Online Appendix for

"On adjusting the one-sided Hodrick-Prescott filter"

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A Closed form expressions for filter polynomials

For HP-1s, the filter polynomial for the cyclical component is given by

$$W_{t|t,\lambda}(L) = e_t' Q_t' \left(Q_t Q_t' + \frac{1}{\lambda} I_{t-2} \right)^{-1} Q_t \eta_t, \tag{6}$$

for $t = 3, \dots, T$ and where $\eta_t = (L^{t-1}, \dots, L^1, 1)'$, $e_t = (0, \dots, 0, 1)'$ is a t-dimensional column vector, I_{t-2} is an identity matrix whose size is $(t-2) \times (t-2)$, and Q_t is second order differencing matrix whose size is $(t-2) \times t$. Specifically,

$$Q_t = \begin{pmatrix} 1 & -2 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 1 & -2 & 1 \end{pmatrix}. \tag{7}$$

To show this, we use insights of Hamilton (2018) and Phillips and Jin (2021). Specifically, the minimization problem of Equation (1) has the matrix representation

$$\hat{\tau}_{1:T,\lambda} = \underset{\tau_{1:T}}{\arg\min} \left\{ (y_{1:T} - \tau_{1:T})'(y_{1:T} - \tau_{1:T}) + \lambda \tau_{1:T}' Q_T' Q_T \tau_{1:T} \right\}, \tag{8}$$

where $\hat{\tau}_{1:T,\lambda} = (\hat{\tau}_{1|T,\lambda}, \dots, \hat{\tau}_{T|T,\lambda})'$, $\tau_{1:T} = (\tau_1, \dots, \tau_T)'$, and $y_{1:T} = (y_1, \dots, y_T)'$. Taking the derivative with respect to $\tau_{1:T}$ yields

$$-2(y_{1:T} - \tau_{1:T}) + 2\lambda Q_T' Q_T \tau_{1:T}. \tag{9}$$

Setting this expression to zero and rearranging gives the solution

$$\hat{\tau}_{1:T,\lambda} = (I_T + \lambda Q_T' Q_T)^{-1} y_{1:T} = \left\{ I_T - Q_T'(Q_T Q_T' + \lambda^{-1} I_{T-2})^{-1} Q_T \right\} y_{1:T}. \tag{10}$$

where the second equality is given by the Woodbury matrix identity. Since the estimated cyclical component is $\hat{\psi}_{1:T,\lambda} = y_{1:T} - \hat{\tau}_{1:T,\lambda}$, similarly defining $\hat{\psi}_{1:T,\lambda} = (\hat{\psi}_{1|T,\lambda}, \dots, \hat{\psi}_{1|T,\lambda})'$, it follows that

$$\begin{aligned} \hat{\psi}_{1:T,\lambda} &= \left\{ I_T - \left(I_T - Q_T'(Q_T Q_T' + \lambda^{-1} I_{T-2})^{-1} Q_T \right) \right\} y_{1:T} && (11) \\ &= Q_T'(Q_T Q_T' + \lambda^{-1} I_{T-2})^{-1} Q_T y_{1:T} && (12) \\ &= Q_T'(Q_T Q_T' + \lambda^{-1} I_{T-2})^{-1} Q_T \eta_T y_T, && (13) \end{aligned}$$

where $\eta_T = (L^{T-1}, \dots, L^1, 1)'$.

Finally, the cyclical component of HP-1s $\hat{\psi}_{t|t,\lambda}$ is obtained by setting the information set of $\hat{\psi}_{1:T,\lambda}$ to t (for $t = 3, \dots, T$) and picking its last element, i.e.

$$\hat{\psi}_{t|t,\lambda} = e_t' \hat{\psi}_{1:t,\lambda} = e_t' Q_t' (Q_t Q_t' + \lambda^{-1} I_{t-2})^{-1} Q_t \eta_t y_t, \tag{14}$$

where $e_t = (0, \dots, 0, 1)'$ is a t-dimensional column vector.

We use the filter weights of HP-1s given a sample size of $t = T = 1{,}000$ so as to avoid problems related to the small sample properties of the filter weights (Schüler (2020)). We ensure that increasing the sample size does not alter the frequency domain properties of the filter polynomial for a given value of the smoothing parameter. In an extension of our methodology, not discussed further in this paper, we also take into account HP-1s filter weights derived from smaller samples, however. We do so such that the adjustment of HP-1s can take finite sample issues of HP-1s into account. For more information, please see the computer programs that are available from the authors' website (see Footnote 1 in the main paper).

