Loyalty Rewards Programs in the Face of Entry

Ross A. Brater*

July 2013

Abstract

I consider a two-period model of a horizontally differentiated product market which features an incumbent firm and a potential entrant. In the first period, the incumbent firm may elect to establish a loyalty rewards program that provides a discount to customers who are repeat purchasers in the second period. Although such a program generates switching costs for consumers, I find that the entrant cannot be adversely affected by the implementation of rewards, irrespective of the choice of reward. Furthermore, such a program necessarily reduces the incumbent's second-period profits. Rewards programs are, however, of interest to incumbents; an incumbent firm can increase its total profits by implementing a rewards program and then increasing both first- and second-period prices relative to the no-rewards equilibrium levels.

1 Introduction

Customer loyalty rewards have long been popular with firms and consumers. Casual observation reveals numerous examples of programs that serve to make lower prices available to repeat customers. Airlines provide frequent-flyer programs, while automobile companies offer lower prices for customers who trade in same-make vehicles. Supermarkets mail coupons to shoppers who have previously purchased groceries and offer discounts on gasoline purchases after a level of purchases has been reached. Big box retail stores award gift certificates after a certain dollar amount has been spent, and credit card companies offer cash-back payments after different increments of charges are made. Even coffee shops and pizzerias use frequent-shopper cards to award a free espresso or slice of pizza after a certain number have been purchased.

One effect of such programs is that consumers are saddled with a switching cost after making an initial purchase. When a consumer has qualified for a reward (or has made purchases which will accrue towards a reward), it becomes costly for them to purchase from a different supplier in the future because they will lose the value of the reward. For instance,

*Department of Economics, The Ohio State University, 1945 N. High Street, Columbus, OH 43210; brater.1@osu.edu

a consumer interested in purchasing a new car will not only consider cars' sticker prices, but also will take into account any loyalty discount offered by the manufacturer of her current vehicle. In order to lure a consumer away from a rival firm, a firm must offer a price below its rival's price by a margin large enough to compensate for the discount the consumer would otherwise receive.

Some switching costs are not the result of any action by a firm but rather that exist exogenously: examples range from the cost of establishing a relationship with a new provider to inefficiencies that arise from adopting a new operating system on a personal computer. The effect of such a switching cost on consumers typically provides an incentive for firms to initially set low prices in order to "lock in" consumers, who can then be exploited in the periods that follow due to the switching cost. Additionally, switching costs may allow an existing firm to lock in enough of the market that entry by another firm, which would be profitable in the absence of switching costs, is prevented.[1] However, the switching costs generated by loyalty rewards differ from these types of switching costs in several important respects: they are chosen by the firms that offer the rewards and they are costly to those firms in future periods. That is, when a firm makes a choice of how large a reward or discount to offer, they determine precisely the level of the switching costs that consumers will face rather than taking it as given. Additionally, by implementing such a rewards program, the firm must follow through with a lower price to those consumers who qualify. Equivalently, the reward may be viewed as a cash payout, in which case the firm must pay those who redeem the reward. These differences have important implications. In particular, a firm engaging in a rewards program needs to carefully weigh the benefit it gains from the switching cost against the cost of paying out the reward. Importantly, the reward payout reduces the benefit of lock-in, as locked-in consumers are necessarily charged lower prices or are paid a reward. This may reduce the feasibility of limit pricing in early periods for the purposes of entry deterrence.

I consider rewards programs in the context of a two-period environment where a monopolist faces potential entry by a rival firm. Specifically, an incumbent firm is a monopolist in an initial period armed with the knowledge that an entrant who offers a horizontally differentiated good may enter in the following period. The incumbent might be interpreted, for example, as an airline that offers the only flight on a particular route out of a given airport. The entrant might be a rival airline who is considering serving that route. Following the example of differentiated-product models featuring switching costs such as those of von Weizäcker (1984), Klemperer (1987b), and Caminal and Matutes (1990), I make the assumption that consumers are uncertain about how their preferences might change between the two periods. That is, their preferences change over time such that they are not sure which firm's product will be more appealing in the second period. Extending the airline example, this can be interpreted as a traveler knowing which airline offers the flight which best fits their schedule at the moment but not knowing which will offer a more convenient flight the next time she will fly. A simple reason for this may be that the she merely has not made travel plans yet even though she knows she will take a vacation within the next

[1]See Farrell and Klemperer (2007) for an excellent review of the literature on switching costs.

year. Likewise, the automobile shopper has full information about which cars are currently in production, but likely cannot predict which manufacturer might be producing the vehicle she will like best ten years down the line.

The purpose of this paper is to study how an incumbent firm might benefit from implementing a rewards program. In particular, I focus on so-called "coupon rewards," which specify a discount for repeat consumers from the to-be-determined publicly available period-two price, but do not guarantee any specific price. Alternatively, these coupons could be viewed as a cash payout received upon a repeat purchase.[2] If a rewards program is adopted, the incumbent imposes switching costs upon its previous customers in the following period. An interesting question is then whether this will allow the incumbent to sell to enough of the market in the second period to forestall the entry and maintain the monopoly. Perhaps surprisingly, the answer is no. If the entrant were to find entry profitable in the absence of rewards, then, regardless of the rewards program implemented, the incumbent is powerless to deter entry. Furthermore, the second-period profits of the incumbent actually fall due to the cost of implementing rewards. A second question then arises: given that the incumbent is made strictly worse off in the second period, can using a rewards program actually be beneficial? The answer to this question is yes. Rewards increase the value of purchasing the incumbent's good in the first period for all consumers due to those consumers' uncertainty about their preferences in the following period. The incumbent extracts some of this surplus while at the same time increasing its first-period sales, and the resulting increase in profit is more than enough to compensate for the period-two loss. In contrast to the normal behavior under standard switching costs, rather than pricing low and then later exploiting those facing the switching cost, the incumbent using rewards must extract surplus up front via a higher first-period price to make up for an unavoidable loss in the future.

