+(IA_2/IA_3)*(Iza-Izss); % A value of m(0)
Im=[]; Im=[Im;Ima];
Ica=((1-tau_w)/(1+tau_c))*share*effic*(1-Ila)^(share-1)*Iza^(share*(1-epsiron))*Ila/beta; % A value of c(0)
Ic=[]; Ic=[Ic;Ica];
Ira=(1-share)*effic*(1-Ila)^share*Iza^(share*(1-epsiron)); % A value of r(0)
Ir=[]; Ir=[Ir;Ira];
linfl_a=etha*(1+tau_c)*Ica/Ima-(1-tau_k)*Ira; % A value of π(0)
linfl=[]; linfl=[linfl;linfl_a];

Iya=effic*(1-Ila)^share*Iza^(share*(1-epsiron)); % A value of y(0)
Iy=[]; Iy=[Iy;Iya];
IG_ka=(1-g_l-g_c-g_w)*Iya-Ica; % Growth rate of private capital in period 0
IG_k=[]; IG_k=[IG_k;IG_ka];
IG_kpa=g_l*Iya/Iza; % Growth rate of public capital in period 0
IG_kp=[]; IG_kp=[IG_kp;IG_kpa];
Itheta_a=((g_l+g_c+g_w)-(1-share)*tau_k-share*tau_w)*Iya-(tau_c-kusai*etha*(1+tau_c))*Ica+(1+(1+kusai)*Ima)*t_ss)/((1+kusai)*Ima); % Money growth
rate in period 0
Itheta=[]; Itheta=[Itheta;Itheta_a];
IG_la=(Ila/Itheta_L)*(etha*gamma*(Itheta-linfl_a-IG_ka)-IPai*IG_ka-share*(1-epsiron)*(1-gamma-phai*gamma)*(IG_kpa-IG_ka)+(1-tau_k)*Ira-rho); %
Growth rate of leisure in period 0
IG_l=[]; IG_l=[IG_l;IG_la];
IG_ya=share*(IG_la/(1-Ila))+share*(1-epsiron)*IG_kpa+(1-share*(1-epsiron))*IG_ka; % Grwoth rate of real GDP in period 0
IG_y=[]; IG_y=[IG_y;IG_ya];
IPiketa=(1-tau_k)*Ira-IG_ya; % (1-tau_k)r-G_y in period 0
IPiket=[]; IPiket=[IPiket;IPiketa];
IG_wa=((1+kusai)*Ima/(1+(1+kusai)*Ima))*(Itheta-linfl_a)+(1/(1+(1+kusai)*Ima))*IG_ka; % Grwoth rate of wealth A(0) in period 0
IG_w=[]; IG_w=[IG_w;IG_wa];

% Inequality in a balnaced growth path
nyu=beta/(1+beta+etha); % ν in (17)
Iwss_tild=share*effic*(1-I/ss)^(share-1)*Izss^(share*(1-epsiron)); % w*/K*, Real wage-private capital ratio in a balanced growth
Iw0_tild=share*effic*(1-Ila)^(share-1)*Iza^(share*(1-epsiron)); % w(0)/K(0), Real wage-private capital ratio in period 0
Iwtild=[]; Iwtild=[Iwtild;Iw0_tild];

Ialpha_a=1+(1-tau_w)*(Iwss_tild/(1+(1+kusai)*Imss))*(1-Ila/I/ss)/(-Ilambda(1,1)-(1-tau_w)*(Iwss_tild/(1+(1+kusai)*Imss)))*(1-
I/ss/nyu)); % A value of α(0) in (D-4);
Ialpha=[]; Ialpha=[Ialpha;Ialpha_a];

Iomega_a=1+(Ima/(1+(1+kusai)*Ima))*(Ialpha_a-
1+nyu/I/ss)/Ialpha_a; % A value of ω(0) in (D-6), Coefficient
of (D-6)
Iomega=[]; Iomega=[Iomega;Iomega_a];

