Generalizing Determinacy under Monetary and Fiscal Policy Switches: The Case of the Zero Lower Bound: Online Appendix

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Abstract

This online appendix shows additional results related to our paper entitled "Generalizing Determinacy under Monetary and Fiscal Policy Switches: The Case of the Zero Lower Bound". It first analyzes the determinacy regions of the Bianchi and Melosi (2017) model. It then shows sensitivity robustness analyses for this model. Finally, it includes discussion and analysis implied by the introduction of economic growth in our benchmark model. It shows the associated impact in the determinacy regions.

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1 Bianchi and Melosi (2017) Model and Determinacy

This appendix analyzes the Bianchi and Melosi (2017) model, which is very similar to the extended version of our benchmark model. The estimated model of Bianchi and Melosi (2017) turns out to be much more against the fiscal theory than our benchmark model.

1.1 The Model

To proceed, we keep their notation as much as possible except for a couple of things. Their regimes are specified by a combination of two independent variables, one representing policy stances, ξ_t^p, and the other one governing changes in preferences, ξ_t^d. First, let us define the regime variable s_t in terms of their regimes.

$$s_t = [\xi_t^p, \xi_t^d] = \{[Z, l], [M, h], [F, h]\} = \{0, 1, 2\}. \tag{1}$$

This adjustment is to make the comparison with ours easy: Regime $[Z, l]$, labeled as regime 0, stands for a ZLB economy whereas the non-ZLB regime has two sub-regimes: a monetary (**M**) and a fiscal (**F**) regime, represented by regimes 1 and 2. Regimes 0 and 1 correspond to ours. Hence, there are essentially two different types of **F** regimes – the ZLB case and the non-ZLB case – and one **M** regime. As we will show, this is one of the sources that sets the model against the fiscal theory. To summarize, their three regimes and the corresponding transition probabilities are specified as:[1]

$$P = \begin{bmatrix} p_{ll} & (1-p_{ll})p_{ZM} & (1-p_{ll})(1-p_{ZM}) \\ (1-p_{hh}) & p_{hh}p_{MM} & p_{hh}(1-p_{MM}) \\ (1-p_{hh}) & p_{hh}(1-p_{FF}) & p_{hh}p_{FF} \end{bmatrix} \tag{2}$$

Next, we write their model only with their endogenous variables, ignoring the exogenous parts because they do not affect the determinacy of the model. The eliminated variables include $\tilde{P}_t^m, \tilde{R}_{t,t+1}^m, \hat{y}_t^*, d_t, \bar{d}_{\xi_t^d}, \tilde{tr}_t^*, \bar{g}_t, \tilde{g}_t, a_t, \mu_t, tp_t$ (refer to their paper for the definitions of these variables). Note that the regime-dependent variable $\bar{d}_{\xi_t^d}$ can be removed because it enters the linearized Euler equation in an additive fashion, therefore, it is also exogenous, not affecting determinacy result. Hence, the preference regime variable ξ_t^d only affects the monetary policy rule. The model can then be succinctly written as:

$$\begin{aligned} \tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \kappa\vartheta_1 \hat{y}_t - \kappa\vartheta_2 \hat{y}_{t-1}, & (3) \\ \hat{y}_t &= \mu E_t \hat{y}_{t+1} + (1-\mu)\hat{y}_{t-1} - \tilde{\varphi}(\hat{R}_t - E_t\tilde{\pi}_{t+1}), & (4) \\ \tilde{R}_t &= (1 - Z_{\xi_t^d}(s_t)) \left\{ \rho_R(s_t)\tilde{R}_{t-1} + (1-\rho_R(s_t))(\psi_\pi(s_t)\tilde{\pi}_t + \psi_y(s_t)\hat{y}_t) \right\} & (5) \\ &\quad + Z_{\xi_t^d}(s_t)\rho_{R,Z}\tilde{R}_{t-1}, & \\ \tilde{b}_t^m &= \beta^{-1}\tilde{b}_{t-1}^m + b^m\beta^{-1}\left[\tilde{R}_{t-1} - \tilde{\pi}_t - \hat{y}_t + \hat{y}_{t-1}\right] - \tilde{\tau}_t + \tilde{tr}_t, & (6) \\ \tilde{\tau}_t &= \rho_\tau(s_t)\tilde{\tau}_{t-1} + (1-\rho_\tau(s_t))\left[\delta_b(s_t)\tilde{b}_{t-1}^m + \delta_y \hat{y}_t\right], & (7) \\ \tilde{tr}_t &= \rho_{tr}\tilde{tr}_{t-1} + (1-\rho_{tr})\phi_y \hat{y}_t, & (8) \end{aligned}$$

where $\vartheta_1 = \frac{1}{1-\Phi M_a^{-1}} + \frac{\alpha}{1-\alpha}$, $\vartheta_2 = \frac{\Phi M_a^{-1}}{1-\Phi M_a^{-1}}$, $\mu = \frac{1}{1+\Phi M_a^{-1}}$, $\tilde{\varphi} = \frac{(1-\Phi M_a^{-1})}{1+\Phi M_a^{-1}}$ and $M_a = \exp(\gamma)$. In our notation, $[\tilde{\pi}_t\ \hat{y}_t\ \tilde{R}_t\ \tilde{b}_t^m\ \tilde{\tau}_t]$ correspond to $[\pi_t\ y_t\ r_t\ b_t\ \tau_t]$. Next we write their estimated

[1] Our transition probability matrix P is specified as $\sum_{j=1}^{S} p_{ij} = 1$ while they use the transposed version such that $\sum_{i=1}^{S} p_{ij}$. It does not matter how to specify P as long as the algebraic operation is correct.

or calibrated parameters in terms of our regime variable s_t as follows:

	$s_t = 0$	$s_t = 1$	$s_t = 2$
	$[\xi_t^p, \xi_t^d] = [Z, l]$	$[\xi_t^p, \xi_t^d] = [M, h]$	$[\xi_t^p, \xi_t^d] = [F, h]$
$\psi_\pi(s_t)$	0	$\psi_{\pi,\xi_t^p} = 1.6019$	$\psi_{\pi,\xi_t^p} = 0.6356$
$\psi_y(s_t)$	0	$\psi_{y,\xi_t^p} = 0.5065$	$\psi_{y,\xi_t^p} = 0.2709$
$\rho_R(s_t)$	0	$\rho_{R,\xi_t^p} = 0.8652$	$\rho_{R,\xi_t^p} = 0.6663$
$\delta_b(s_t)$	0	$\delta_{b,\xi_t^p} = 0.0712$	$\delta_{b,\xi_t^p} = 0$
$\rho_\tau(s_t)$	0	$\rho_{\tau,\xi_t^p} = 0.9652$	$\rho_{\tau,\xi_t^p} = 0.6874$
$Z_{\xi_t^d}(s_t)$	1	0	0

$$
\begin{aligned}
\beta &= 0.9985,\ \kappa = 0.0073,\ \Phi = 0.8628,\ \gamma = 0.004185,\ \rho_{R,Z} = 0.2. \\
b^m &= 0.2672 \times 4,\ \rho_{\tau,Z} = 0.6874. \\
\rho_{tr} &= 0.4599,\ \delta_y = 0.2766,\ \delta_e = 0.3661,\ \phi_y = -0.2910. \\
p_{ll} &= 0.9306,\ p_{hh} = 0.9995,\ p_{MM} = p_{FF} = 0.9923,\ p_{ZM} = 0.9225.
\end{aligned}
$$

Note that the New-Keynesian block of the first three equations is very similar to an extended version of our benchmark model, incorporating endogenous persistence. The remaining parts constitute the fiscal block, which is more involved, but the core factor is the same as ours: the coefficient of $\tilde{b}_{t-1}$ is given by β^{-1} and that of the tax policy is $\delta_b(s_t)$. By letting the vector of endogenous variables be $x_t = [\tilde{\pi}_t\ \hat{y}_t\ \tilde{R}_t\ \tilde{b}_t^m\ \tilde{\tau}_t\ \tilde{tr}_t]'$, it is straightforward to write their model in the form of a general MSRE model, (see equation (8) in the paper).