The literature on switching costs is extensive; however, given the inherent differences mentioned above between rewards programs and standard models of switching costs, the majority of this work has little bearing on the topic at hand. Of primary interest are the models that examine entry deterrence. One noteworthy example is Klemperer (1987a), who shows that forestalling entry is possible in a two-period model of Cournot competition even when switching costs are low. By contrast, I demonstrate that an incumbent has no hope of preventing entry, even when imposing a large switching cost via rewards. The difference comes down to the fact that, when switching costs are generated by rewards programs, the incumbent is burdened with a higher marginal cost in the second period due to the reward payout. Subgame perfection then rules out the possibility of the incumbent hurting the profit level of the entrant, regardless of what commitment has been made in the prior period.

It should be clear that loyalty rewards programs are a form of price discrimination, based upon the past behavior of consumers. There are several interesting studies that examine such behavior-based price discrimination. Fudenberg and Tirole (2000) consider a two-period duopoly market for a horizontally differentiated product. In their model, firms are able to price discriminate in the second period based on the purchasing decisions of consumers in the

[2] This is in contrast to other rewards programs which might feature nonlinear pricing within a single period.

first period. They find that firms will optimally charge higher prices to repeat consumers and lower prices to potential "new" consumers. Villas-Boas (1999) reports similar results for a model where firms are infinitely-lived and face overlapping generations of consumers. These findings seem to contradict the notion of loyalty rewards, which have precisely the opposite effect on prices. In contrast to these results, I find that charging a lower price to repeat customers is, in fact, optimal, even if doing the opposite were feasible. The reason for this discrepancy lies in that, in the current model, consumers preferences change over time. When consumers' preferences do not change, higher prices to repeat customers are optimal because consumers' preferences are partially revealed to firms. Firms then take advantage of this information by extracting additional surplus from those consumers with higher willingness to pay. However, when consumers preferences are not consistent from period to period, less information about current preferences is revealed by past purchasing decisions. This results in an environment where loyalty rewards, hence lower prices, are optimal.

Given the seeming ubiquitousness of rewards programs in our everyday lives, studies that focus on coupon loyalty rewards programs are surprisingly sparse. Banerjee and Summers (1987) examine a duopoly market for an undifferentiated product, and find that firms, when given the option of adopting a coupon rewards program, independently elect to implement rewards programs and that doing so serves as a collusive device for firms. Caminal and Matutes (1990) consider a two-period differentiated duopoly model. They also find that using coupon rewards increases profits for firms. However, they show that when firms are given the option to use a different form of rewards, namely, precommittment in the first period to period-two prices for repeat consumers, a prisoner's dilemma is created which results in both firms precommitting and in lower equilibrium profits than when no rewards are used. The latter result is averted in this paper; a first-period monopolist would not adopt any strategy that results in lower profits than the no-rewards equilibrium, hence rewards do survive. To my knowledge, this is the first paper to examine the use of coupon rewards by a monopolist, and the first to study the effects of such a rewards program on entry.

The remainder of the paper proceeds as follows. Section 2 formally establishes the model, and Section 3 establishes the benchmark result when the incumbent firm does not use a rewards program. Section 4 solves the model of rewards and presents the main results. Section 5 concludes.

2 The Model

The framework I adopt consists of a market which operates for two periods. An incumbent firm, I, is active in the market during both periods, while an entrant, E, is not present in the first period but may enter the market in the second period should it anticipate doing so to be profitable. The entrant faces a fixed cost of entry. Each firm i has marginal cost of production c^i, and the entrant's marginal cost is assumed to be less than the incumbent's, i.e., $c^E \leq c^I$.[3] These firm characteristics are common knowledge. The incumbent sets a

[3] This assumption guarantees that the benchmark market will be fully covered in the second period for any of the monopoly sales levels I allow for the incumbent in the first period. Additionally, because entry

price p_1^I in the first period and a price p_2^I in period two. Should the entrant decide to enter, it sets a price p_2^E in the second period. In the first period, the incumbent firm also sets a reward amount $r > 0$. The reward is a coupon reward; if a consumer purchases from the incumbent in the first period, she receives a discount of r off of the incumbent's second-period price should she elect to purchase from the incumbent again in the second period. That is, consumers who have previously purchased from the incumbent are eligible for a price $p_2^I - r$ in the second period, while new customers must pay the full price p_2^I. That is, first-period buyers are assured that they will receive a discount from the price paid by the incumbent's new consumers in the following period, and are made aware of the dollar amount of the discount.

The firms' products are horizontally differentiated. Specifically, consumer preferences are modeled as in the familiar linear city model of Hotelling (1929): there is a continuum of consumers (normalized to have a mass of 1) uniformly distributed over the unit interval; the incumbent is located at 0, and the entrant, when and if it chooses to enter, becomes located at 1.[4] A consumer at postition θ who purchases from firm i in period $\tau \in \{1, 2\}$ realizes utility

$$U(\theta, i) = R - |\theta - i| \, t - p_\tau^i,$$

where p_τ^i is the (effective) price she faces offered by firm i in period τ and $t > 0$ measures consumers' "transportation costs," so that costs increase linearly in the distance "travelled."[5] Consumers have unit demand; that is, in each period each consumer will purchase at most one unit of one firm's product, and will purchase if doing so results in nonnegative expected utility. Consumers live for both periods; however, consumers' locations are not fixed across periods. In the first period, consumers are uncertain about their relative willingness to pay for the two firms' products in the following period. Specifically, consumers are repositioned in the second period independently of their original positions.[6] Consumers have rational expectations; that is, they correctly anticipate the second-period equilibrium prices of firms.

Firms maximize total expected profits across periods, while consumers maximize total expected utility across periods. For simplicity, the discount factor of both the incumbent

is more profitable for lower-cost firms, it is more likely to occur when the potential entrant is of low cost compared to the entrant than when the opposite is true.

[4]The purpose of this paper is not to study the strategic choice of location, or product, by the incumbent or the entrant, which are taken as exogenously determined. Rather, I envision a scenario where firms have characteristics which are known to consumers before the game begins. For instance, Chrysler and Ford will likely have difficulty tailoring their products to a specific geographic market, but a dealer of one may very well consider entry in a market where the other is present, and participation in rewards programs may vary by region. Similarly, Delta and Southwest will not be able to change their product for a specific route, and some routes may be ineligible for free or discounted flights.