For the cyclical component of HP-2s, we use the large sample result for the filter polynomial given in King and Rebelo (1993), i.e.

$$W_{t|T,\lambda}(L) := \frac{\lambda[1-L]^2[1-L^{-1}]^2}{1+\lambda[1-L]^2[1-L^{-1}]^2}. \tag{15}$$

This assumes that $T \to \infty$ and t being far away from the beginning and the end of the sample.

B Filtering from a frequency-domain perspective: The power transfer function

Power transfer functions allow us to analyze the extent to which different variants of the HP filter succeed in eliminating lower frequencies and preserving higher frequencies. For the HP filters, PTFs summarize the cyclical properties of the HP filter independently of any data generating process (DGP). This is because the filter weights do not depend on the data that is being filtered.[4]

Assume that $\mathcal{W}(L)$ is a linear filter polynomial with $\mathcal{W}(L) = \sum_{j=-\infty}^{\infty} w_j L^j$ and absolutely summable polynomial coefficients. The PTF of the linear filter polynomial $\mathcal{W}(L)$ is defined as

$$PTF_{\mathcal{W}}(\omega) = |\mathcal{W}(e^{-\mathrm{i}\omega})|^2, \tag{16}$$

with $\mathrm{i}^2 = -1$. $PTF_{\mathcal{W}}(\omega)$ is a non-negative and real-valued scalar function that measures how $\mathcal{W}(L)$ dampens ($PTF_{\mathcal{W}}(\omega) < 1$), passes ($PTF_{\mathcal{W}}(\omega) = 1$), or amplifies ($PTF_{\mathcal{W}}(\omega) > 1$)

[4]A counterexample for this is the Hamilton (2018) filter. The PTF of this filter varies with the DGP of the series to be filtered (see Schüler (2021)).

movements at specific frequencies ω of the time series the researcher aims to filter.

To illustrate the interpretation of $PTF_{\mathcal{W}}(\omega)$, assume y_t is a stationary stochastic process with autocovariances $\gamma_k = \text{Cov}(y_t, y_{t-k})$ and define the autocovariance-generating function of y_t as $g_y(z) = \sum_{k=-\infty}^{\infty} \gamma_k z^k$, where z denotes a complex scalar and $\sum_{k=-\infty}^{\infty} |\gamma_k| < \infty$. Evaluating the autocovariance-generating function at $z = e^{-i\omega}$ and dividing by 2π yields the spectral density of y_t:

$$S_y(\omega) = \frac{1}{2\pi} g_y(e^{-\mathrm{i}\omega}). \tag{17}$$

Integrating the spectral density over the interval $[-\pi, \pi]$ gives the variance of y_t, i.e.

$$\text{Var}(y_t) = \int_{-\pi}^{\pi} S_y(\omega) d\omega = 2 \int_{0}^{\pi} S_y(\omega) d\omega,$$

where we can interpret the value of $\omega \in [0, \pi]$ as a cycle frequency measured in radians.[5] This suggests that we can decompose the variance of y_t into portions related to movements at different frequencies. For instance, integrating the spectral density over the interval $[0, \omega_1]$ with $\omega_1 < \pi$, i.e. $2 \int_0^{\omega_1} S_y(\omega) d\omega$, would give the portion of variance related to movements at frequencies less than or equal to ω_1.

From this, it is possible to show that the spectral densities of y_t and the filtered series $x_t = \mathcal{W}(L) y_t$ are related through

$$S_x(\omega) = PTF_{\mathcal{W}}(\omega) \cdot S_y(\omega). \tag{18}$$

It is in this sense that $PTF_{\mathcal{W}}(\omega)$ dampens ($PTF_{\mathcal{W}}(\omega) < 1$), passes ($PTF_{\mathcal{W}}(\omega) = 1$), or amplifies ($PTF_{\mathcal{W}}(\omega) > 1$) movements at specific frequencies ω of y_t.

[5] Given the periodicity of $e^{-i\omega}$, cycles with frequencies higher than two periods are indistinguishable, a phenomenon commonly called the aliasing effect (see, for example, Hamilton (1994)). Hence, the analysis of any PTF is limited to $[0, \pi]$.