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Ifey_a=(Ira*(1+kusai*Ima)/(Ira*(1+kusai*Ima)+Iw0_tild*(1-Ila))*Iomega_a-(Iw0_tild*Ila/(Ira*(1+kusai*Ima)+Iw0_tild*(1-Ila)))*(1-nyu/I1ss)/Ialpha_a: % A value of  $\phi(0)$  in (D-8a)
Ifey=[]; Ifey=[Ifey;Ifey_a];

Ipusai_a=((1-tau_k)*Ira*(1+kusai*Ima)/((1-tau_k)*Ira*(1+kusai*Ima)+(1-tau_w)*Iw0_tild*(1-Ila))*Iomega_a-((1-tau_w)*Iw0_tild*Ila/((1-tau_k)*Ira*(1+kusai*Ima)+(1-tau_w)*Iw0_tild*(1-Ila)))*(1-nyu/I1ss)/Ialpha_a: % A value of  $\psi(0)$ , Coefficient of after-tax income inequality
Ipusai=[]; Ipusai=[Ipusai;Ipusai_a];

Ikapper=0.406; % (bonds+money)/wealth in the benchmark
parameter values

Isigma_aa=1+((2*Ikapper-1)*(1-Ikapper)*Ikapper/((1-Ikapper)^2+Ikapper^2))*((1+kusai)*Ima/0.6835-1); % After-shock value of  $\sigma_a(0)$ 
Isigma_ass=Isigma_aa/Ialpha_a % Balanced growth value of  $\sigma_a$ , i.e.,
 $\sigma_a^*$ 
Isigma_a=[]; Isigma_a=[Isigma_a;Isigma_aa];

Isigma_va=Iomega_a*Isigma_aa/1.1634; % A value of  $\sigma_v(0)$  relative to the
benchmark in period 0
Isigma_v=[]; Isigma_v=[Isigma_v;Isigma_va];

Isigma_ya=Ifey_a*Isigma_aa/0.2837; % A value of  $\sigma_y(0)$  relative to the
benchmark in period 0
Isigma_y=[]; Isigma_y=[Isigma_y;Isigma_ya];

Isigma_yfa=Ipusai_a*Isigma_aa/0.2544; % A value of  $\sigma_{ya}(0)$  relative to the
benchmark in period 0
Isigma_yf=[]; Isigma_yf=[Isigma_yf;Isigma_yfa];

% Transitional dynamics of average variables and standard deviations

nobs=200;

for t=1:nobs

    Izt=Izss+exp(Ilambda(1,1))*(Iza-Izss); % Dynamics of z(t)
    Iza=Izt;
    Iz=[Iz;Izt];

    Ilt=I1ss+exp(Ilambda(1,1))*(Ila-I1ss); % Dynamics of l(t)
    Ila=Ilt;
    Il=[Il;Ilt];

    Imt=Imss+exp(Ilambda(1,1))*(Ima-Imss); % Dynamics of m(t)
    Ima=Imt;
    Im=[Im;Imt];

    Ict=((1-tau_w)/(1+tau_c))*share*effic*(1-Ilt)^(share-1)*Izt^(share*(1-epsiron))*Ilt/beta; % Dynamics of c(t)
    Ic=[Ic;Ict];

    Irt=(1-share)*effic*(1-Ilt)^(share)*Izt^(share*(1-epsiron)); % Dynamics of r(t)
    Ir=[Ir;Irt];

    Iinfl_t=etha*(1+tau_c)*Ict/Imt-(1-tau_k)*Irt; % Dynamics of inflation rate
    Iinfl=[Iinfl;Iinfl_t];

    Iyt=effic*(1-Ilt)^(share)*Izt^(share*(1-epsiron)); % Dynamics of real GDP-capital ratio, y(t)
    Iy=[Iy;Iyt];

    IG_kt=(1-g_I-g_C-g_W)*Iyt-Ict; % Growth rate of private capital in period t
    IG_k=[IG_k;IG_kt];

    IG_kpt=g_I*Iyt/Izt; % Growth rate of public capital in period t
    IG_kp=[IG_kp;IG_kpt];