[5]As is the case for any such differentiated-product model, the position θ of a consumer in this model need not correspond to a physical location in a city, nor does t have to represent a literal transportation cost. This aspect of the model merely captures the degree to which consumers prefer one firm's product to the other.

[6]I argue below that the central results of the paper do not hinge on the fact that consumers' locations in period two are independent of those in period one; the same goes for the fact that all consumers are active in both periods.

and the consumers is assumed to be equal to one.[7] The solution concept to be used is subgame-perfect Nash equilibrium. The timing of the game is as follows. At the beginning of period one, the incumbent announces a first-period price and reward amount. Consumers then make their purchasing decisions. First-period profits and utility are realized, then the entrant makes its decision of whether to enter or not. At the beginning of the second period, consumers are repositioned, then any active firms simultaneously announce their second-period prices. Finally, consumers make their purchasing decisions and second-period profits and utilities are realized.

3 No Rewards: A Benchmark

I first consider a two-period model where the incumbent firm does not offer a rewards program (i.e., $r = 0$). The purpose of this section is to establish a benchmark against which the optimal rewards program may be compared. Because consumers' purchasing decisions in the first period have no impact on the second period of the game, consumers maximize utility by acting myopically. Similarly, the incumbent's pricing decision in period one in no way affects the second period of the game, so the incumbent also acts myopically. Because I wish to examine how the implementation of a rewards program can impact the profits that an entrant would realize in the absence of such a program, I assume that the entrant's cost of entry is such that it can profitably enter the market when rewards are not used; that is, there will be two firms in the market in the second period.[8] I first analyze the incumbent's optimal first-period strategy before proceeding to the familiar Hotelling setup of the second period.

3.1 Period One: Monopoly

Facing the monopolist who offers a price of p_M^I and no rewards program, each consumer has an outside option of zero, and their decisions whether or not to purchase will have no impact on the second period. A consumer at position θ, then, will buy from the monopolist if and only if

$$R - \theta t - p_M^I \geq 0 \Longleftrightarrow \theta \leq \frac{R - p_M^I}{t}.$$

[7]This does not affect the qualitative results of the paper. Period two is completely unaffected by this assumption, while the qualitative results pertaining to period one hold provided that the discount factor is sufficiently high.

[8]It turns out that the entrant enters in the presence of rewards if and only if it would have entered when rewards were not used, so this assumption is necessary if the entrant is to have any impact on the outcome of the game.

The measure of consumers who buy from the monopolist, n_M^I, is then given by

$$n_M^I\left(p_M^I\right)=\begin{cases}1 & \text{if } p_M^I<R-t\\ \dfrac{R-p_M^I}{t} & \text{if } R-t\le p_M^I\le R\\ 0 & \text{if } p_M^I>R\end{cases}$$

The monopolist's profits are given by

$$\Pi_M^I\left(p_M^I\right)=\left(p_M^I-c^I\right)n_M^I\left(p_M^I\right).$$

I assume that the monopolist can earn positive profits. To earn positive profits, the monopolist must sell to a mass of consumers greater than zero (i.e. $p_M^I<R$), and the price charged by the monopolist must exceed its marginal cost (i.e. $p_M^I>c^I$). Thus, I make the following assumption.

Assumption 1. *$R-c^I>0$.*

Because I wish to examine how the introduction of a rewards program in the presence of a potential entrant will affect the pricing strategy and sales level of the incumbent firm, I focus on the interesting case where incumbent does not cover the entire market as a monopolist not using a rewards program. The necessary and sufficient condition for this to hold is presented in the following assumption.

Assumption 2. *$R-c^I<2t$.*

Trivially, charging any price $p_M^I\ge R$ yields zero profits, and charging any price $p_M^I<R-t$ yields strictly lower profits than charging $p_M^I=R-t$, which yields profits of $R-c^I-t$. If $p_M^I\in(R-t,R)$, profits are given by

$$\left(p_M^I-c^I\right)\frac{R-p_M^I}{t}.$$

This expression is strictly concave in p_M^I, and the first-order condition implies an optimal price in this interval of

$$p_M^I=\frac{R+c^I}{2}$$

which yields profits of

$$\Pi_M^I\left(\frac{R+c^I}{2}\right)=\frac{\left(R-c^I\right)^2}{4t}.$$

Assumption 1 implies that $\frac{R+c^I}{2}<R$ and Assumption 2 implies that $R-t<R-\frac{R-c^I}{2}=\frac{R+c^I}{2}$; therefore, $\frac{R+c^I}{2}\in(R-t,R)$. Furthermore,

$$\Pi_M^I\left(\frac{R+c^I}{2}\right)=\frac{\left(R-c^I\right)^2}{4t}>0=\Pi_M^I(R)$$

by Assumption 1, and

$$\Pi_M^I\left(\frac{R+c^I}{2}\right)=\frac{\left(R-c^I\right)^2}{4t}>R-c^I-t=\Pi_M^I\left(R-t\right)$$

by Assumptions 1 and 2.[9] Therefore, the price, sales and profit level of the (monopolist) incumbent firm in the first period with no rewards program are given by

$$\begin{aligned}p_M^I&=\frac{R+c^I}{2}\\ n_M^I&=\frac{R-c^I}{2t}\\ \Pi_M^I&=\frac{1}{t}\left(\frac{R-c^I}{2}\right)^2.\end{aligned}$$

Henceforth, I will refer to these values as the "monopoly" or "no-rewards" values for period one.

3.2 Period Two: Price Competition

Period two, in the absence of a rewards program, is simply the standard differentiated product model of competition in which the incumbent sets a price p_H^I and the entrant sets a price p_H^E. I assume that the market will be fully covered in equilibrium; that is, the every consumer purchases in equilibrium and gains strictly positive utility by doing so. If this is not the case, then each firm would set its price as if it were a monopolist, so entry into the market has no impact on the incumbent firm's price or sales level. The following assumption is the necessary and sufficient condition to ensure market coverage for all $c^E \leq c^I$.

Assumption 3. *$R-c^I>t$.*

This assumption implies that the sum of the incumbent firm's monopoly sales level and the hypothetical monopoly sales level of any entrant with equal or lower marginal cost is greater than one. In other words, the market is fully covered upon entry, and firms must strategically alter prices from monopoly levels. To ensure that equilibrium sales levels for both firms are strictly positive, I make the following assumption.