C Adjustment parameters for the one-sided Hodrick-Prescott filter

Table 3: Given a value of λ, i.e. the HP-2s smoothing parameter, HP-1s* uses the value of λ^* and κ

λ	λ^*	κ	λ	λ^*	κ	λ	λ^*	κ	λ	λ^*	κ
1.00	0.35	2.7174	55.00	22.15	1.3921	1000	406	1.1718	55000	22411	1.0598
1.25	0.45	2.5372	57.50	23.16	1.3869	1250	508	1.1617	57500	23431	1.0591
1.50	0.55	2.4111	60.00	24.17	1.3820	**1600**	**650**	**1.1513**	60000	24450	1.0584
1.75	0.65	2.3165	62.50	25.19	1.3774	1750	711	1.1477	62500	25469	1.0578
2.00	0.75	2.2420	65.00	26.20	1.3730	2000	813	1.1425	65000	26488	1.0572
2.25	0.85	2.1814	67.50	27.21	1.3688	2250	915	1.1381	67500	27507	1.0567
2.50	0.95	2.1308	70.00	28.23	1.3648	2500	1017	1.1342	70000	28526	1.0562
2.75	1.04	2.0877	72.50	29.24	1.3610	2750	1118	1.1309	72500	29546	1.0557
3.00	1.14	2.0503	75.00	30.25	1.3574	3000	1220	1.1279	75000	30565	1.0552
3.25	1.24	2.0176	77.50	31.26	1.3540	3250	1322	1.1252	77500	31584	1.0547
3.50	1.34	1.9885	80.00	32.28	1.3506	3500	1424	1.1227	80000	32603	1.0543
3.75	1.44	1.9625	82.50	33.29	1.3475	3750	1525	1.1205	82500	33623	1.0538
4.00	1.55	1.9390	85.00	34.30	1.3444	4000	1627	1.1184	85000	34642	1.0534
4.25	1.65	1.9177	87.50	35.32	1.3415	4250	1729	1.1166	87500	35661	1.0530
4.50	1.75	1.8981	90.00	36.33	1.3387	4500	1831	1.1148	90000	36680	1.0527
4.75	1.85	1.8802	92.50	37.35	1.3359	4750	1933	1.1132	92500	37700	1.0523
5.00	1.95	1.8636	95.00	38.36	1.3333	5000	2034	1.1116	95000	38719	1.0519
5.25	2.05	1.8483	97.50	39.37	1.3308	5250	2136	1.1102	97500	39738	1.0516
5.50	2.15	1.8339	100	40	1.3283	5500	2238	1.1089	100000	40758	1.0512
5.75	2.25	1.8206	125	51	1.3077	5750	2340	1.1076	125000	50951	1.0484
6.00	2.35	1.8080	150	61	1.2919	6000	2442	1.1064	150000	61145	1.0462
6.25	**2.45**	**1.7962**	175	71	1.2792	6250	2543	1.1053	175000	71340	1.0444
6.50	2.55	1.7851	200	81	1.2688	6500	2645	1.1042	200000	81534	1.0429
6.75	2.65	1.7746	225	91	1.2599	6750	2747	1.1032	225000	91730	1.0416
7.00	2.75	1.7647	250	101	1.2523	7000	2849	1.1022	250000	101925	1.0405
7.25	2.85	1.7552	275	111	1.2456	7250	2951	1.1012	275000	112120	1.0396
7.50	2.95	1.7463	300	122	1.2397	7500	3053	1.1003	300000	122316	1.0387
7.75	3.05	1.7377	325	132	1.2343	7750	3154	1.0995	325000	132512	1.0379
8.00	3.15	1.7296	350	142	1.2295	8000	3256	1.0986	350000	142708	1.0372
8.25	3.25	1.7218	375	152	1.2251	8250	3358	1.0978	375000	152904	1.0366
8.50	3.35	1.7143	400	162	1.2211	8500	3460	1.0971	**400000**	**163101**	**1.0360**
8.75	3.45	1.7072	425	172	1.2174	8750	3562	1.0964	425000	173297	1.0354
9.00	3.55	1.7004	450	183	1.2140	9000	3664	1.0956	450000	183494	1.0349
9.25	3.65	1.6938	475	193	1.2108	9250	3765	1.0950	475000	193691	1.0344
9.50	3.76	1.6875	500	203	1.2078	9500	3867	1.0943	500000	203887	1.0340
9.75	3.86	1.6814	525	213	1.2051	9750	3969	1.0937	525000	214084	1.0336
10.00	3.96	1.6755	550	223	1.2024	10000	4071	1.0930	550000	224281	1.0332
12.50	4.96	1.6265	575	233	1.2000	12500	5089	1.0878	575000	234478	1.0328
15.00	5.97	1.5897	600	243	1.1976	15000	6108	1.0837	600000	244675	1.0324
17.50	6.98	1.5607	625	254	1.1954	17500	7127	1.0804	625000	254873	1.0321
20.00	7.99	1.5370	650	264	1.1933	20000	8145	1.0776	650000	265070	1.0318
22.50	9.00	1.5172	675	274	1.1913	22500	9164	1.0753	675000	275267	1.0315
25.00	10.01	1.5001	700	284	1.1894	25000	10183	1.0733	700000	285465	1.0312
27.50	11.02	1.4853	725	294	1.1876	27500	11202	1.0715	725000	295662	1.0309
30.00	12.03	1.4723	750	304	1.1859	30000	12221	1.0699	750000	305860	1.0307
32.50	13.04	1.4606	775	315	1.1842	32500	13240	1.0685	775000	316057	1.0304
35.00	14.05	1.4502	800	325	1.1826	35000	14259	1.0672	800000	326255	1.0302
37.50	15.06	1.4407	825	335	1.1811	37500	15278	1.0660	825000	336453	1.0299
40.00	16.08	1.4320	850	345	1.1796	40000	16297	1.0649	850000	346650	1.0297
42.50	17.09	1.4241	875	355	1.1782	42500	17316	1.0639	875000	356848	1.0295
45.00	18.10	1.4167	900	365	1.1768	45000	18335	1.0629	900000	367046	1.0293
47.50	19.11	1.4099	925	376	1.1755	47500	19354	1.0621	925000	377244	1.0291
50.00	20.12	1.4036	950	386	1.1743	50000	20373	1.0612	950000	387442	1.0289
52.50	21.14	1.3977	975	396	1.1730	52500	21392	1.0605	975000	397640	1.0287
									1000000	407838	1.0285