    Itheta_t=((g_I+g_C+g_W-(1-share)*tau_k-share*tau_w)*Iyt-(tau_c-kusai*etha*(1+tau_c))*Ict+(1+(1+kusai)*Imt)*t_ss)/((1+kusai)*Imt); % Money
growth rate in period t
    Itheta=[Itheta;Itheta_t];

    IG_lt=(Ilt/Itheta_L)*(etha*gamma*(Itheta_t-Iinfl_t-IG_kt)-IPai*IG_kt-share*(1-epsiron)*(1-gamma-phai*gamma)*(IG_kpt-IG_kt)+(1-tau_k)*Irt-rho); %  $d\dot{l}/dt$ 
    IG_l=[IG_l;IG_lt];

    IG_yt=-share*(IG_lt/(1-Ilt))+share*(1-epsiron)*IG_kpt+(1-share*(1-epsiron))*IG_kt; % Grwoth rate of real GDP

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IG_y=[IG_y;IG_yt];

IPiket_t=(1-tau_k)*Irt-IG_yt; % (1-tau_k)r-G_y
IPiket=[IPiket;IPiket_t];

IG_wt=((1+kusai)*Imt/(1+(1+kusai)*Imt))*(ltheta_t-linfl_t)+1/(1+(1+kusai)*Imt))*IG_kt; % Growth rate of wealth A(t)
IG_w=[IG_w;IG_wt];

Ialpha_t=1+(1-tau_w)*(lwss_tild/(1+(1+kusai)*Imss))*(1-lft/lss)/(-lambd(1,1)-(1-tau_w)*(lwss_tild/(1+(1+kusai)*Imss))*(1-lfss/nyu)); %
Dynamics of  $\alpha(t)$ ;
Ialpha=[Ialpha;Ialpha_t];

Isigma_at=Ialpha_t*Isigma_ass; % Dynamics of  $\sigma_a(t)$ 

Iomega_t=1+(lmt/(1+kusai*Imt))*(Ialpha_t-1+nyu/lss)/Ialpha_t; % Dynamics of  $\omega(t)$ 
Iomega=[Iomega;Iomega_t];

Isigma_vt=Iomega_t*Isigma_at/1.1634; % Dynamics of  $\sigma_v(t)$  relative to the benchmark
value

Iwtild_t=share*effic*(1-lft)^(share-1)*Izt^(share*(1-epsiron)); % Dynamics of  $w_{tild}$ 

Ifey_t=(Irt*(1+kusai*Imt)/(Irt*(1+kusai*Imt)+Iwtild_t*(1-lft))*Iomega_t-(Iwtild_t*Ilt/(Irt*(1+kusai*Imt)+Iwtild_t*(1-lft)))*(1-
nyu/lss)/Ialpha_t; % Dynamics of  $\phi(t)$ 
Isigma_yt=Ifey_t*Isigma_at/0.2837;

Ipusai_t=((1-tau_k)*Irt*(1+kusai*Imt)/((1-tau_k)*Irt*(1+kusai*Imt)+(1-tau_w)*Iwtild_t*(1-lft))*Iomega_t-((1-tau_w)*Iwtild_t*Ilt/((1-
tau_k)*Irt*(1+kusai*Imt)+(1-tau_w)*Iwtild_t*(1-lft)))*(1-nyu/lss)/Ialpha_t; % Dynamics of  $\psi(t)$ 
Isigma_yft=Ipusai_t*Isigma_at/0.2544;

Iwtild=[Iwtild;Iwtild_t];
Ifey=[Ifey;Ifey_t];
Ipusai=[Ipusai;Ipusai_t];

Isigma_a=[Isigma_a;Isigma_at];
Isigma_v=[Isigma_v;Isigma_vt];
Isigma_y=[Isigma_y;Isigma_yt];
Isigma_yf=[Isigma_yf;Isigma_yft];

end

% g_I=4.5% (Government investment-GDP ratio=4.5%) and  $\xi=1.10$  (Bonds-money ratio=1.35)