Assumption 4. *$t>\frac{1}{3}\left(c^I-c^E\right)$.*

[9] To see this, note that Assumptions 1 and 2 hold if and only if there exists some $k\in(0,1)$ such that $R-c^I=2kt$. Then $\frac{\left(R-c^I\right)^2}{4t}=k^2t$ and $R-c^I-t=2kt-t$. The desired inequality thus holds if and only if

$$k^2t>2kt-t,$$

which is equivalent to $(k-1)^2>0$.

Because the market is fully covered, the mass of consumers who buy from firm i given prices p_H^i and p_H^j is given by

$$n_H^i\left(p_H^i, p_H^j\right) = \frac{1}{2} + \frac{p_H^j - p_H^i}{2t},$$

so that $n_H^I = 1 - n_H^E$ corresponds to the location of the consumer who is indifferent between the two firms. Firm i's profits are given by

$$\Pi_H^i\left(p_H^i, p_H^j\right) = \left(p_H^i - c^i\right) n_H^i\left(p_H^i, p_H^j\right).$$

This expression is strictly concave in p_H^i. The first-order conditions for firms I and E yield, respectively, the following reaction functions:

$$\begin{aligned} p_H^I &= \frac{1}{2}\left(c^I + t + p_H^E\right) \\ p_H^E &= \frac{1}{2}\left(c^E + t + p_H^I\right). \end{aligned}$$

Solving the reaction functions simultaneously yields firms' equilibrium pricing strategies:

$$\begin{aligned} p_H^I &= c^I - \frac{1}{3}\left(c^I - c^E\right) + t \\ p_H^E &= c^E + \frac{1}{3}\left(c^I - c^E\right) + t. \end{aligned}$$

Using these prices, one can easily derive the equilibrium market shares

$$\begin{aligned} n_H^I &= \frac{1}{2} - \frac{1}{6t}\left(c^I - c^E\right) \\ n_H^E &= \frac{1}{2} + \frac{1}{6t}\left(c^I - c^E\right) \end{aligned}$$

and profit levels

$$\begin{aligned} \Pi_H^I &= \frac{1}{2t}\left(t - \frac{1}{3}\left(c^I - c^E\right)\right)^2 \\ \Pi_H^E &= \frac{1}{2t}\left(t + \frac{1}{3}\left(c^I - c^E\right)\right)^2. \end{aligned}$$

In what follows, I will refer to these equilibrium values as the "Hotelling" or "no-rewards" values for period two. This equilibrium is presented as a benchmark and has been studied extensively; hence, I do not discuss its properties further here. I now turn to the model which is the emphasis of this paper.

4 Rewards Programs

This section considers the two-period model where the incumbent firm, as a first-period monopolist, announces a reward amount at the beginning of the first period, along with a first period price. If the entrant elects to enter in the second period, firms engage in standard duopoly price competition, with one exception: the market is segmented into two groups of consumers, those who qualify for the discounted price (or reward payout) and those who face the higher, undiscounted price of the incumbent. Recall that consumers change their locations between periods, so that these groups are both uniformly distributed over the unit interval.[10] Because a subgame-perfect equilibrium is desired, I first consider the second period of the game and find that the entrant cannot be deterred from entering. Using backward induction, I then consider the first period where the incumbent firm anticipates entry in the ensuing period.[11]

4.1 Period Two

In the second period, the market is segmented into two groups: consumers who did elect to purchase from the incumbent in the previous period and those who did not. A consumer who elected not to purchase in period one faces a price of p_2^I offered by the incumbent and a price of p_2^E offered by the entrant. For now, assume that the market is fully covered in period two (I derive the necessary and sufficient condition for this to be the case in the following section). In the second period, a consumer located at position θ who did not purchase in period one purchases from the incumbent in period two if and only if

$$\theta \leq \underline{\theta} = \frac{1}{2} + \frac{p_2^E - p_2^I}{2t}. \tag{1}$$

However, a consumer who did purchase in period one faces a price of $p_2^I - r$ offered by the incumbent, which is lower than that offered to potential "new" customers. Hence, a consumer located at position θ in period two who did purchase from the incumbent in period one will purchase again from the incumbent if and only if

$$\theta \leq \overline{\theta} = \frac{1}{2} + \frac{p_2^E - p_2^I + r}{2t}. \tag{2}$$

Because the market is fully covered in period two, the consumers mentioned above will purchase from the entrant if the relevant inequality is not satisfied. Switching costs for consumers are generated by the rewards program in the following sense. In the absence of rewards programs, a consumer located at θ such that $\underline{\theta} < \theta < \overline{\theta}$ would strictly prefer to purchase from the entrant, regardless of her prior purchasing decision. However, under the

[10]Note that whether the groups are actually distributed uniformly or are merely distributed uniformly in expectation does not affect the analysis.

[11]Formally, the equilibrium strategy profile must specify a pricing strategy by the incumbent when entry does not occur. However, this outcome is always off of the equilibrium path, so I do not discuss it.

rewards program, a repeat customer of the incumbent who is located in this interval gains the additional benefit of the reward; the opportunity cost of switching, the foregoing of the reward, prevents such switching from taking place.

Letting n_1^I denote the proportion of the market who purchased in period one, the total period-two market share of the incumbent is

$$n_2^I = n_1^I \left(\frac{1}{2} + \frac{p_2^E - p_2^I + r}{2t} \right) + \left(1 - n_1^I\right) \left(\frac{1}{2} + \frac{p_2^E - p_2^I}{2t} \right), \tag{3}$$

while the period-two market share of the entrant is

$$n_2^E = n_1^I \left(\frac{1}{2} + \frac{p_2^I - p_2^E - r}{2t} \right) + \left(1 - n_1^I\right) \left(\frac{1}{2} + \frac{p_2^I - p_2^E}{2t} \right).$$

The incumbent firm's second-period profit function can be written as

$$\Pi_2^I = \left(p_2^I - r - c^I\right) n_1^I \left(\frac{1}{2} + \frac{p_2^E - p_2^I + r}{2t} \right) + \left(p_2^I - c^I\right) \left(1 - n_1^I\right) \left(\frac{1}{2} + \frac{p_2^E - p_2^I}{2t} \right). \tag{4}$$