Some notable differences in parameter values are remarked as follows before the analysis. First, their model considers a longer history dating back to 1960s. Therefore, the empirical evidence is in favor of the existence of another **F** regime apart from the ZLB regime. Second, fiscal policy in this model is more passive in regime 1 and it is passive in regime 2 with the same probability as regime 1. This implies that fiscal policy is much more passive in non-ZLB regimes than our benchmark model on average. Third, the implied probability of staying in a ZLB regime when the previous regime is also a ZLB regime is 0.9306, implying an average duration of 14 to 15 quarters, which is much shorter than ours. As long as determinacy is concerned, these differences actually weaken the fiscal theory.

1.2 Determinacy Analysis

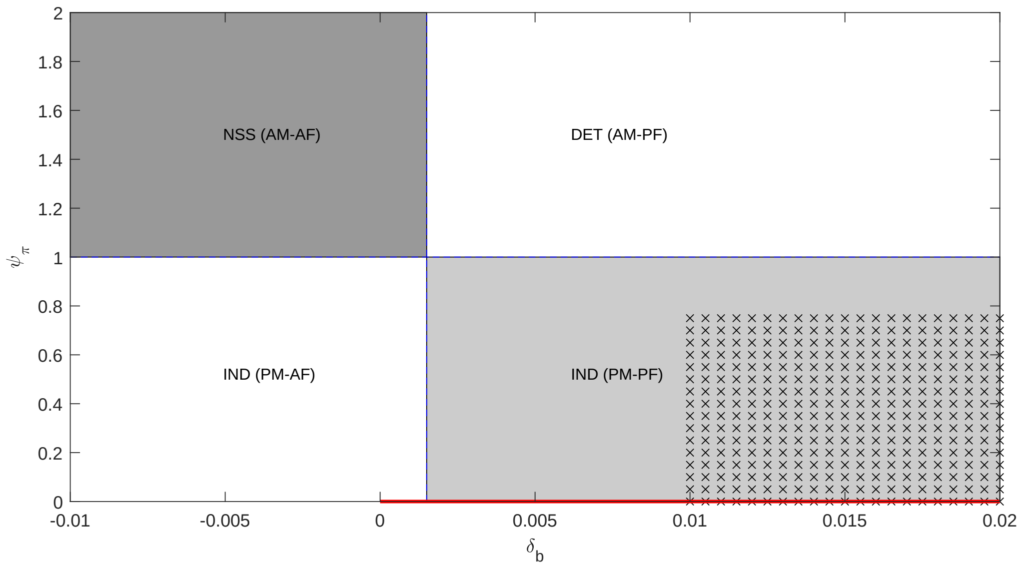
Because there is an additional **F** regime in this model, we fix the parameter values at this regime and examine the determinacy property in terms of $\psi_\pi(s_t)$ and $\delta_b(s_t)$, as in the benchmark model. This does not mean that the model has only ZLB and **M** regimes. The three regimes are all taken into account in the analysis.

First, Figure 8 represents the taxonomy of the model under fixed regime. This figure is exactly the same as our benchmark counterpart (Figure 1 in the paper) with a notable exception. The indeterminacy region contains an area with a PM-PF policy mix denoted by $\times$ representing that the model solution is non-unique, since it is associated with a pair of complex-valued MOD solutions and therefore the model cannot be determinate.

Second, Figure 9 replicates Figure 2 in the paper.[2] Left panel shows which policy mixes would lead to determinacy in regime 1 when a ZLB regime 0 is given by $\psi_\pi(0) = 0$ and $\delta_b(0) = 0$, and **F** in regime 2 is given by $\psi_\pi(2) = 0.6356$ and $\delta_b(2) = 0$. Similar to the

[2] Areas with complex-valued MOD solutions, denoted by $\times$, also exist under regime-switching, as Figure 9 shows, but only in the region of indeterminacy and no stable solution in the neighborhood of the PM-PF and/or AM-AF regions, therefore it does not affect the determinacy region. Refer to Cho (2021) for technical aspects related to this phenomenon.

Figure 8: Bianchi-Melosi Model Classification under Fixed Regime

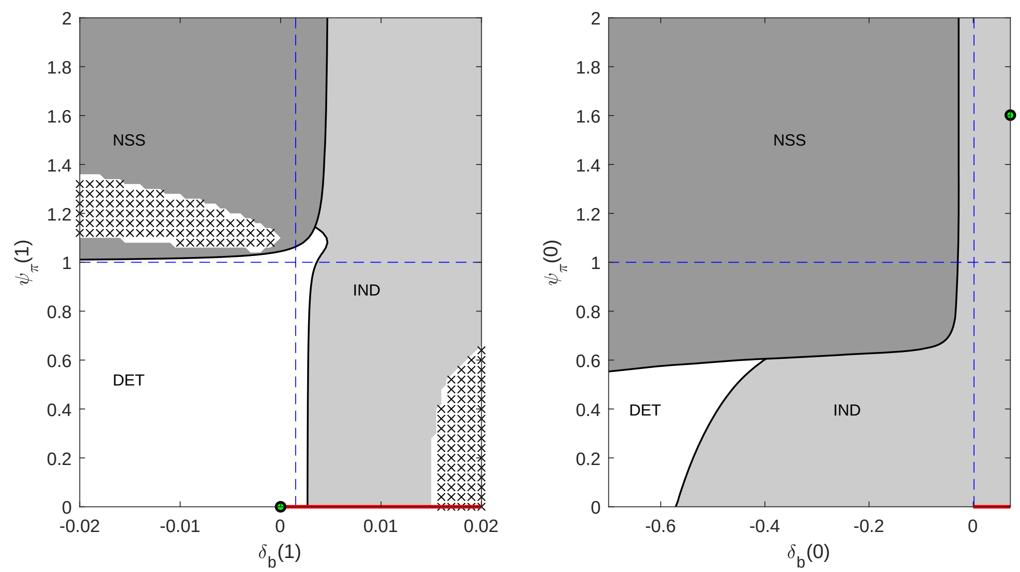


This figure depicts determinacy/indeterminacy/no stable solution regions for Bianchi and Melosi model in terms of ψ_π and δ_b under fixed-regimes. The thick solid line in red depicts a range for the fiscal policy $\delta_b \geq 0$ that is used in the literature. The indeterminacy region with $\times$ represents a region in which the MOD solution is complex-valued.

benchmark case, this model turns out to be indeterminate for almost all combinations of $\mathbf{M}$ policy mixes in regime 1. There does exist a very small area in which determinacy prevails: a very mildly active monetary policy such as $\psi_\pi(1) < 1.1$ and a very mildly passive fiscal policy with $\delta_b(1) \in (0.0015,\ 0.0025)$. However, the model is clearly indeterminate when evaluated with their estimated policy parameters, $\psi_\pi(1) = 1.6019$ and $\delta_b(1) = 0.0712$. This finding can also be confirmed from the right panel. Specifically, this figure fixes the $\mathbf{M}$ regime 1 at $\psi_\pi(1) = 1.6019$ and $\delta_b(1) = 0.0712$, and the $\mathbf{F}$ regime at the estimated parameter values, and seek for the combinations $\psi_\pi(0)$ and $\delta_b(0)$ that leads to determinacy where regime 0 represents any regime. Therefore, according to our taxonomy under regime-switching, the policy mix in this case is overall PM-PF.