% Parameter values
tau_k=0.276; % Capital income tax rate
tau_w=0.224; % Labor income tax rate
tau_c=0.08; % Consumption tax rate
g_I=0.045; % Ratio of government investment to output
g_C=0.150; % Ratio of government consumption to output
g_W=0; % Ratio of wastful government expenditure to output
beta=1.4; % Weight of leisure in utility
etha=0.12; % Weight of real balances in utility
phai=0.22; % Weight of government consumption in utility
share=0.6; % Labor share in output
effic=0.9; % TFP
rho=0.04; % Rate of time preference
gamma=-1.5; % Elasticity of intertemporal substitution in consumption
epsiron=0.6; % Geometric weight of average externalities derived from private capital
kusai=1.10; % Bonds-money ratio
t_ss=0.0137; % Lump-sum-tax-wealth ratio

nobs=200; % periods

% Steady state values of z and l
fun = @rootGT1;
x0 = [0.7,0.7];

param=[tau_k tau_w tau_c g_I g_C g_W beta etha phai share effic rho gamma epsiron];

```

```

x = fsolve(fun, x0, optimset, param);

Inzss=x(1): % Steady state ratio of public capital to private capital, z*
Inlss=x(2): % Steady state value of leisure, l*

% Balanced growth equilibrium values

Incss=((1-tau_w)/(1+tau_c))*share*effic*(1-Inlss)^(share-1)*Inzss^(share*(1-epsiron))*Inlss/beta; % Consumption-capital ratio, c*
Inyss=effic*(1-Inlss)^share*Inzss^(share*(1-epsiron)); % Output-capital ratio, y*
(1-g_l-g_c-g_w)*Inyss;
Inrss=(1-share)*effic*(1-Inlss)^share*Inzss^(share*(1-epsiron)); % Before-tax return on private capital, r*
(1-tau_k)*Inrss; % After-tax return on private capital
InPai=1-gamma-etha*gamma-phai*gamma; % Π in (12c)
InGss=((1-tau_k)*Inrss-rho)/InPai; % Balanced growth rate
Inmss=((1+etha)*tau_c+etha)*Incss+(tau_k*(1-share)+tau_w*share-g_l-g_c-g_w)*Inyss-t_ss)/((t_ss+(1-1/InPai)*(1-
tau_k)*Inrss-rho/InPai)*(1+kusai)); % Real balances-capital ratio, m*
Inmss/Incss; % Money-bonnds ratio
Intheta_ss=(etha*(1+tau_c)*Incss/Inmss-((1-1/InPai)*(1-tau_k)*Inrss-rho/InPai)); % Money growth rate
Ininfl=etha*(1+tau_c)*Incss/Inmss-(1-tau_k)*Inrss; % Inflation rate
InPiketty=(InPai-1)*(1-tau_k)*Inrss/InPai-rho/InPai; % (1-tau_k)rss-Gss

% Local stability
Intheta_L=((1-share)*(1-gamma-phai*gamma)+phai*gamma)*Inlss/(1-Inlss)+(1-gamma-beta*gamma); % Δ in (12c)

Iny_z=share*(1-epsiron)*Inyss/Inzss; % dy/dz
Iny_l=-share*Inyss/(1-Inlss); % dy/dl

Inc_z=(1-epsiron)*share*Incss/Inzss; % dc/dz
Inc_l=((1-share)*Inlss/(1-Inlss)+1)*Incss/Inlss; % dc/dl

Inr_z=share*(1-epsiron)*Inrss/Inzss; % dr/dz
Inr_l=-share*Inrss/(1-Inlss); % dr/dl

InG_k=(1-g_l-g_c-g_w)*Inyss-Incss; % Growth rate of private capital
InG_kp=g_l*effic*(1-Inlss)^share*Inzss^(share*(1-epsiron)-1); % Growth rate of Public capital

InG_kl=-share*(1-g_l-g_c-g_w)*Inyss/(1-Inlss)-((1-share)*Inlss/(1-Inlss)+1)*Incss/Inlss; % d(G_k)/dl
InG_kz=share*(1-epsiron)*InG_k/Inzss; % d(G_k)/dz

InG_kpz=-(1-share*(1-epsiron))*InG_kp/Inzss; % d(G_kp)/dz
InG_kpl=-share*InG_kp/(1-Inlss); % d(G_kp)/dl