Notes: λ denotes the smoothing parameter of the two-sided HP filter. λ^* is the corresponding adjusted smoothing parameter, used as an input to the one-sided HP filter. κ is the scaling factor by which the extracted cyclical component of the one-sided HP filter is multiplied. For instance, consider HP-2s with λ =1,600 (column three, in bold). Instead of 1,600, the adjusted HP-1s employs a smoothing parameter with value of 650 (λ^*). In parallel, it multiplicatively rescales the extracted cyclical component by a factor of 1.1513. We also offer software implementing the adjusted HP-1s for a given HP-2s smoothing parameter (see `https://sites.google.com/site/yvesschueler/research`).

D Applying the adjusted one-sided HP filter

D.1 Data

The data used in this study was collected from three different sources. Yearly data was obtained from the Jordà-Schularick-Taylor Macrohistory Database (Jordà et al. (2017)). Quarterly GDP data was sourced from the OECD Main Economic Indicators Database. The quarterly credit-to-GDP ratio data was obtained from the BIS.

Our yearly (quarterly) sample extends from 1880 to 2020 (1952Q2 to 2022Q2). At the time of writing, the yearly database only covers the time period until 2020.

It is important to note that the availability of data varies across countries, resulting in different starting dates for analysis. For the yearly dataset, the sample period begins in 1990 for France. In the case of Japan, the last available credit data point from 2017 is extended to 2020 using BIS data.

Canada's quarterly data starts from 1991Q1, France's data begins from 1969Q4, and Germany and Italy's data covers the period from 1960Q4. Japan's quarterly data starts from 1964Q4, while the United Kingdom's data begins from 1963Q1. Lastly, the United States' quarterly data spans from 1952Q2.

D.2 Simulation exercise

To fit the ARIMA(p,d,q) models, we first determine the order of integration using ADF tests. We find that all series are I(1) in levels. These findings are robust to different specifications of the test equation (i.e. with and without a time trend). Subsequently, we fit ARMA(p,q) models to the stationary series in first differences. We choose the lag orders of the AR and MA polynomials based on the Akaike Information Criterion searching over a grid with a maximum lag order of three lags for each polynomial. Table 4 gives the estimated ARMA equations fitted for each series. We cumulate the simulated data from the estimated ARMA processes to recover the levels. These are the non-stationary series we filter in the simulation exercise.