We have argued that a negative response of a tax policy to the previous debt to GDP

Figure 9: Bianchi-Melosi Model Classification under Regime-Switching



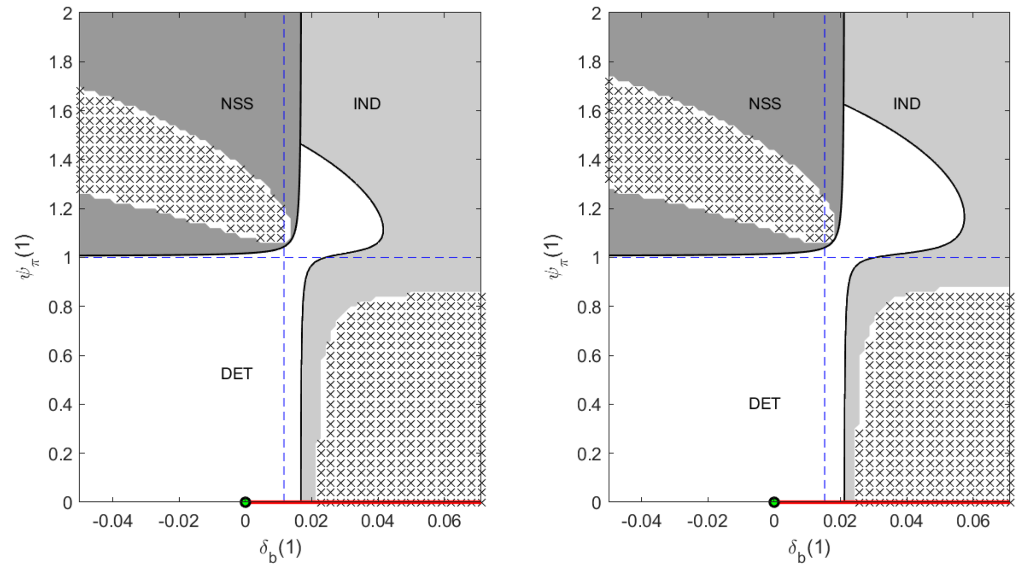
This figure depicts determinacy/indeterminacy/no stable solution regions for the Bianchi-Melosi model. Left panel displays these partitions in terms of $\psi_\pi(1)$ and $\delta_b(1)$ when regime 0 is a ZLB regime with $\psi_\pi(0) = 0$ and $\delta_b(0) = 0$, and regime 2 is regime **F** with $\psi_\pi(2) = 0.6356$ and $\delta_b(2) = 0$. Right panel displays them in terms of $\psi_\pi(0)$ and $\delta_b(0)$ when $\psi_\pi(1) = 1.6019$ and $\delta_b(1) = 0.0712$ in regime 1, and regime 2 is regime **F** with $\psi_\pi(2) = 0.6356$ and $\delta_b(2) = 0$.

hardly makes economic sense. Right panel reveals that for uniqueness of equilibrium, $\delta_b(0)$ must be less than -0.57 , extremely lower than our benchmark counterpart -0.012. A 57% tax cut policy in response to the previous period debt-to-GDP ratio is clearly not acceptable by any economic standard. Moreover, the model is also indeterminate even when regime 0 as well as regime 1 are monetary. This might be due to the existence of an additional **F** regime 2 in this model. To see this, we reexamine the model by assuming away the **F** regime and adjusting the transition probabilities such that $p_{01} = p_{hh}p_{MM}$, $p_{ZM} = 1$ and $p_{hh} = 0.9923$. Indeed, the right panel of Figure 9 without **F** regime (not reported here) resembles that of Figure 2 in the paper very closely: determinacy is recovered when the economy is switching in the neighborhood of two **M** regimes. Also $\delta_b(0) < -0.027$ in the ZLB regime 0 ensures determinacy when regime 1 is given by

$\psi_\pi(1) = 1.6019$ and $\delta_b(1) = 0.0712$. This exercise implies that if the three-regime case is a more realistic description of the U.S. economy, then the fiscal theory is much harder to hold.

We have also performed various robustness tests, reproducing this figure with other model parameter values. As in our benchmark model, the determinacy area is almost invariant to almost all of the parameters. Again, the equilibrium determinacy critically depends on the discount factor in this model as well. Figure 10 replicates Figure 7 in the paper compatible with a real natural rate of 6%. As in the benchmark model, a lower value of the time discount factor expands the determinacy region in the **M** regime, although there is still a large area leading to indeterminacy. This is due to several reasons. For instance, the existence of another **F** regime 2 or the shorter duration of the ZLB regime 0 are responsible for the differences between our benchmark model and this model. The main implication of the determinacy analysis for this model is essentially the same as that of the benchmark model: a lower value of the time discount factor is a necessary ingredient to revive the role of the fiscal theory in a regime-switching environment.

Figure 10: Bianchi-Melosi Model Classification under Regime-Switching from a ZLB regime with Lower Time Discount Factors.



This figure reproduces the left panel of Figure 9 with $\hat{\beta} = 0.9885$ in the left panel and $\hat{\beta} = 0.985$ in the right panel.

2 Robustness Analysis

For ease of exposition, let us reproduce the extended model:

$$\pi_t = \beta E_t \vartheta_1 \pi_{t+1} + \beta(1-\vartheta_1)\pi_{t-1} + \kappa y_t + z_t^{AS}, \tag{9a}$$

$$c_t = \mu E_t c_{t+1} + (1-\mu)c_{t-1} - \varphi(r_t - E_t \pi_{t+1}) + z_t^{IS}, \tag{9b}$$

$$r_t = \varsigma(s_t)\left[(1-\rho)[\psi_\pi(s_t)\pi_t + \psi_y(s_t)y_t] + \rho r_{t-1}\right] + \varsigma(s_t) z_t^{MP}, \tag{9c}$$

$$y_t = f_c c_t + (1-f_c) g_t, \tag{9d}$$

$$b_t = \beta^{-1} b_{t-1} + \bar{b}\beta^{-1}(r_{t-1} - \pi_t) - \tau_t + g_t - \bar{b}\beta^{-1}(y_t - y_{t-1}) + z_t^b, \tag{9e}$$

$$\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau)(\delta_b(s_t) b_{t-1} + \delta_y y_t), \tag{9f}$$

$$g_t = \rho_g g_{t-1} - (1-\rho_g)(\zeta_b(s_t) b_{t-1} + \zeta_y y_t). \tag{9g}$$

The parameter ranges we consider for sensitivity analysis are as follows. $p_{00} \in (0.975, 0.99)$, $p_{11} \in (0.85, 0.99)$, $\bar{b} \in (0.5, 2.5)$, $\psi_y \in [0, 1]$, $\vartheta_1 \in [0.6, 1], \mu \in [0.6, 1]$, $\rho \in [0, 0, 9]$, and $\rho_\tau, \rho_g \in [0, 0, 9]$, $f_c = 1$ or 0.8, $\zeta_b(s_t) = 0$ or $\zeta_b(s_t) = \delta_b(s_t)$. $\delta_y, \zeta_y \in [0, 0, 1]$, and additionally $\kappa \in (0, 0.3]$, $\varphi \in [0.5, 1.5]$. We replicate Figure 2, the main result, using the following specifications.