% Elements of (15)
Ina_31=Inmss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*Iny_l+(1-tau_k)*Inr_l*(1+kusai)*Inmss-(tau_c+etha*(1+tau_c))*Inc_l)/((1+kusai)*Inmss)-
InG_kl*Inmss; % a_31
Ina_32=Inmss*((g_l+g_c-(1-share)*tau_k-share*tau_w)*Iny_z+(1-tau_k)*Inr_z*(1+kusai)*Inmss-(tau_c+etha*(1+tau_c))*Inc_z)/((1+kusai)*Inmss)-
InG_kz*Inmss; % a_32
Ina_33=-((g_l+g_c-(1-share)*tau_k-share*tau_w)*Inyss-
(tau_c+etha*(1+tau_c))*Incss+t_ss)/((1+kusai)*Inmss); % a_33

Ina_21=(InG_kpl-InG_kl)*Inzss; % a_21
Ina_22=(InG_kpz-InG_kz)*Inzss; % a_22
Ina_23=0; % a_23

Ina_11=(etha*gamma*Ina_31/Inmss-InPai*InG_kl-share*(1-epsiron)*(1-gamma-phai*gamma)*Ina_21/Inzss+(1-tau_k)*Inr_l)*(Inlss/Intheta_L); % a_11
Ina_12=(etha*gamma*Ina_32/Inmss-InPai*InG_kz-share*(1-epsiron)*(1-gamma-phai*gamma)*Ina_22/Inzss+(1-tau_k)*Inr_z)*(Inlss/Intheta_L); % a_12
Ina_13=(etha*gamma*Ina_33/Inmss)*(Inlss/Intheta_L); % a_13

InA=[Ina_11 Ina_12 Ina_13; Ina_21 Ina_22 Ina_23; Ina_31 Ina_32 Ina_33]; % Jacobian in (15)
Ine=eig(InA); % Three eigenvalues in the Jacobian

% Saddle path
InMeig=sort(Ine);
Inlambda=diag(InMeig);
Inlambda(1,1); % Unique stable root

InA_1=(Inlambda(1,1)-Ina_22)/Ina_21; % Coefficient of saddle path of leisure
InA_2=Ina_32+Ina_31*InA_1; % Coefficient of saddle path of money, Numerator
InA_3=Inlambda(1,1)-Ina_33; % Coefficient of saddle path of money, Denominator

Inza=0.5759; % A value of z(0) equal to pre-shock value, Public capital-private capital ratio
Inz=[]; Inz=[Inz;0.5759];
Inla=Inlss+InA_1*(Inza-Inzss); % A value of l(0)

```

```

Inl=[]: Inl=[Inl;Inla];
Inma=Inmss+(InA_2/1A_3)*(Inza-Inzss); % A value of m(0)
Inm=[]: Inm=[Inm;Inma];
Inca=((1-tau_w)/(1+tau_c))*share*effic*(1-Inla)^(share-1)*Inza^(share*(1-epsiron))*Inla/beta; % A value of c(0)
Inco=[]: Inco=[Inco;Inca];
Inra=(1-share)*effic*(1-Inla)^share*Inza^(share*(1-epsiron)); % A value of r(0)
Inr=[]: Inr=[Inr;Inra];
Ininfl_a=etha*(1+tau_c)*Inca/Inma-(1-tau_k)*Inra; % A value of inflation rate in period 0
Ininfl=[]: Ininfl=[Ininfl;Ininfl_a];