Similarly, the entrant's profit function is given by

$$\Pi_2^E = \left(p_2^E - c^E\right) \left(n_1^I \left(\frac{1}{2} + \frac{p_2^I - p_2^E - r}{2t} \right) + \left(1 - n_1^I\right) \left(\frac{1}{2} + \frac{p_2^I - p_2^E}{2t} \right) \right).$$

Again, both profit functions are strictly concave in firms' own prices, and the first-order conditions yield firms' reaction functions:

$$\begin{aligned} p_2^I &= \frac{1}{2} \left(c^I + t + p_2^E + 2rn_1^I\right) \\ p_2^E &= \frac{1}{2} \left(c^E + t + p_2^I - rn_1^I\right). \end{aligned}$$

Firms' period-two best response functions are very similar to those in the model without rewards. The differences, represented by the final term in each function, can be explained as follows. There are two effects of introducing a rewards program. The first is the switching cost that the reward generates. From the perspective of either firm, this effect can be interpreted as a virtual change in the other firm's price. Fixing the firms' prices, introducing a reward of amount r has the same effect on the incumbent as if the entrant increased its price by rn_1^I. The reward causes the incumbent to further attract proportion $\frac{r}{2t}$ of the n_1^I consumers who qualify for it; that is, the incumbent's second-period market share increases by $\frac{r}{2t}n_1^I$. This is identical to the effect that would result if the entrant increased its price by rn_1^I in the absence of a rewards program. Thus, with rewards, the incumbent increases its price by the same amount as it would if the entrant made such a price increase and there were no rewards. The entrant makes a similar adjustment downward, as if the entrant had reduced his price by rn_1^I.

The second effect of the rewards program can be interpreted as a change in the marginal cost of the incumbent. The rewards program, which has been discussed as a discount for repeat consumers, can alternatively be viewed as a cash payment to repeat consumers. This increases the marginal cost of attracting additional consumers. That is, selling to additional consumers who did purchase in the first period incurs upon the incumbent an additional cost of r per unit sold. Fixing the entrant's price, a reduction in price by the incumbent in the model with rewards increases sales by the same amount as the model without rewards, with proportion n_1^I of these sales being to repeat customers and the remainder being to new customers. Hence, the incumbent's marginal cost incurred by attracting new consumers has increased from c^I in the model without rewards to $c^I + rn_1^I$ when rewards are used. Thus, the incumbent increases his price to compensate in the same manner it would if its cost c^I were to rise by rn_1^I. The entrant, however, experiences no change in cost and makes no such adjustment. This is a crucial difference between the model of this paper and models where switching costs exist exogenously rather than being created via rewards, in which case the incumbent firm would not have to make the reward payments. In the former environment, this change in marginal cost would be negated for both firms, whereas only the entrant is unaffected in the latter.

The preceding discussion can be summarized via the following annotated reaction functions which illustrate the differences between the Hotelling reaction functions and those under a rewards program:

$$p_2^I = \frac{1}{2}\big(c^I + \underbrace{rn_1^I}_{\text{change in cost}} + t + p_2^E + \underbrace{rn_1^I}_{\text{virtual change in } p_2^E}\big); \qquad p_2^E = \frac{1}{2}\big(c^E + t + p_2^I - \underbrace{rn_1^I}_{\text{virtual change in } p_2^I}\big)$$

Solving the reaction functions simultaneously yields firms' subgame equilibrium prices:

$$\hat{p}_2^I = c^I - \frac{1}{3}\left(c^I - c^E\right) + t + rn_1^I \tag{5}$$

$$\hat{p}_2^E = c^E + \frac{1}{3}\left(c^I - c^E\right) + t. \tag{6}$$

In equilibrium, the incumbent increases his price from the level without rewards. In light of the previous discussion, this is not surprising. The incumbent has both a higher marginal cost and some of its customers face a switching cost; both of these effects cause the incumbent to increase his price.

Due to the asymmetric effects of the implementation of a rewards program on firms' reaction functions, such a program does not affect the equilibrium price of the entrant. As can be seen from the reaction functions above, that is not to say that the entrant does not consider the impact of the reward on consumers' decisions. If the incumbent were to deviate from equilibrium by changing the reward amount but not changing its second period price, the entrant would not be best responding by charging its Hotelling equilibrium price. Rather, in equilibrium, the entrant's reaction to the two adjustments made by the incumbent exactly counteract the single adjustment the entrant would make on its own. As might be expected, this is different from the environment where the incumbent does not create switching costs by using rewards. Instead, if switching costs exist exogenously, the equilibrium price of the entrant does not correspond to the Hotelling price.[12]

While all consumers face the Hotelling price offered by the entrant, the incumbent's potential new customers are faced with a price p_2^I that lies above the Hotelling price. The incumbent's potential repeat customers, however, face an effective price $p_2^I - r$ which is lower than the Hotelling price. That is, $p_2^I \geq p_H^I \geq p_2^I - r$. Not surprisingly, then, the proportion of consumers eligible for the reward who buy from the incumbent is greater than the incumbent's no-rewards market share, while the proportion of consumers not eligible for the reward who buy from the incumbent is less than the incumbent's no-rewards market share, i.e., $\overline{\theta} \geq n_H^I \geq \underline{\theta}$.

Substituting the period-two equilibrium prices into (3) yields the incumbent's equilibrium period-two market share:

$$\begin{aligned} n_2^I &= n_1^I \left(\frac{1}{2} + \frac{\frac{1}{3}\left(c^E - c^I\right) - r n_1^I + r}{2t} \right) + \left(1 - n_1^I\right) \left(\frac{1}{2} + \frac{\frac{1}{3}\left(c^E - c^I\right) - r n_1^I}{2t} \right) \\ &= \frac{1}{2} - \frac{1}{6t}\left(c^I - c^E\right) = n_H^I. \end{aligned}$$

Noting that $n_2^E = 1 - n_2^I$ and recalling that the entrant's equilibrium price is the same whether or not rewards are used, the following results has been proven.

