Bianchi Melosi Model

Appendix B of Cho and Moreno(2019,working paper) entitled "Has the Fiscal Policy Saved the Great Recession?" examines determinacy of the model of the Bianchi and Melosi(2019,AER in the mean-square stability sense. This note explains how to rewrite the original model ignoring exogenous shocks that do not affect determinacy properties.

A. Original Bianchi Melosi Model

$$\begin{aligned}
\tilde{\pi}_t &= \beta E_t \pi_{t+1} + \kappa\vartheta_1 \hat{y}_t + \kappa\vartheta_2 \hat{y}_{t-1}, && (1)\\
(1+\Phi M_a^{-1})\hat{y}_t &= E_t y_{t+1} + \Phi M_a^{-1}\hat{y}_{t-1} - (1-\Phi M_a^{-1})(\hat{R}_t - E_t\tilde{\pi}_{t+1}) && (2)\\
&\quad +(1-\Phi M_a^{-1})[(1-\rho_d)d_t - \bar{d}_{\xi_t^d} + E_{\xi_t^d}\bar{d}_{\xi_{t+1}^d} && (3)\\
\tilde{R}_t &= (1-Z_{\xi_t^d})\left[\rho_{R,\xi_t^d}\tilde{R}_{t-1} + (1-\rho_{R,\xi_t^d})(\psi_{\pi,\xi_t^p}\tilde{\pi}_t + \psi_{y,\xi_t^p}(\hat{y}_t - \hat{y}_t^*))\right] && (4)\\
&\quad +Z_{\xi_t^d}\rho_{R,Z}\tilde{R}_{t-1} && (5)\\
\tilde{R}_t &= E_t[\tilde{R}_{t,t+1}^m] && (6)\\
\tilde{R}_{t-1,t}^m &= R^{-1}\rho\tilde{P}_t^m - \tilde{P}_{t-1}^m && (7)\\
\vartheta_1\hat{y}_t^* &= \vartheta_2\hat{y}_{t-1}^* && (8)\\
\tilde{b}_t^m &= \beta^{-1}\tilde{b}_{t-1}^m + b^m\beta^{-1}\left[\hat{R}_{t-1,t}^m - \tilde{\pi}_t - \hat{y}_t + \hat{y}_{t-1}\right] - \tilde{\tau}_t + \tilde{t}r_t && (9)\\
\tilde{\tau}_t &= \rho_{\tau,\xi_t^p}\tilde{\tau}_{t-1} + (1-\rho_{\tau,\xi_t^p})\left[\delta_{b,\xi_t^p}\tilde{b}_{t-1}^m + \delta_e\tilde{t}r_t^* + \delta_y(\hat{y}_t - \hat{y}_t^*)\right] && (10)\\
\tilde{t}r_t &= \rho_{tr}\tilde{t}r_{t-1} + (1-\rho_{tr})\phi_y(\hat{y}_t - \hat{y}_t^*) && (11)
\end{aligned}$$

$$\begin{aligned}
[\xi_t^p, \xi_t^d] &= [M,h] = \{[Z,l],[M,h],[F,h],\} = \{1,2,0\} && (12)\\
P\{1,2,0\} &= \begin{bmatrix} p_{hh}p_{MM} & p_{hh}(1-p_{FF}) & (1-p_{ll})p_{ZM} \\ p_{hh}(1-p_{MM}) & p_{hh}p_{FF} & (1-p_{ll})(1-p_{ZM}) \\ (1-p_{hh}) & 1-p_{hh} & p_{ll} \end{bmatrix}' && (13)
\end{aligned}$$

The model is simplified by eliminating $\tilde{P}_t^m, \tilde{R}_{t,t+1}^m, \hat{y}_t^*, d_t, \bar{d}_{\xi_t^d}$ further.