Inya=effic*(1-Inla)^share*Inza^(share*(1-epsiron)); % A value of y(0)
Iny=[]: Iny=[Iny;Inya];
InG_ka=(1-g_1-g_C-g_W)*Inya-Inca; % Growth rate of privagte capital in period 0
InG_k=[]: InG_k=[InG_k;InG_ka];
InG_kpa=g_1*Inya/Inza; % Growth rate of public capital in period 0
InG_kp=[]: InG_kp=[InG_kp;InG_kpa];
Intheta_a=(g_1+g_C+g_W-(1-share)*tau_k-share*tau_w)*Inya-(tau_c-kusai*etha*(1+tau_c))*Inca+(1+(1+kusai)*Inma)*t_ss/((1+kusai)*Inma); % Money growth rate in period 0
Intheta=[]: Intheta=[Intheta;Intheta_a];
InG_la=(Inla/Intheta_L)*(etha*ganmma*(Intheta-Ininfl_a-InG_ka)-InPai*InG_ka-share*(1-epsiron)*(1-gamma-phai*ganmma)*(InG_kpa-InG_ka)+(1-tau_k)*Inra-rho); % Growth rate of leisure in period 0
InG_l=[]: InG_l=[InG_l;InG_la];
InG_ya=-share*(InG_la/(1-Inla))+share*(1-epsiron)*InG_kpa+(1-share*(1-epsiron))*InG_ka; % Grwoth rate of GDP in period 0
InG_y=[]: InG_y=[InG_y;InG_ya];
InPiketa=(1-tau_k)*Inra-InG_ya; % (1-tau_k)r-G_y in period 0
InPiket=[]: InPiket=[InPiket;IPiketa];
InG_wa=((1+kusai)*Inma/(1+(1+kusai)*Inma))*(Intheta-Ininfl_a)+(1/(1+(1+kusai)*Inma))*InG_ka; % Grwoth rate of wealth in period 0
InG_w=[]: InG_w=[InG_w;InG_wa];

% Inequality in a bairnaced growth path
nyu=beta/(1+beta+etha); % v in (17)
Inwss_tild=share*effic*(1-Inlss)^(share-1)*Inzss^(share*(1-epsiron)); % w*/K*, Real wage-private capital ratio in a balanced growth
Inw0_tild=share*effic*(1-Inla)^(share-1)*Inza^(share*(1-epsiron)); % wtild in period 0
Inwtild=[]: Inwtild=[Inwtild;Inw0_tild];

Inalpha_a=1+(1-tau_w)*(Inwss_tild/(1+(1+kusai)*Inmss))*(1-Inla/Inlss)/(-Inlambda(1,1)-(1-tau_w)*(Inwss_tild/(1+(1+kusai)*Inmss))*(1-Inlss/nyu)); %
A value of alpha(0);
Inalpha=[]: Inalpha=[Inalpha;Inalpha_a];

Inomega_a=1+(Inma/(1+kusai)*Inma))*(Inalpha_a-1+nyu/Inlss)/Inalpha_a; %
A value of omega(0)
Inomega=[]: Inomega=[Inomega;Inomega_a];

Infey_a=(Inra*(1+kusai)*Inma)/(Inra*(1+kusai)*Inma+Inw0_tild*(1-Inla))*Inomega_a-(Inw0_tild*Inla/(Inra*(1+kusai)*Inma+Inw0_tild*(1-Inla)))*(1-nyu/Inlss)/Inalpha_a; % A value of phi(0)
Infey=[]: Infey=[Infey;Infey_a];

Inpusai_a=((1-tau_k)*Inra*(1+kusai)*Inma)/((1-tau_k)*Inra*(1+kusai)*Inma+(1-tau_w)*Inw0_tild*(1-Inla))*Inomega_a-((1-tau_w)*Inw0_tild*Inla/((1-tau_k)*Inra*(1+kusai)*Inma+(1-tau_w)*Inw0_tild*(1-Inla)))*(1-nyu/Inlss)/Inalpha_a; % A value of psi(0)
Inpusai=[]: Inpusai=[Inpusai;Inpusai_a];

Inkapper=0.406; % (bonds+money)/wealth in the
benchmark parameter values

Insigma_aa=1+((2*Inkapper-1)*(1-Inkapper)*Inkapper/((1-Inkapper)^2+Inkapper^2))*((1+kusai)*Inma/0.6835-1); % After-shock value of sigma_a(0)
Insigma_ass=Insigma_aa/Inalpha_a; % Balanced growth value of sigma_a,
i.e., sigma_a*
Insigma_a=[]: Insigma_a=[Insigma_a;Insigma_aa];