Table 4: Estimated ARIMA models

	log GDP
Yearly	$\Delta y_t = 3.27 + 0.83\Delta y_{t-1} - 0.82\Delta y_{t-2} - 0.08\Delta y_{t-3} + \varepsilon_t - 0.4\varepsilon_{t-1} + 0.9\varepsilon_{t-2}$
Quarterly	$\Delta y_t = 0.74 - 0.36\Delta y_{t-1} + 0.33\Delta y_{t-2} + \varepsilon_t + 0.68\varepsilon_{t-1}$
	Credit-to-GDP ratio
Yearly	$\Delta y_t = 0.32 + 0.57\Delta y_{t-1} - 0.84\Delta y_{t-2} + 0.29\Delta y_{t-3} + \varepsilon_t + 0.07\varepsilon_{t-1} + 0.93\varepsilon_{t-2}$
Quarterly	$\Delta y_t = 0.38 + 0.92\Delta y_{t-1} + 0.99\Delta y_{t-2} - 0.93\Delta y_{t-3} + \varepsilon_t - 0.64\varepsilon_{t-1} - 0.92\varepsilon_{t-2} + 0.57\varepsilon_{t-3}$

Notes: Estimated ARIMA models for log US GDP and the US credit-to-GDP ratio. Quarterly data was obtained from the FRED and BIS databases, yearly series were retrieved from the Jordà-Schularick-Taylor Macrohistory Database (see Jordà et al. (2017)). The sample period of the yearly (quarterly) data is 1880 to 2019 (1952Q2 to 2019Q4).

Table 5: Simulation exercise (continued)

	Relative difference in SD		Difference in autocorrelations	
λ	HP-1s*	HP-1s	HP-1s*	HP-1s
T = 50				
White noise				
6.25	**0.01** [1.00]	**-0.49** [0.00]	**0.03** [0.96]	**0.08** [0.04]
1,600	**0.01** [0.95]	**-0.10** [0.05]	**0.03** [0.80]	**0.04** [0.20]
400,000	**-0.01** [0.88]	**-0.04** [0.12]	**0.03** [0.59]	**0.03** [0.41]
Random walk				
6.25	**0.05** [0.97]	**-0.32** [0.03]	**0.08** [1.00]	**0.21** [0.00]
1,600	**0.08** [0.51]	**0.06** [0.49]	**0.08** [0.95]	**0.14** [0.05]
400,000	**0.06** [0.43]	**0.04** [0.57]	**0.05** [0.62]	**0.05** [0.38]
ARIMA				
Yearly:				
6.25	**0.08** [0.84]	**-0.18** [0.16]	**0.00** [1.00]	**0.00** [0.00]
1,600	**0.12** [0.65]	**0.13**[0.35]	**0.11** [0.96]	**0.17** [0.04]
Quarterly:				
1,600	**0.12** [0.64]	**0.13** [0.45]	**0.10** [0.95]	**0.16** [0.05]
400,000	**0.14** [0.41]	**0.13** [0.59]	**0.19** [0.50]	**0.20** [0.50]
T = 500				
White noise				
6.25	**0.00** [1.00]	**-0.51** [0.00]	**0.01** [1.00]	**0.07** [0.00]
1,600	**0.00** [1.00]	**-0.11** [0.00]	**0.00** [1.00]	**0.02** [0.00]
400,000	**0.00** [1.00]	**-0.03** [0.00]	**0.00** [0.91]	**0.01** [0.09]
Random walk				
6.25	**0.04** [1.00]	**-0.33** [0.00]	**0.06** [1.00]	**0.20** [0.00]
1,600	**0.04** [0.49]	**0.04** [0.51]	**0.02** [1.00]	**0.07** [0.00]
400,000	**0.04** [1.00]	**0.11** [0.00]	**0.01** [1.00]	**0.02** [0.00]
ARIMA				
Yearly:				
6.25	**0.06** [1.00]	**-0.20** [0.00]	**0.00** [0.92]	**0.00** [0.08]
1,600	**0.05** [0.93]	**0.08** [0.07]	**0.07** [1.00]	**0.16** [0.00]
Quarterly:				
1,600	**0.05** [0.90]	**0.07** [0.10]	**0.07** [1.00]	**0.16** [0.00]
400,000	**0.02** [0.99]	**0.07** [0.10]	**0.02** [1.00]	**0.04** [0.00]
T = 1000				
White noise				
6.25	**0.00** [1.00]	**-0.51** [0.00]	**0.00** [1.00]	**0.07** [0.00]
1,600	**0.00** [1.00]	**-0.11** [0.00]	**0.00** [1.00]	**0.02** [0.00]
400,000	**0.00** [1.00]	**-0.03** [0.00]	**0.00** [0.98]	**0.01** [0.02]
Random walk				
6.25	**0.04** [1.00]	**-0.33** [0.00]	**0.06** [1.00]	**0.20** [0.00]
1,600	**0.04** [0.50]	**0.04** [0.50]	**0.02** [1.00]	**0.07** [0.00]
400,000	**0.04** [1.00]	**0.11** [0.00]	**0.01** [1.00]	**0.02** [0.00]
ARIMA				
Yearly:				
6.25	**0.06** [1.00]	**-0.20** [0.00]	**0.00** [0.85]	**0.00** [0.15]
1,600	**0.05** [0.97]	**0.07** [0.03]	**0.05** [1.00]	**0.13** [0.00]
Quarterly:				
1,600	**0.05** [0.97]	**0.07** [0.03]	**0.05** [1.00]	**0.13** [0.00]
400,000	**0.01** [1.00]	**0.05** [0.00]	**0.03** [1.00]	**0.10** [0.00]