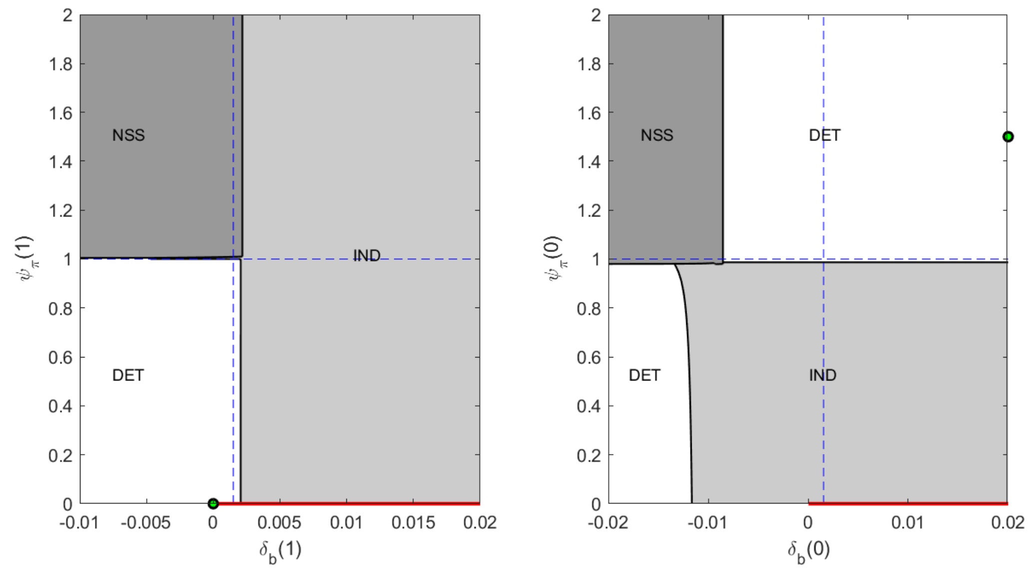
Parameters	$\psi_\pi(s_t)$	$\delta_b(s_t)$	(p_{00}, p_{11})	$\bar{b}$	ψ_y	ϑ_1	μ	ρ	ρ_τ	δ_y	f_c	$\zeta_b(s_t)$	ρ_g	ζ_y
Baseline			(0.975,0.99)	1	0	1	1	0	0	0	1	$(0, 0)$	0	0
SpecP0			(0.99,0.99)											
SpecP1			(0.99,0.975)											
SpecP2			(0.99,0.95)											
SpecP3			(0.99,0.9)											
SpecP4			(0.99,0.85)											
SpecA1				0.6										
SpecA2				2.5										
SpecB1					0.5									
SpecB2								0.7						
SpecB3						0.6	0.6	0.7						
SpecC1									0.7	0.1				
SpecC2						0.6	0.6	0.7	0.7	0.1				
SpecD1											0.8	$\delta_b(s_t)$		
SpecD2											0.8	$\delta_b(s_t)$	0.7	0.1
SpecD3		(0,0)									0.8	$\zeta_b(s_t)$	0.7	0.1

In all of the specifications, except the last one, we analyze the model in the same way as we do with $\psi_\pi(s_t)$ and $\delta_b(s_t)$. In the last specification, $\delta_b(s_t)$ is fixed at 0 in both regimes and $\zeta_b(s_t)$ plays the same role as $\delta_b(s_t)$. The empty entries in the table represent the parameter values of the baseline model.

2.1 Baseline Model

Here Figure 2 is reproduced for ease of comparison.

Figure 2: Model Classification under Regime-Switching



The left panel depicts determinacy/indeterminacy/non-stable solution regions for the baseline model in terms of $\psi_\pi(1)$ and $\delta_b(1)$ under a benchmark regime-switching when the current regime is a ZLB regime with $\psi_\pi(0) = 0$ and $\delta_b(0) = 0$ (denoted by a green dot). The right panel shows the same regions when the current regime is a monetary dominance (M) regime with $\psi_\pi(1) = 1.5$ and $\delta_b(1) = 0.02$.

The main result of the paper is that switching between **F** –including the **ZLB** regime– and **M** regimes does not lead to determinacy. The benchmark switching is the switching between the two regimes represented by a green dot: $\psi_\pi(0) = 0, \delta_b(0) = 0$ and $\psi_\pi(1) = 1.5, \delta_b(1) = 0.02$. Clearly, one can find that the model is indeterminate.

We report 15 different specifications listed in the table above. Across the alternative sensitivity analyses, results are very different to the ones shown in this figure. Figure numbers are indexed from 21 through 35.

2.2 Sensitivity Analysis: Transition Probabilities

Switching between **F** and **M** regimes leads to indeterminacy in the following three cases, similar to the benchmark case.