Insigma_va=Inomega_a*Insigma_aa/1.1634; % A value of sigma_v(0)
Insigma_v=[]: Insigma_v=[Insigma_v;Insigma_va];

Insigma_ya=Infey_a*Insigma_aa/0.2837; % A value of sigma_y(0)
Insigma_y=[]: Insigma_y=[Insigma_y;Insigma_ya];

Insigma_yfa=Inpusai_a*Insigma_aa/0.2544; % A value of sigma_ya(0)
Insigma_yf=[]: Insigma_yf=[Insigma_yf;Insigma_yfa];

% Transitional dynamics of average variables and standard deviations
nobs=200;

```

```

for t=1:nobs

    Inzt=Inzss+exp(Inlambd(1,1))*(Inza-Inzss);    % Dynamics of z(t)
    Inza=Inzt;
    Inz=[Inz;Inzt];

    Inlt=Inlss+exp(Inlambd(1,1))*(Inla-Inlss);    % Dynamics of l(t)
    Inla=Inlt;
    Inl=[Inl;Inlt];

    Inmt=Inmss+exp(Inlambd(1,1))*(Inma-Inmss);    % Dynamics of m(t)
    Inma=Inmt;
    Inm=[Inm;Inmt];

    Inct=((1-tau_w)/(1+tau_c))*share*effic*(1-Inlt)^(share-1)*Inzt^(share*(1-epsilon))*Inlt/beta;    % Dynamics of c(t)
    Inc=[Inc;Inct];

    Inrt=(1-share)*effic*(1-Inlt)^share*Inzt^(share*(1-epsilon));    % Dynamics of r(t)
    Inr=[Inr;Inrt];

    Ininfl_t=etha*(1+tau_c)*Inct/Inmt-(1-tau_k)*Inrt;    % Dynamics of inflation rate
    Ininfl=[Ininfl;Ininfl_t];

    Inyt=effic*(1-Inlt)^share*Inzt^(share*(1-epsilon));    % Dynamics of y(t)
    Iny=[Iny;Inyt];

    InG_kt=(1-g_l-g_C-g_W)*Inyt-Inct;    % Growth rate of private capital
    InG_k=[InG_k;InG_kt];

    InG_kpt=g_l*Inyt/Inzt;    % Growth rate of public capital
    InG_kp=[InG_kp;InG_kpt];

    Intheta_t=((g_l+g_C+g_W-(1-share)*tau_k-share*tau_w)*Inyt-(tau_c-kusai*etha*(1+tau_c))*Inct+(1+(1+kusai)*Inmt)*t_ss)/((1+kusai)*Inmt); %
    Money growth rate in period t
    Intheta=[Intheta;Intheta_t];

    InG_lt=(Inlt/Intheta_L)*(etha*gamma*(Intheta_t-Ininfl_t-InG_kt)-InPai*InG_kt-share*(1-epsilon)*(1-gamma-phi*gamma)*(InG_kpt-InG_kt)+(1-
    tau_k)*Inrt-rho); % dl^dot/dt
    InG_l=[InG_l;InG_lt];

    InG_yt=-share*(InG_lt/(1-Inlt))+share*(1-epsilon)*InG_kpt+(1-share*(1-epsilon))*InG_kt;    % Growth rate of GDP
    InG_y=[InG_y;InG_yt];

    InPiket_t=(1-tau_k)*Inrt-InG_yt;    % (1-tau_k)r-G_y
    InPiket=[InPiket;InPiket_t];

    InG_wt=((1+kusai)*Inmt/(1+(1+kusai)*Inmt))*(Intheta_t-Ininfl_t)+(1/(1+(1+kusai)*Inmt))*InG_kt;    % Growth rate of wealth
    InG_w=[InG_w;InG_wt];

    Inalpa_t=1+(1-tau_w)*(Inwss_tild/(1+(1+kusai)*Inmss))*(1-Inlt/Inlss)/(-Inlambd(1,1)-(1-tau_w)*(Inwss_tild/(1+(1+kusai)*Inmss))*(1-
    Inlss/nyu)); % Dynamics of alpha(t);
    Inalpa=[Inalpa;Inalpa_t];