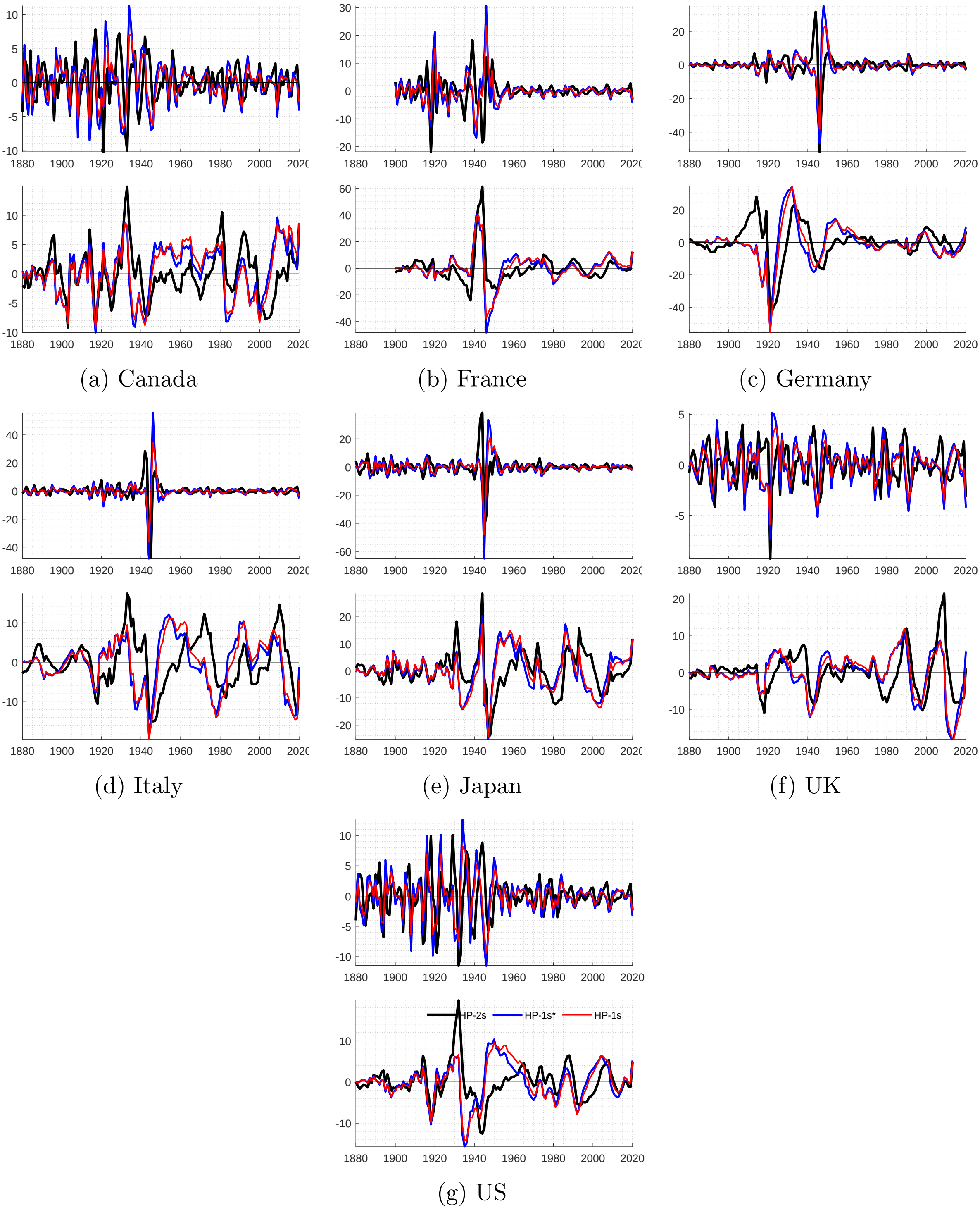
Notes: The table compares properties of cyclical components extracted using the adjusted one-sided HP filter (columns "HP-1s*") and the unadjusted one-sided HP filter (columns "HP-1s"), both relative to the two-sided HP filter. Column λ gives the value of the smoothing parameter used with the two-sided HP filter and the unadjusted one-sided HP filter. The adjusted one-sided HP filter uses the corresponding smoothing and scaling parameter given in Table 3. For a description of the summary statistics ("relative difference in standard deviations" and "difference in autocorrelations"), please see Section 5.1. Numbers are the averages of the summary statistics across simulations. In square brackets, we report the fraction that the summary statistic of a one-sided filter is smaller, in absolute terms, than the summary statistic of the other one-sided filter, i.e. matches HP-2s more closely.