Figure 21: $(p_{00}, p_{11}) = (0.99, 0.99)$

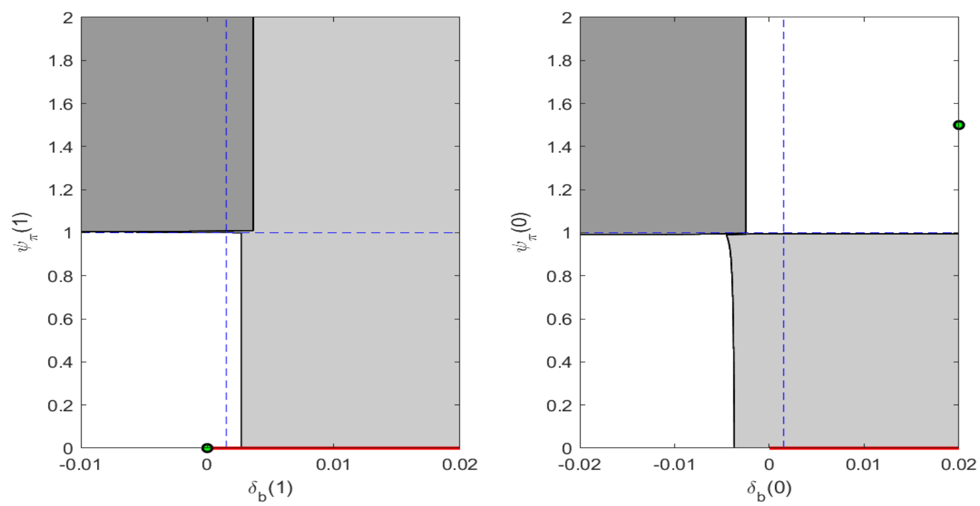


Figure 22: $(p_{00}, p_{11}) = (0.99, 0.975)$

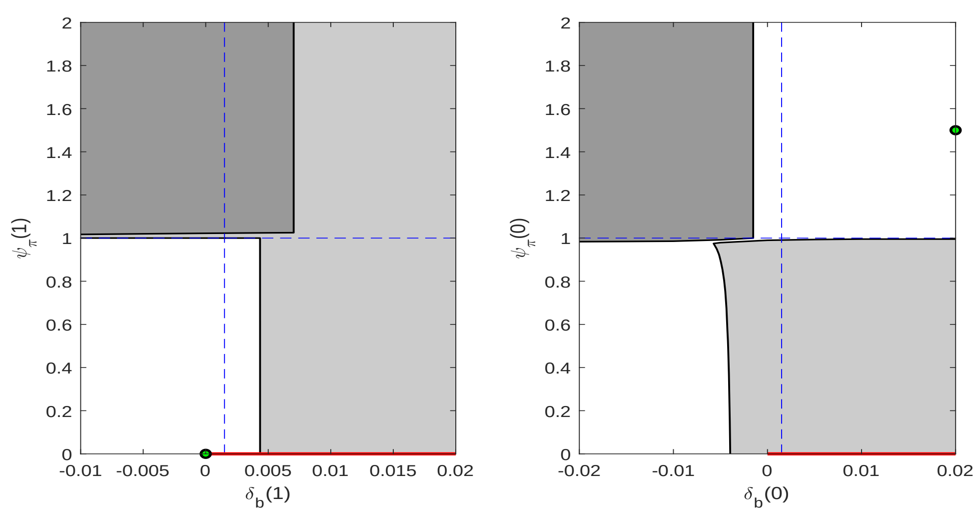
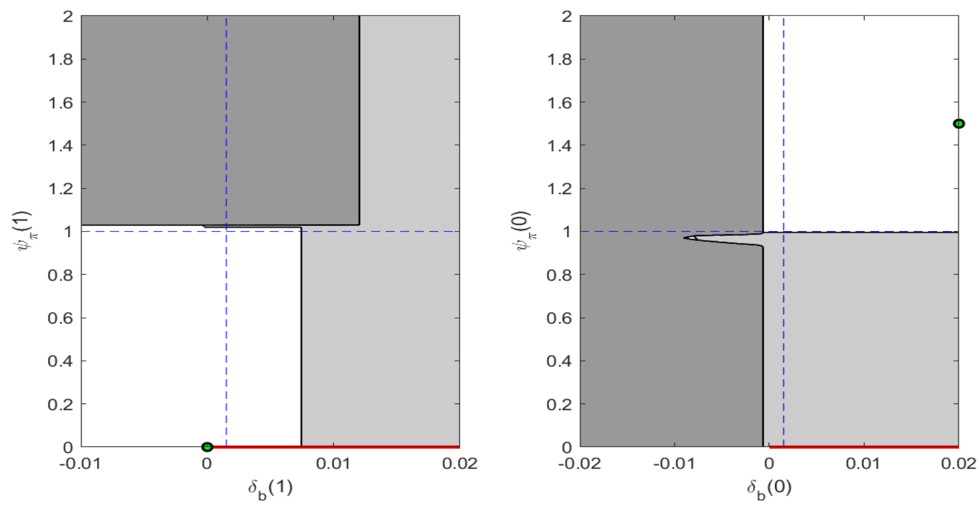


Figure 23: $(p_{00}, p_{11}) = (0.99, 0.95)$



In the following two cases where the duration of the **M** regime becomes much shorter, the model has no stable solution.

Figure 24: $(p_{00}, p_{11}) = (0.99, 0.9)$

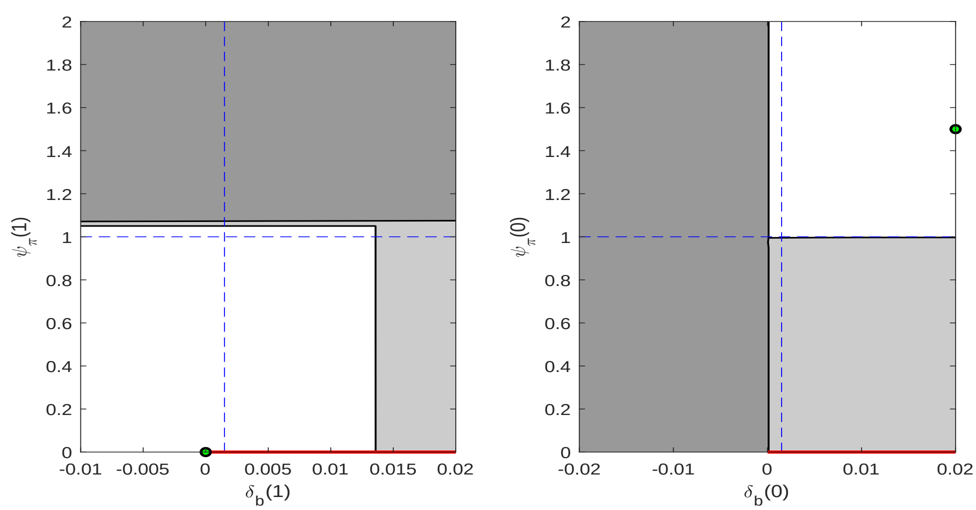
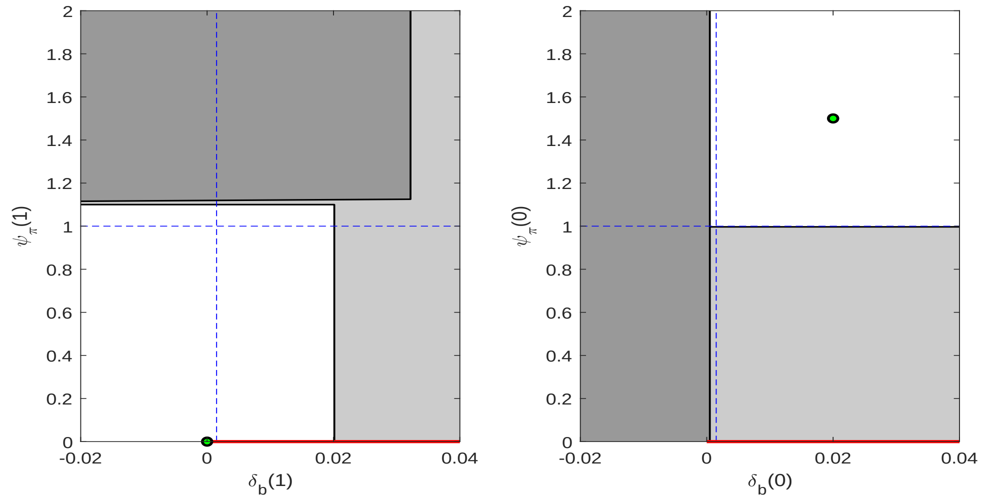


Figure 25: $(p_{00}, p_{11}) = (0.99, 0.85)$



2.3 Sensitivity Analysis: $\bar{b}$

This exercise shows that our main result is invariant to changes in the steady state debt to GDP ratio.