Proposition 1. *In any equilibrium where the incumbent firm uses a rewards program, firms' market shares in the second period are identical to those in the second period when no rewards are used. Furthermore, the entrant's equilibrium second-period price and profit level are also unaffected by rewards.*

Corollary 1. *There is no rewards program which allows the incumbent to prevent entry.*

The implication of this result is that if the entrant can profitably enter when rewards are not used, then it also enters profitably when a rewards program is in place. In other words, the incumbent is not able to forestall entry by implementing a rewards program prior to entry occurring in an effort to generate switching costs for consumers. This stands in

[12] In fact, in this environment, a switching cost r that can is costless for the incumbent results in equilibrium prices

$$\begin{aligned} p_2^I &= c^I - \frac{1}{3}\left(c^I - c^E\right) + t + \frac{1}{3} r n_1^I \\ p_2^E &= c^E + \frac{1}{3}\left(c^I - c^E\right) + t - \frac{1}{3} r n_1^I; \end{aligned}$$

that is, the incumbent would increase its price over the Hotelling level by less than it does when it must pay the amount of switching cost, and the entrant would actually shade its price down from the Hotelling level by the same amount as the incumbent increases its price. This agrees with the intuition behind the reaction functions discussed above. Also note that the entrant's price is now decreasing in the incumbent's first-period sales level. This means that the incumbent's action in period one will impact the profits of the entrant in period two. As shall shortly be demonstrated, this feature is absent in the model with rewards programs.

contrast to models with exogenous switching costs, where entry may be deterred even in the presence of very low switching costs. The intuition for this difference lies in the added cost of the reward payout. When switching costs exist exogenously, the incumbent firm is able to prevent the entrant from gaining a critical mass of market share if it has sold to enough consumers in the first period because consumers are reluctant to switch. However, under the rewards program, the incumbent has to pay each repeat customer the amount of the reward. This provides the incumbent with an incentive to concede more of its old consumers to the entrant. This incentive grows stronger when the incumbent's first-period sales level is larger because the effective marginal cost of the incumbent increases in the first-period sales level (recall that the incumbent must pay the reward amount to a proportion n_1^I of the consumers who buy from it in the second period). As a result, subgame perfection requires that the entrant's profit level be unaffected by the incumbent's first-period sales, unlike the situation where switching costs are costless for the incumbent.

The result of the impossibility of entry deterrence is more robust than might be immediately apparent. Appendix A shows that the relaxation of several assumptions which have been made for simplicity do not alter this finding. In particular, this result does not depend critically upon the assumption that all consumers remain in the market in both periods, or that the distributions of consumers' locations are independent from one period to the next. Furthermore, the above results under rewards are unaffected by allowing for consumers to have quadratic transportation costs, rather than linear costs.[13] Although not explicitly addressed in Appendix A, it should be clear that the assumption of no discounting does not affect the analysis of period two, so relaxing this assumption renders the above results unchanged.

In what follows, it will be helpful to express the incumbent firm's period-two profits in terms of the parameters of the model, the first-period sales level, and the reward amount. Substituting the subgame equilibrium prices into (4) and rearranging yields

$$\begin{aligned} \Pi_2^I &= \frac{1}{2t}\left(t - \frac{1}{3}\left(c^I - c^E\right)\right)^2 - \frac{r^2}{2t} n_1^I \left(1 - n_1^I\right) \\ &= \Pi_H^I - \frac{r^2}{2t} n_1^I \left(1 - n_1^I\right). \end{aligned} \tag{7}$$

The following result follows from this expression.

Proposition 2. *In any equilibrium where the incumbent firm uses a rewards program, the incumbent firm's equilibrium second-period profits are strictly less than the second-period profits when no rewards are used.*

The interpretation of the above profit function, and therefore this proposition, is as follows. The implementation of the rewards program has two effects on the second period profit, which are easily understood if the reward is viewed as a cost: a cash payout. First,

[13] The qualitative results of the rest of the model will be unaffected by allowing for any of these changes, so long as the picture does not change too drastically.

revenue increases above the Hotelling level due to the higher price the incumbent charges as a result of the switching costs consumers face. The increase in revenue is precisely the price markup multiplied by the period-two market share, or $rn_1^I \cdot n_2^I$. The second effect is that profits fall due to the cost of the reward payout. The actual amount that is paid out is the reward amount r multiplied by the mass of repeat purchasers. Of the n_1^I consumers eligible for the reward, $\overline{\theta}$ actually buy from the incumbent again, hence, the mass of repeat purchasers is given by $n_1^I \cdot \overline{\theta}$; the total reward payout is thus $rn_1^I \cdot \overline{\theta}$. This means that the total change in profit is $rn_1^I \left(n_2^I - \overline{\theta}\right)$. From (2) and (3), this is seen to be precisely the difference from the Hotelling profit level given in (7). In particular, because $\overline{\theta} \geq n_H^I = n_2^I$, this total change in profits is always nonpositive. Rewards programs, in effect, actually put the incumbent at a disadvantage in the second period due to the reward payout. Note, however, that this loss is decreasing in the first-period sales level as long as that sales level is greater than the no-rewards monopoly sales level.[14] The reason for this is that when more consumers buy in the first period, the second-period price increases, which increases revenue in the second period as the total market share is unaffected. Although more consumers are eligible for the reward, a smaller portion of these actually purchase again and claim the reward due to the higher price; this, in turn, further reduces the loss. That is, this loss provides the incumbent with an incentive to increase sales in the first period. This point will be returned to later.

From the preceding discussion, it is clear that the implementation of a rewards program hurts the incumbent's period-two profits, regardless of the reward amount or period-one sales. This underscores another key difference between this model and other models that feature switching costs: the incumbent cannot exploit locked-in consumers in order to increase its profit in the second period and instead is placed at a disadvantage because it must pay the reward amount to those who are locked in. Any increase in total profit must instead come from increased profits in the first period, where the firm is made more attractive to consumers because of the potential reward in the second period coupled with consumers' uncertainty about their future preferences. Trivially, setting the reward amount equal to zero results in Hotelling profits in the second period, and normal monopoly profits in the first period. The important question is then whether the incumbent can increase its profits in period one by enough to offset the period-two loss incurred by utilizing a rewards program. This question is addressed in the next section, where the first period of the game is considered.