D.3 Application: How different are financial and business cycles for G7 countries?

Table 6: Business and financial cycle statistics for G7 countries

	Standard deviation (%)						
	Canada	France	Germany	Italy	Japan	UK	US
log GDP							
Yearly data							
HP-2s	3.06	4.90	6.35	7.03	7.26	1.93	3.49
HP-1s*	3.29	5.32	6.86	7.19	7.55	2.04	3.65
HP-1s	2.49	4.10	5.33	5.20	5.58	1.56	2.74
Quarterly data							
HP-2s	1.56	1.67	1.67	1.89	1.58	2.29	1.58
HP-1s*	1.60	1.68	1.68	1.91	1.68	2.32	1.61
HP-1s	1.58	1.60	1.63	1.81	1.73	2.23	1.59
Credit-to-GDP ratio							
Yearly data							
HP-2s	4.09	11.34	11.26	6.27	6.95	5.35	4.35
HP-1s*	4.30	11.37	11.49	6.37	7.16	5.52	4.57
HP-1s	4.43	10.82	11.60	6.65	7.17	5.52	4.72
Quarterly data							
HP-2s	5.99	5.25	5.34	8.02	11.48	8.94	5.78
HP-1s*	5.93	5.32	5.33	8.14	11.81	9.38	5.75
HP-1s	6.17	5.79	5.62	9.18	13.47	10.64	6.03
	Autocorrelation						
	Canada	France	Germany	Italy	Japan	UK	US
log GDP							
Yearly data							
HP-2s	0.27	0.21	0.39	0.29	0.28	0.29	0.34
HP-1s*	0.35	0.31	0.47	0.32	0.34	0.38	0.39
HP-1s	0.49	0.48	0.58	0.41	0.45	0.51	0.50
Quarterly data							
HP-2s	0.70	0.50	0.69	0.67	0.71	0.59	0.77
HP-1s*	0.72	0.51	0.70	0.69	0.76	0.61	0.78
HP-1s	0.77	0.57	0.74	0.73	0.81	0.66	0.82
Credit-to-GDP ratio							
Yearly data							
HP-2s	0.78	0.84	0.89	0.90	0.76	0.88	0.87
HP-1s*	0.80	0.84	0.90	0.90	0.77	0.90	0.88
HP-1s	0.85	0.86	0.92	0.92	0.82	0.91	0.91
Quarterly data							
HP-2s	0.96	0.94	0.98	0.98	1.00	0.98	0.99
HP-1s*	0.97	0.95	0.98	0.98	0.99	0.98	0.99
HP-1s	0.97	0.96	0.98	0.99	1.00	0.99	0.99

Notes: This table shows the underlying inputs used in the comparison of financial and business cycles in Table 2. "HP-2s" refers to the cyclical component obtained via the two-sided HP filter, "HP-1s*" the adjusted one-sided HP filter, and "HP-1s" the unadjusted one-sided HP filter. For more details, please see Section 5.2. "Autocorrelation" refers to the first-order autocorrelation.