Figure 26: $\bar{b} = 0.6$

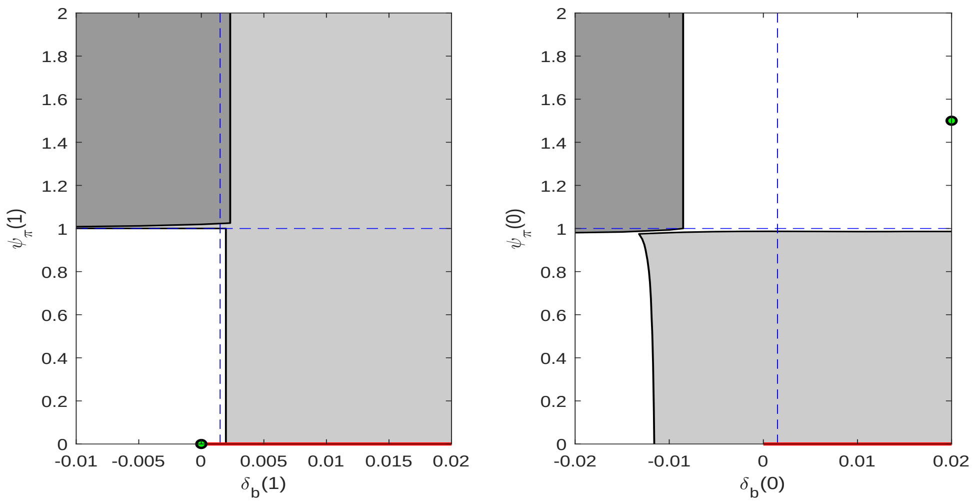
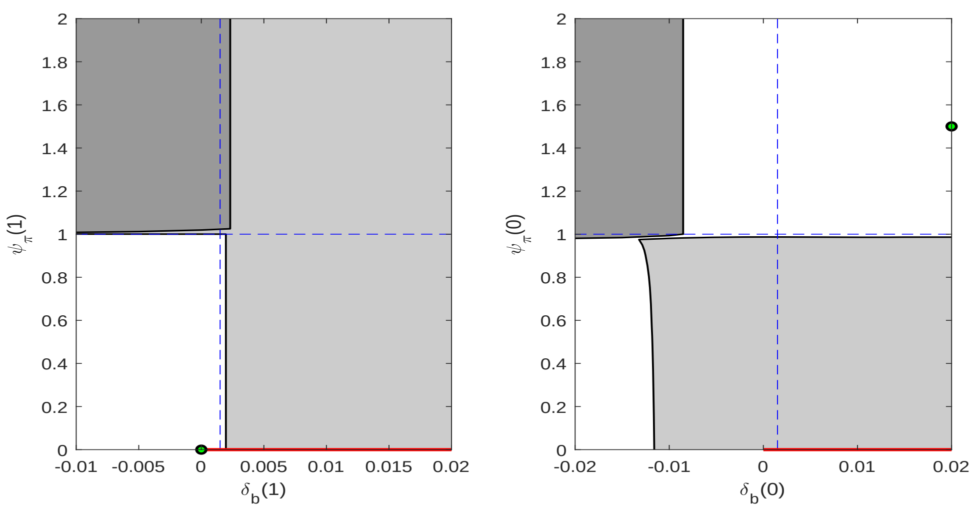


Figure 27: $\bar{b} = 2.5$



2.4 Sensitivity Analysis: $\psi_y, \rho, \vartheta_1$ and μ

This is the case where the monetary policy responses to the output gap, and/or private sector dynamics exhibit endogenous persistence. In all these cases, the main results barely change.

Figure 28: $\psi_y = 0.5$

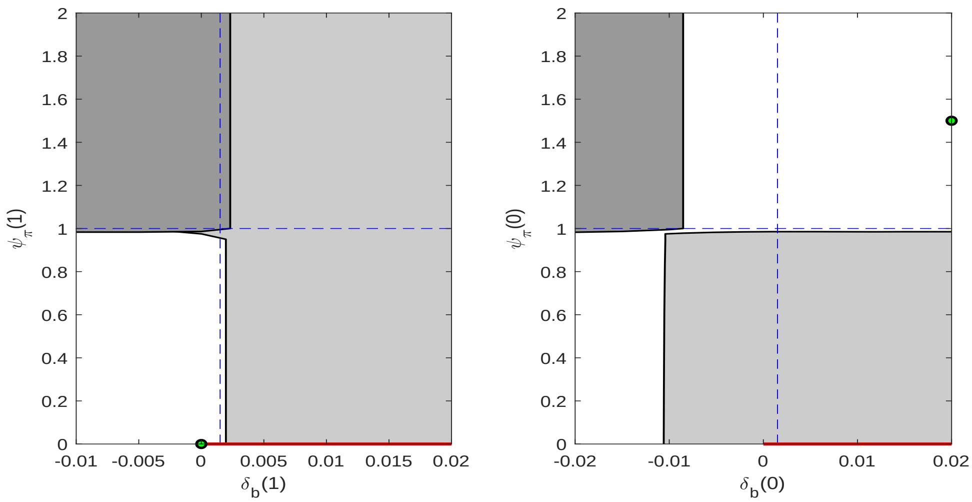


Figure 29: $\rho = 0.7$

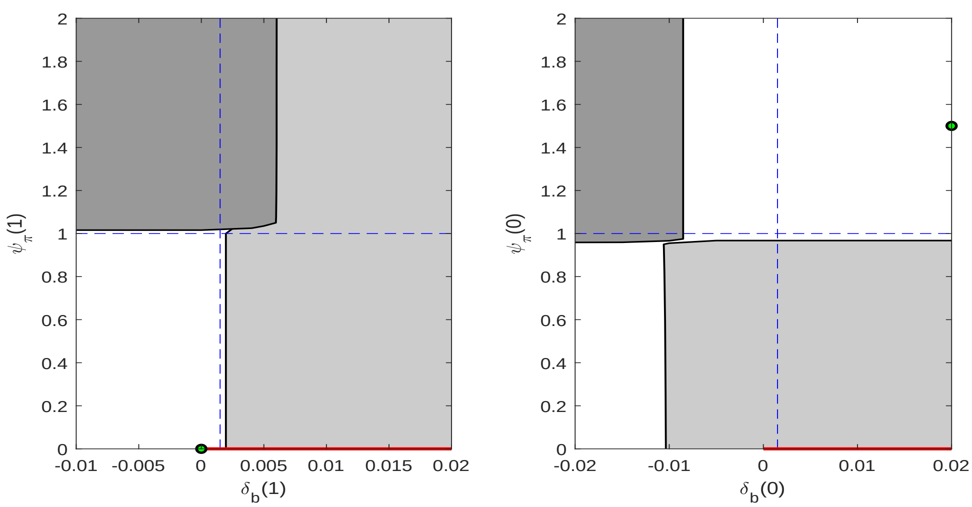
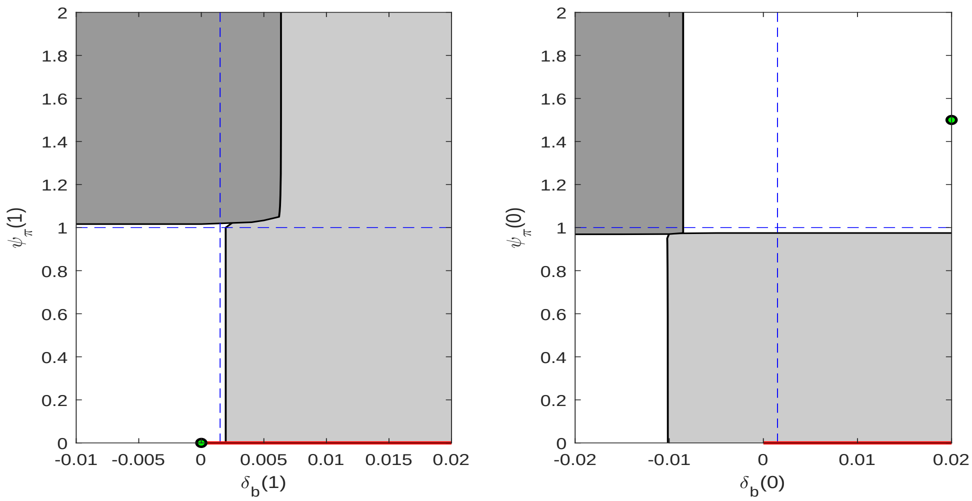


Figure 30: $\vartheta_1 = \mu = 0.6, \rho = 0.7$



2.5 Sensitivity Analysis: ρ_τ δ_y

This is the case of endogenous persistence of the tax policy and/or the private sector. Once again, the main result does not change much.