$$\begin{aligned}
\tilde{\pi}_t &= \beta E_t \pi_{t+1} + \kappa\vartheta_1 \hat{y}_t + \kappa\vartheta_2 \hat{y}_{t-1}, && (14)\\
(1+\Phi M_a^{-1})\hat{y}_t &= E_t y_{t+1} + \Phi M_a^{-1}\hat{y}_{t-1} - (1-\Phi M_a^{-1})(\hat{R}_t - E_t\tilde{\pi}_{t+1}) && (15)\\
\tilde{R}_t &= (1-Z_{\xi_t^d})\left[\rho_{R,\xi_t^d}\tilde{R}_{t-1} + (1-\rho_{R,\xi_t^d})(\psi_{\pi,\xi_t^p}\tilde{\pi}_t + \psi_{y,\xi_t^p}\hat{y}_t)\right] && (16)\\
&\quad +Z_{\xi_t^d}\rho_{R,Z}\tilde{R}_{t-1} && (17)\\
\tilde{b}_t^m &= \beta^{-1}\tilde{b}_{t-1}^m + b^m\beta^{-1}\left[\hat{R}_{t-1,t}^m - \tilde{\pi}_t - \hat{y}_t + \hat{y}_{t-1}\right] - \tilde{\tau}_t + \tilde{t}r_t && (18)\\
\tilde{\tau}_t &= \rho_{\tau,\xi_t^p}\tilde{\tau}_{t-1} + (1-\rho_{\tau,\xi_t^p})\left[\delta_{b,\xi_t^p}\tilde{b}_{t-1}^m + \delta_y\hat{y}_t\right] && (19)\\
\tilde{t}r_t &= \rho_{tr}\tilde{t}r_{t-1} + (1-\rho_{tr})\phi_y\hat{y}_t && (20)
\end{aligned}$$

Parameters

$$\begin{array}{llll} & s_t=0 & s_t=1 & s_t=2 \\ & [\xi_t^p,\xi_t^d]=[Z,l] & [\xi_t^p,\xi_t^d]=[M,h] & [\xi_t^p,\xi_t^d]=[F,h] \\ \psi_\pi(s_t) & 0 & \psi_{\pi,\xi_t^p}=1.6019 & \psi_{\pi,\xi_t^p}=0.6356 \\ \psi_y(s_t) & 0 & \psi_{y,\xi_t^p}=0.5065 & \psi_{y,\xi_t^p}=0.2709 \\ \rho_R(s_t) & 0 & \rho_{R,\xi_t^p}=0.8652 & \rho_{R,\xi_t^p}=0.6663 \\ \delta_b(s_t) & 0 & \delta_{b,\xi_t^p}=0.0712 & \delta_{b,\xi_t^p}=0 \\ \rho_\tau(s_t) & 0 & \rho_{\tau,\xi_t^p}=0.9652 & \rho_{\tau,\xi_t^p}=0.6874 \\ Z_{\xi_t^d}(s_t) & 1 & 0 & 0 \end{array} \quad (21)$$

$$\begin{aligned} \beta &= 0.9985, \kappa=0.0073,\ \Phi=0.8628,\quad \gamma=0.004185, \rho_{R,Z}=0.2; & (22)\\ b^m &= 0.2672*4, \rho_{\tau,Z}=0.6874 & (23)\\ \rho_{tr} &= 0.4599, \delta_y=0.2766, \delta_e=0.3661, \phi_y=-0.2910, & (24)\\ p_{ll} &= 0.9306,\ p_{hh}=0.9995,\ p_{MM}=p_{FF}=0.9923, p_{ZM}=0.9225 & (25) \end{aligned}$$

where

$$\begin{aligned} \vartheta_1 &= \frac{1}{1-\Phi M_a^{-1}}+\frac{\alpha}{1-\alpha},\quad \vartheta_2=\frac{\Phi M_a^{-1}}{1-\Phi M_a^{-1}} & (26)\\ \mu &= \frac{1}{1+\Phi M_a^{-1}},\ \tilde{\varphi}=\frac{(1-\Phi M_a^{-1})}{1+\Phi M_a^{-1}} & (27)\\ M_a &= \exp(\gamma) & (28) \end{aligned}$$