4.2 Period One

In period one, the incumbent announces a first-period price p_1^I and a reward amount r which maximizes its total profits, taking into consideration the period-two equilibrium prices. To properly define this optimization problem, I first derive the first-period sales level of the incumbent in terms of the period-one price and the reward amount.

Because consumers get zero utility from not purchasing, the location of the indifferent

[14]Note that Assumption 3 implies that the monopolist sells to at least half the market when rewards are not used.

consumer in period one, and therefore the incumbent's sales level in the first period, is the position where the sum of the utility of buying in period one and the expected utility in period two conditional on having done so is equal to the expected utility in period two conditional on not having purchased. Using (2), a consumer calculates her expected period-two utility conditional on having purchased in period one as

$$\int_0^{\overline{\theta}} \left(R - \theta t - p_2^I + r\right) \mathrm{d}\theta + \int_{\overline{\theta}}^1 \left(R - (1-\theta)\, t - p_2^E\right) \mathrm{d}\theta$$
$$= \overline{\theta}\left(R - \frac{\overline{\theta}}{2} t - p_2^I + r\right) + \left(1 - \overline{\theta}\right)\left(R - \frac{\left(1-\overline{\theta}\right)}{2} t - p_2^E\right). \tag{8}$$

Likewise, using (1), a consumer's expected period-two utility conditional on having not purchased is

$$\int_0^{\underline{\theta}} \left(R - \theta t - p_2^I\right) \mathrm{d}\theta + \int_{\underline{\theta}}^1 \left(R - (1-\theta)\, t - p_2^E\right) \mathrm{d}\theta$$
$$= \underline{\theta}\left(R - \frac{\underline{\theta}}{2} t - p_2^I\right) + \left(1 - \underline{\theta}\right)\left(R - \frac{\left(1-\underline{\theta}\right)}{2} t - p_2^E\right). \tag{9}$$

From the above expressions, the location of the indifferent consumer, n_1^I, is then implicitly defined by the following equation:

$$R - n_1^I t - p_1^I + \overline{\theta}\left(R - \frac{\overline{\theta}}{2} t - p_2^I + r\right) + \left(1 - \overline{\theta}\right)\left(R - \frac{\left(1-\overline{\theta}\right)}{2} t - p_2^E\right)$$
$$= \underline{\theta}\left(R - \frac{\underline{\theta}}{2} t - p_2^I\right) + \left(1 - \underline{\theta}\right)\left(R - \frac{\left(1-\underline{\theta}\right)}{2} t - p_2^E\right)$$

Substituting the subgame equilibrium prices, of which p_2^I depends on n_1^I, into the expressions for $\underline{\theta}$ and $\overline{\theta}$ then substituting for the four expressions in the above equation allows it to be solved for the incumbent's first-period sales level in terms of the reward amount, the first-period price, and the parameters of the model:

$$n_1^I\left(p_1^I, r\right) = \frac{4t\left(R - p_1^I\right) + 2r\left(t - \frac{1}{3}\left(c^I - c^E\right)\right) + r^2}{2\left(r^2 + 2t^2\right)}. \tag{10}$$

Inspection of $n_1^I\left(p_1^I, r\right)$ yields an important result.

Proposition 3. *Implementation of a rewards program decreases the price sensitivity of consumers in the first period. Moreover, a rewards program permits the incumbent to increase sales in the first period over the monopoly sales level while charging a price higher than the monopoly price.*

Proof. The price sensitivity of consumers is the marginal loss in sales that results from an incremental price increase, given by $\left|\frac{\partial n_1^I}{\partial p_1^I}\right| = \frac{2t}{r^2+2t^2}$. For any $r > 0$, this is strictly less than $\frac{1}{t}$, the price sensitivity of consumers in the first-period of the no-rewards model. To complete the proof, first consider the first-period sales level that results when the no-rewards first-period price $p_M^I = \frac{R+c^I}{2}$ is charged by the incumbent and the reward amount $\bar{r} = t - \frac{1}{3}\left(c^I - c^E\right)$ is used (which is strictly positive by Assumption 3):

$$n_1^I\left(p_M^I, \bar{r}\right) = \frac{4t\left(\frac{R-c^I}{2}\right) + 3\bar{r}^2}{2\left(\bar{r}^2 + 2t^2\right)}.$$

This is greater than $n_M^I = \frac{R-c^I}{2t}$ if and only if

$$4t^2\left(\frac{R-c^I}{2}\right) + 3t\bar{r}^2 > \left(2\bar{r}^2 + 4t^2\right)\left(\frac{R-c^I}{2}\right),$$

or, equivalently (as $\bar{r}$ is strictly positive), $3t > R - c^I$. This is true by Assumption 2. Therefore, $n_1^I\left(p_M^I, \bar{r}\right) > n_M^I$. Because n_1^I is continuous in p_1^I, there exists a price $p' > p_M^I$ such that $n_1^I\left(p', \bar{r}\right) > n_M^I$. □

The first statement of this proposition says that if the incumbent increases its price, it loses fewer customers when it uses a rewards program than when it does not use rewards. This might initially seem somewhat counterintuitive, as the increase in price has the same effect on consumers' first-period utility whether or not rewards are present. The intuition lies in the fact that, as a result of the rewards program, the incumbent's period-two price is increasing in the first-period sales level. When the incumbent increases his first-period price, it will result in a lower sales level in the first period. Consumers correctly anticipate a lower period-two price from the incumbent (while the entrant's price remains constant) and also realize that by purchasing from the incumbent they increase the probability that they will end up buying from the incumbent in the second period thus benefitting from the period-two price reduction. Thus, the decrease in purchasers' expected utility as a result of a price increase is actually reduced when rewards are used.

The second statement of this proposition says that implementing a rewards program permits the incumbent firm to charge a higher price in the first period while, at the same time, increasing first-period sales. The appeal of the rewards program to consumers is thus: unsure of which firm's product they will find more attractive in the second period, they are willing to spend more on the incumbent's good in the first period because there is a positive probability that they will find the reward useful later. It has been established that the incumbent can increase its first-period profits through rewards. Next to be shown is that the increased first-period profit can be enough to counteract the second-period loss that results from utilizing rewards.