Figure 31: $\rho_\tau = 0.7$ and $\delta = 0.1$

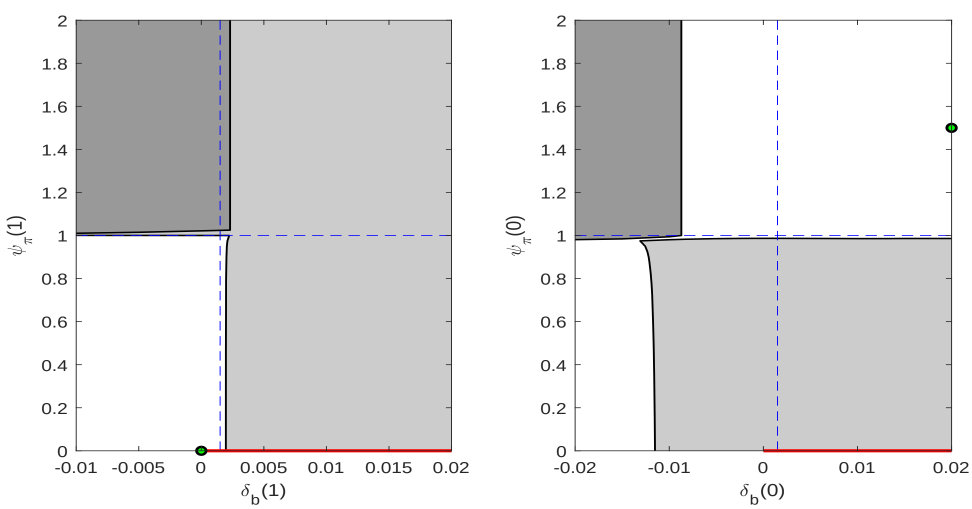
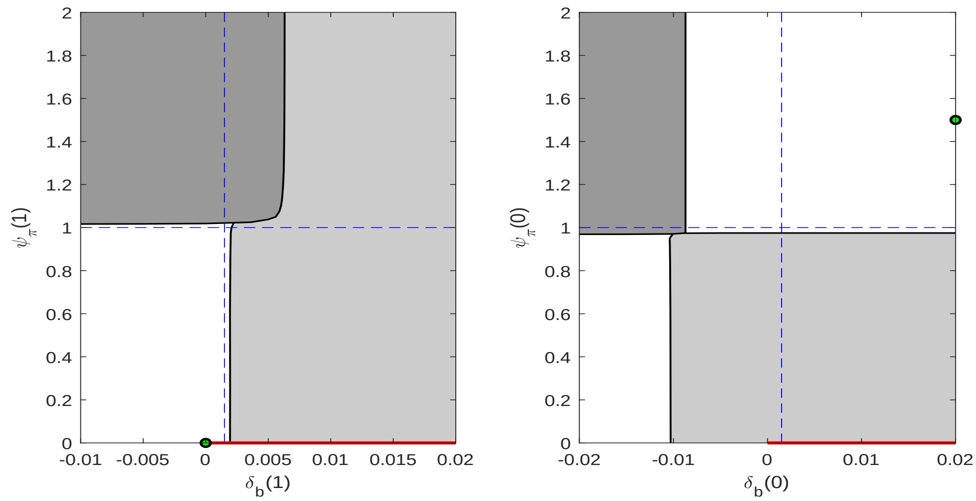


Figure 32: $\vartheta_1 = \mu = 0.6, \rho = 0.7, \rho_\tau = 0.7$ and $\delta = 0.1$



2.6 Sensitivity Analysis: $f_c, \zeta_b(s_t), \rho_g$ and ζ_y

This exercise examines the alternative fiscal policy with government spending. This alteration does not affect the main result much.

Figure 33: $f_c = 0.8$ and $\zeta_b(s_t) = \delta_b(s_t)$

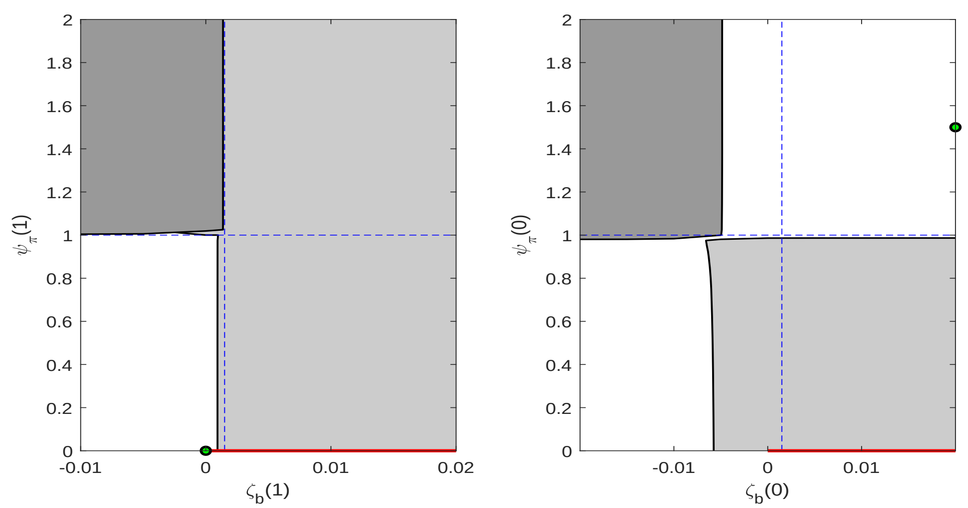
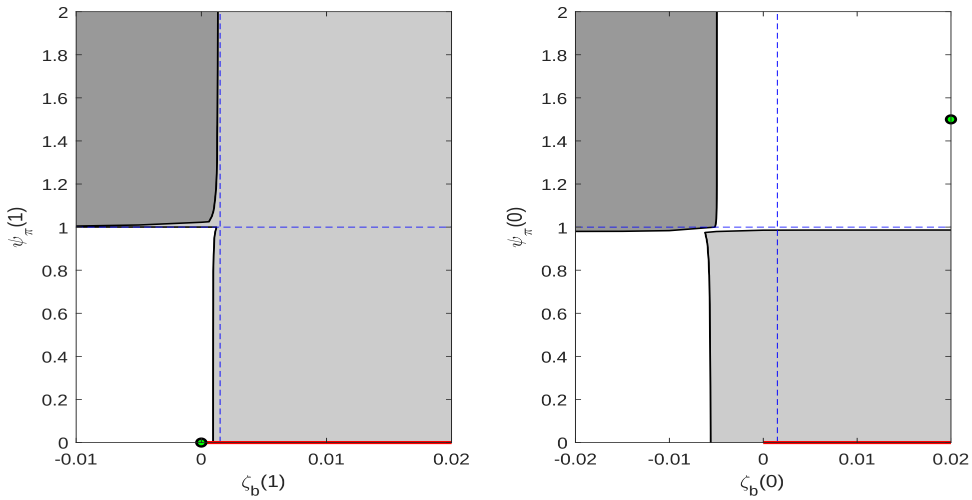


Figure 34: $f_c = 0.8$, $\zeta_b(s_t) = \delta_b(s_t)$, $\rho_g = 0.7$, and $\zeta_y = 0.1$