B. Rewriting the model in our notation (1) Regime

$$\begin{aligned} P\{0,1,2\} &= \begin{bmatrix} p_{ll} & (1-p_{ll})p_{ZM} & (1-p_{ll})(1-p_{ZM}) \\ (1-p_{hh}) & p_{hh}p_{MM} & p_{hh}(1-p_{MM}) \\ (1-p_{hh}) & p_{hh}(1-p_{FF}) & p_{hh}p_{FF} \end{bmatrix} & (29)\\ s_t &= [\xi_t^p,\xi_t^d]=[M,h]=\{[Z,l],[M,h],[F,h],\}=\{0,1,2\} & (30) \end{aligned}$$

(2) Model

$$\begin{aligned} \tilde{\pi}_t &= \beta E_t\pi_{t+1}+\kappa\vartheta_1\hat{y}_t-\kappa\vartheta_2\hat{y}_{t-1}, & (31)\\ \hat{y}_t &= \mu E_t y_{t+1}+(1-\mu)\hat{y}_{t-1}-\tilde{\varphi}(\hat{R}_t-E_t\tilde{\pi}_{t+1}) & (32)\\ \tilde{R}_t &= (1-Z_{\xi_t^d})\left\{\rho_R(s_t)\tilde{R}_{t-1}+(1-\rho_R(s_t))(\psi_\pi(s_t)\tilde{\pi}_t+\psi_y(s_t)\hat{y}_t)\right\}+Z_{\xi_t^d}(s_t)\rho_{R,Z}\tilde{R}_{t-1} & (33)\\ \tilde{b}_t^m &= \beta^{-1}\tilde{b}_{t-1}^m+b^m\beta^{-1}\left[\tilde{R}_{t-1}-\tilde{\pi}_t-\hat{y}_t+\hat{y}_{t-1}\right]-\tilde{\tau}_t+\tilde{t}r_t & (34)\\ \tilde{\tau}_t &= \rho_\tau(s_t)\tilde{\tau}_{t-1}+(1-\rho_\tau(s_t))\left[\delta_b(s_t)\tilde{b}_{t-1}^m+\delta_y\hat{y}_t\right] & (35)\\ \tilde{t}r_t &= \rho_{tr}\tilde{t}r_{t-1}+(1-\rho_{tr})\phi_y\hat{y}_t & (36) \end{aligned}$$

(3) In Matrix form

$x_t = [\tilde{\pi}_t \ \hat{y}_t \ \tilde{R}_t \ \tilde{b}_t^m \ \tilde{\tau}_t \ \tilde{t}r_t]$

$$B_0 = \begin{bmatrix} 1 & -\kappa\vartheta_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & \tilde{\varphi} & 0 & 0 & 0 \\ -(1-\rho_R(s_t))\psi_\pi(s_t) & -(1-\rho_R(s_t))\psi_y(s_t) & 1 & 0 & 0 & 0 \\ b^m\beta^{-1} & b^m\beta^{-1} & 0 & 1 & 1 & -1 \\ 0 & -(1-\rho_\tau(s_t))\delta_y & 0 & 0 & 1 & 0 \\ 0 & -(1-\rho_{tr})\phi_y & 0 & 0 & 0 & 1 \end{bmatrix}, , A_0 =$$

$$\begin{bmatrix} \beta & 0 & 0 & 0 & 0 & 0 \\ \tilde{\varphi} & \mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & \kappa\vartheta_2 & 0 & 0 & 0 & 0 \\ 0 & 1-\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-Z_{\xi_t^d})\rho_R(s_t) + Z_{\xi_t^d}(s_t)\rho_{R,Z} & 0 & 0 & 0 \\ 0 & b^m\beta^{-1} & b^m\beta^{-1} & \beta^{-1} & 0 & 0 \\ 0 & 0 & 0 & (1-\rho_\tau(s_t))\delta_b(s_t) & \rho_\tau(s_t) & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{tr} \end{bmatrix}$$