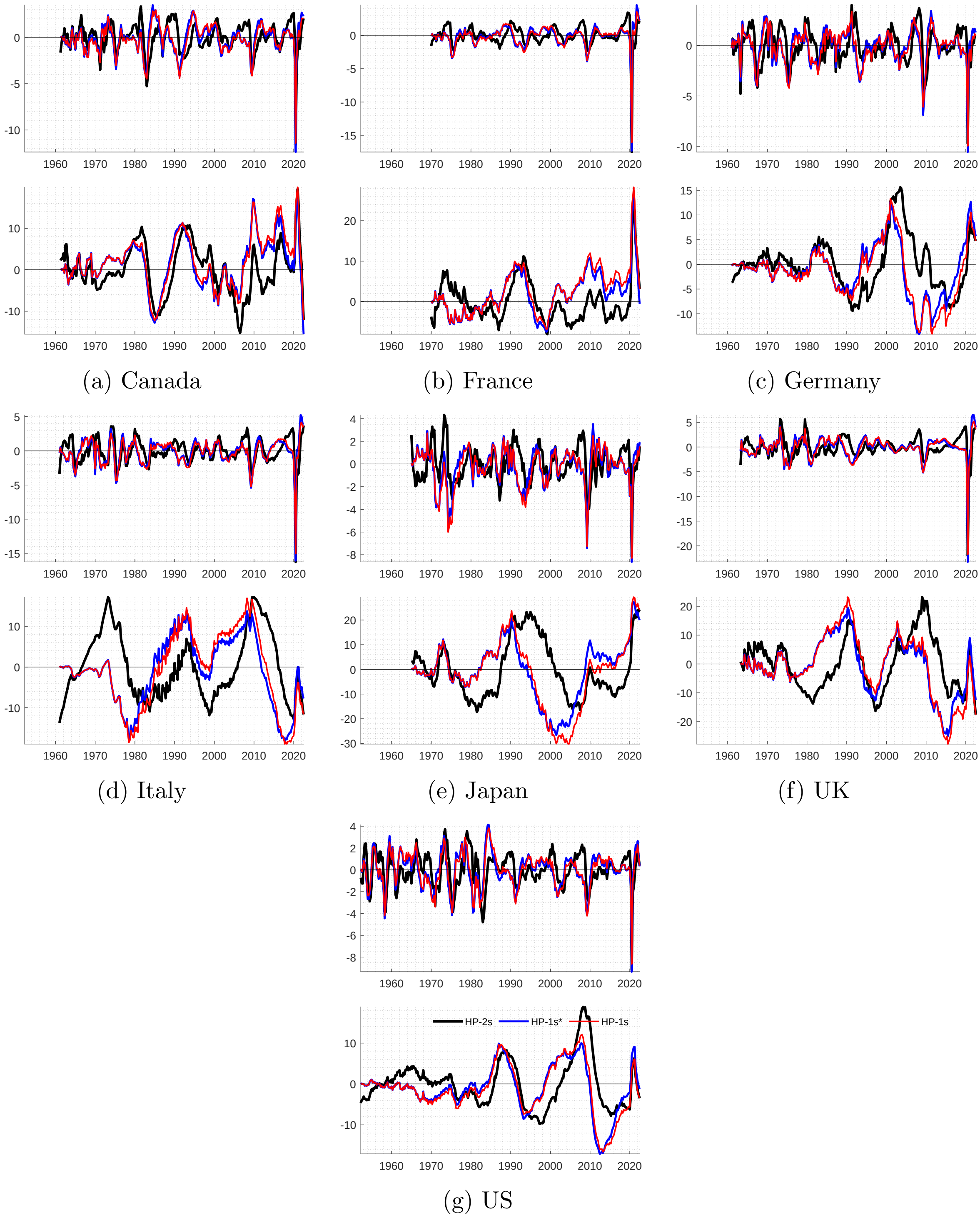
(a) Canada (b) France (c) Germany

(d) Italy (e) Japan (f) UK

(g) US

Figure 2: Yearly Hodrick-Prescott-filtered data

Notes: For each country, the top graph shows filtered log real GDP ($\times 100$). The bottom graph depicts the filtered credit-to-GDP ratio.



(a) Canada (b) France (c) Germany (d) Italy (e) Japan (f) UK (g) US

Figure 3: Quarterly Hodrick-Prescott-filtered data

Notes: For each country, the top graph shows filtered log real GDP ($\times 100$). The bottom graph depicts the filtered credit-to-GDP ratio.