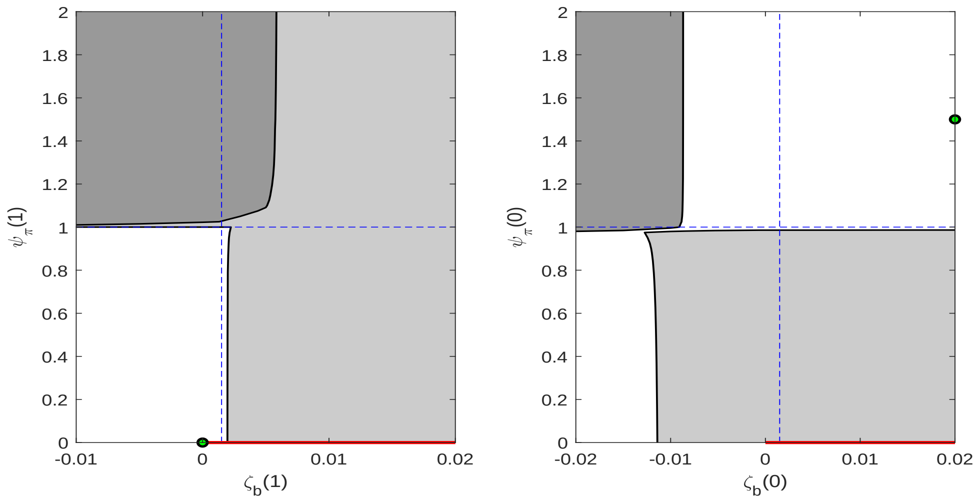
3 The Role of Economic Growth

In subsection 5.2, we showed that a lower value of the discount rate may rescue the fiscal theory as providing a driving force for the exit of the ZLB regime. In this appendix, we suggest an alternative justification for a lower discount rate based on economic growth while preserving the fiscal theory framework. Suppose that the utility function is given by a standard CRRA-type, $U(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ where σ is the inverse of the intertemporal elasticity of substitution φ in equation. Then the Euler equation is given by

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{(1+r_t)}{(1+\pi_{t+1})} \right].$$

In the absence of economic growth and the fact that $C_t = Y_t$ without capital in the model, the steady state value of $\frac{Y_{t-1}}{Y_t}\frac{(1+r_{t-1})}{(1+\pi_t)}$ is β^{-1}. Now suppose that the economy grows at the rate of θ. Then the steady state value of $\frac{Y_{t-1}}{Y_t}\frac{(1+r_{t-1})}{(1+\pi_t)}$ is given by $\tilde{\beta}^{-1}$ where $\tilde{\beta} = (1+\theta)^{1-\sigma}\beta$ may be interpreted as the growth-adjusted discount factor. While economic growth is typically assumed away in theoretical New-Keynesian models, it is now not uncommon

Figure 35: $f_c = 0.8$, $\zeta_b(s_t) = \delta_b(s_t)$, $\rho_g = 0.7$, and $\zeta_y = 0.1$, $\delta_b(s_t)$ is shut off.



to explicitly take it into account in empirical studies. For instance, Smets and Wouters (2007) do introduce economic growth in their fully micro-founded model and show that the effective discount factor is given by $\tilde{\beta}$ defined above. Therefore, if σ is higher than 1, then $\tilde{\beta} < \beta$, and the coefficient of the real debt to GDP ratio, after taking into account the tax policy, is given by:

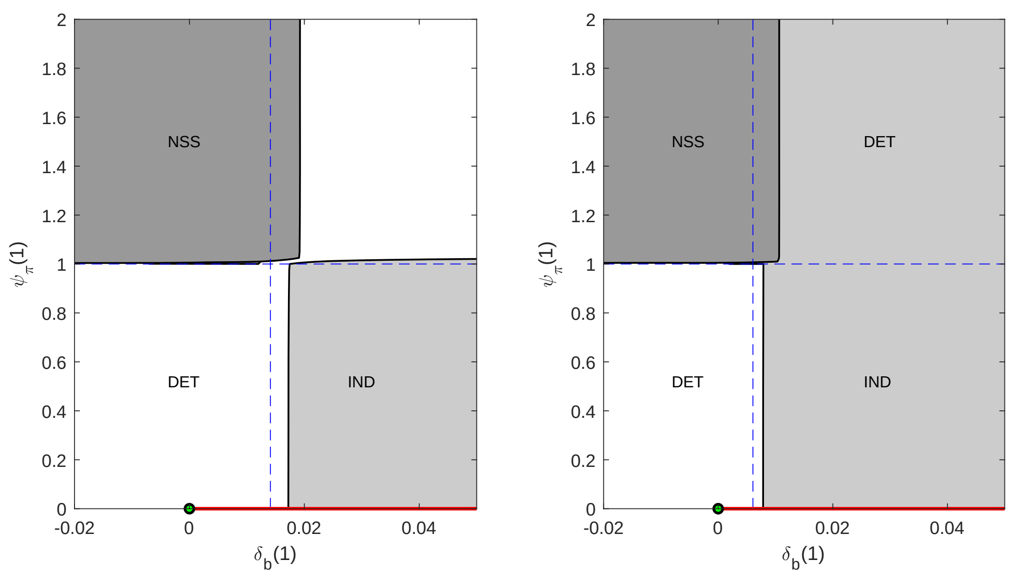
$$\tilde{\delta}(s_t) = \tilde{\beta}^{-1} - \delta_b(s_t).$$

The average annual real GDP growth rate of the U.S. economy since 1992 is about 2.53%, which on a quarterly basis corresponds to $\theta^{HIGH} = 0.0063$. For ease of comparison, we postulate an alternative lower growth rate implying $\theta^{LOW} = 0.0023$.[3] Therefore, the long-run growth rate can have a greater impact on the adjusted discount factor, the higher is the value of risk aversion. A typical parameter value for σ is 1, i.e., the case of log utility function, but a higher value than 1 is also widely used, particularly in asset pricing. The estimated range of this parameter is even more dramatically wide: Havránek (2015) surveyed 2,735 estimates in 169 published articles in the literature and reported

[3] U.S. data is obtained from the Bureau of Economic Analysis.

the suggested value of φ varies around the range of $[0.3,\ 0.5]$. Translating it into σ is then $[2,\ 3.3]$, unless the Epstein and Zin (1989)-type utility function is used. One of the sources for the forward guidance puzzle is also attributed to the excessive sensitivity of consumption to the real interest rate, implying that a benchmark calibration value of φ such as 1 or 0.8 is too large, or that σ is too small. To summarize, both the growth rate θ and the inverse of the elasticity of substitution σ matter significantly in determining $\tilde{\beta}$. When σ is restricted to be larger than 1, the higher θ and σ are, the lower is the adjusted $\tilde{\beta}$. Using $\tilde{\beta}$, Figure 36 replicates Figure 2 in the paper for a high and a low

Figure 36: Model Classification under Regime-Switching from a ZLB regime with $\tilde{\beta}$.



This figure reproduces the left panel of Figure 2 in the paper when $\tilde{\beta} = (1+\theta)^{1-\sigma}\beta$ is used instead of β for the discount rate in equations (1) and (7) in the paper. φ is also replaced with $1/\sigma$ in equation (2) in the paper. The left panel is the result for high growth rate (high θ) compared to the right panel (low θ).

θ. As in the case assuming an ad-hoc lower discount factor, the admissible range of an active fiscal policy is now much larger with $\tilde{\beta}$ than with the pure time discount factor. The determinacy region for the U.S. economy now emerges in the reasonable parameter range representing the **M** policy mix in regime 1, consistent with the fiscal theory with θ^{HIGH}. Even with a standard time discount factor, as long as economic growth is explicitly taken into account, the fiscally-led policy can ensure a unique equilibrium path when θ is sufficiently high. Indeed, $\tilde{\beta} = 0.9861$ for the case of the U.S. Also, the more passive the fiscal policy is, the more active monetary policy is allowed in regime 1 for determinacy. In contrast, under a low growth rate (θ^{LOW}), $\tilde{\beta} = 0.9940$. Indeed, there is no determinacy region in the **M** region in this case (right Panel, Figure 36).

Our suggested modification, which emphasizes the importance of the steady state economic growth rate, is conditional on a value of 3 for σ. Lower and realistic values for σ cannot yield a low discount rate, which is necessary to revive the fiscal theory of the price of level. In this sense, a more serious calibration exercise is called for.

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