    Insigma_at=Inalpa_t*Insigma_ass;    % Dynamics of sigma_a(t)

    Inomega_t=1+(Inmt/(1+kusai)*Inmt))*(Inalpa_t-1+nyu/Inlss)/Inalpa_t; % Dynamics of omega(t)
    Inomega=[Inomega;Inomega_t];

    Insigma_vt=Inomega_t*Insigma_at/1.1634;    % Dynamics of sigma_v(t) relative to the benchmark value

    Inwtild_t=share*effic*(1-Inlt)^(share-1)*Inzt^(share*(1-epsilon));

    Infey_t=(Inrt*(1+kusai)*Inmt)/(Inrt*(1+kusai)*Inmt+Inwtild_t*(1-Inlt))*Inomega_t-(Inwtild_t*Inlt/(Inrt*(1+kusai)*Inmt)+Inwtild_t*(1-
    Inlt))*(1-nyu/Inlss)/Inalpa_t; % Dynamics of phi(t)
    Insigma_yt=Infey_t*Insigma_at/0.2837;    % Dynamics of sigma_y(t) relative to the benchmark value

    Inpusai_t=((1-tau_k)*Inrt*(1+kusai)*Inmt)/((1-tau_k)*Inrt*(1+kusai)*Inmt+(1-tau_w)*Inwtild_t*(1-Inlt))*Inomega_t-((1-
    tau_w)*Inwtild_t*Inlt/((1-tau_k)*Inrt*(1+kusai)*Inmt+(1-tau_w)*Inwtild_t*(1-Inlt)))*(1-nyu/Inlss)/Inalpa_t; % Dynamics of psi(t)
    Insigma_yft=Inpusai_t*Insigma_at/0.2544;    % Dynamics of sigma_yf(t) relative to the benchmark value

    Inwtild=[Inwtild;Inwtild_t];
    Infey=[Infey;Infey_t];
    Inpusai=[Inpusai;Inpusai_t];

```

```

Insigma_a=[Insigma_a;Insigma_at];
Insigma_v=[Insigma_v;Insigma_vt];
Insigma_y=[Insigma_y;Insigma_yt];
Insigma_yf=[Insigma_yf;Insigma_yft];

```

end

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Graphs
figure;
hold on

subplot(5,2,1)
plot([Infl(1:50),Ininfl(1:50)]);
title('1. Inflation');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');
xlabel('time');

subplot(5,2,2)
plot([Infl(1:50),ninfl(1:50)]);
title('2. Inflation');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');
xlabel('time');

subplot(5,2,3)
plot([IG_y(1:50),InG_y(1:50)]);
title('3. Growth rate of income');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');
xlabel('time');

subplot(5,2,4)
plot([IG_y(1:150),nG_y(1:150)]);
title('4. Growth rate of income');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');
xlabel('time');

subplot(5,2,5)
plot([Insigma_a(1:50),Insigma_a(1:50)]);
title('5. Stand. dev. of total wealth');
xlabel('time');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');

subplot(5,2,6)
plot([sigma_a(1:50),nsigma_a(1:50)]);
title('6. Stand. dev. of total wealth');
xlabel('time');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');

subplot(5,2,7)
plot([Insigma_v(1:50),Insigma_v(1:50)]);
title('7. Stand. dev. of income-earning assets');
xlabel('time');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');

subplot(5,2,8)
plot([sigma_v(1:50),nsigma_v(1:50)]);
title('8. Stand. dev. of income-earning assets');
xlabel('time');
legend('  $\xi=1.35$ ','  $\xi=1.1$  ');

subplot(5,2,9)
plot([Insigma_y(1:50),Insigma_y(1:50)]);
title('9. Stand. dev. of before-tax income');
xlabel('time');

```

```
legend('  $\xi=1.35$ ', '  $\xi=1.1$ ' );
```

```
subplot(5,2,10)  
plot([sigma_y(1:50),nsigma_y(1:50)]):  
title('10. Stand. dev. of before-tax income');  
legend('  $\xi=1.35$ ', '  $\xi=1.1$ ' );  
xlabel('time');
```

```
hold off
```