Henceforth, the arguments of n_1^I are suppressed except where necessary. Now, the objective of the incumbent is to maximize total profits, which are the sum of the first- and second-period profits. The first-period profits are simply $\left(p_1^I - c^I\right) n_1^I$, so, using (7), the profit function can be written

$$\Pi^I = \left(p_1^I - c^I\right) n_1^I + \Pi_H^I - \frac{r^2}{2t} n_1^I \left(1 - n_1^I\right). \tag{11}$$

Here, an "interior solution," is a solution $\left(p_1^I, r\right)$ which maximizes the firm's objective function and satisfies $0 < n_1^I < 1$. It can be shown that the unique candidate for an interior solution $\left(p_1^I, r\right)$ to this optimization problem is given by

$$\hat{p}_1^I = \frac{R + c^I}{2} + \frac{\hat{r}^2}{2t}\left(1 - \frac{R - c^I}{2t}\right) + \frac{\hat{r}^2}{8t} - \frac{\hat{r}^4}{16t^3} \tag{12}$$

$$\hat{r} = t - \frac{1}{3}\left(c^I - c^E\right). \tag{13}$$

The derivation of the candidate solution proceeds as follows. First, the first-order condition for the first-period price is solved for the price. This expression for the price is then substituted into the first-order condition for the reward amount. The resulting condition for an interior solution is cubic in the reward amount; however, two of the roots correspond to a first-period sales of zero, which is clearly not optimal (as setting the reward amount equal to zero obviously yields higher total profits). The remaining root yields the candidate optimal reward amount, which is then substituted into the first-order condition for the first-period price, which in turn yields the candidate optimal price. The analytical details are presented in Appendix B.

Note that $\hat{r}$ is positive if and only if Assumption 4 holds.[15] Recall that Assumption 4 is the necessary and sufficient condition for the incumbent firm to realize positive sales in the second period when rewards are not used. That is, there can only be an equilibrium where rewards benefit the incumbent if the incumbent is not completely shut out of the market in the second period when rewards are not used. In other words, if the incumbent cannot realize positive profits in the second period without rewards, there is no rewards program that will help the incumbent increase its total profits. This is not surprising; by Proposition 1, period-two market shares are unaffected by rewards. If all consumers know that they won't be buying from the incumbent in the second period, participating in a rewards program does not benefit them. The best the incumbent can do is set its normal monopoly price in the first period, as its selection of reward amount does not matter.[16]

Because the analysis thus far has not relied upon the fact that the reward amount is positive, it can be seen that even if the incumbent had the option of using a negative reward amount, thereby charging a higher price to repeat consumers, it would not elect to do so. This contrasts with models of observed-behavior price discrimination that support such behavior as an equilibrium strategy. Again, this should not be surprising. In those models, a firm uses

[15] Appendix B shows that the reward amount is the same in the optimal corner solution, so Assumption 4 covers all cases.

[16] Technically speaking, any reward amount paired with the monopoly price would constitute an equilibrium, but no consumers would be swayed by the reward.

this type of price discrimination with the aim of charging a higher price to those consumers who prefer its own good while charging a lower price to customers who otherwise would prefer the product of its opponent in order to "poach" them. In the current model, consumers' locations in the second period are randomized, so prior purchasing decisions reveal nothing about consumers' current preferences.

Substituting the values for $\hat{p}_1^I$ and $\hat{r}$ into (10) yields the incumbent's sales level in the first period for the interior solution:

$$\hat{n}_1^I = \frac{R - c^I}{2t} + \frac{\hat{r}^2}{8t^2}. \tag{14}$$

Noting that this expression is positive, the necessary and sufficient condition for $\left(\hat{p}_1^I, \hat{r}\right)$ to be an valid solution (that is, it results in $0 \leq n_1^I \leq 1$) is given by the following assumption:

Assumption 5. $\hat{r}^2 \leq 4t\left(2t - \left(R - c^I\right)\right)$.

The intuition for this assumption is straightforward. It merely specifies that the incumbent not cover too much of the market in the first period when it doesn't use rewards (i.e., $R - c^I$ is not too close to $2t$). As has been discussed, the incumbent can end up benefitting from rewards by increasing its sales, even at a higher price. If this assumption does not hold, then the monopolist optimally covers the entire market in the first period, necessitating a corner solution.[17]

Importantly, this assumption is not necessary for an equilibrium to exist. In fact, if it fails, the optimal corner solution (where $n_1^I = 1$) is the unique equilibrium. In this situation, the incumbent will charge a first-period price $\tilde{p}_1^I$ which is higher than $\hat{p}_1^I$, and will still use the reward $\hat{r}$.[18] Intuitively, charging the lower price $\hat{p}_1^I$ covers the market, but the consumer located at 1 (as well as every other consumer) still strictly prefers buying to not buying. Thus, the incumbent increases its price to extract this "excess" surplus. This equilibrium is otherwise qualitatively similar to the equilibrium when the solution is in the interior, so the remainder of the analysis will focus on the interior solution. For proofs of the claims made in this paragraph, the reader is referred to the end of Appendix B.

If this assumption holds, then $\left(\hat{p}_1^I, \hat{r}\right)$ is the unique optimal price and reward amount as long as selecting it results in higher profits than choosing $\left(p_M^I, 0\right)$, i.e., electing not to use rewards, and higher profits than choosing the optimal corner solution. As the following proposition points out, this is indeed the case.

Proposition 4. *The incumbent firm's equilibrium first-period price and reward amount are given by $\hat{p}_1^I$ and $\hat{r}$, respectively. Furthermore, the equilibrium first-period price satisfies*

$$\hat{p}_1^I = \frac{R + c^I}{2} + \frac{\hat{r}^2}{2t}\left(1 - \frac{R - c^I}{2t}\right) + \frac{\hat{r}^2}{8t} - \frac{\hat{r}^4}{16t^3} > \frac{R + c^I}{2} = p_M^I, \tag{15}$$

[17]In Appendix B, it is demonstrated that the interior solution and corner solutions coincide when this assumption holds as an equality.

[18]Here, $\tilde{p}_1^I$ denotes the optimal price under the constraint that the market is covered during period one. However, note that $\tilde{p}_1^I > \hat{p}_1^I$ if and only if Assumption 5 fails. It should not be surprising that when Assumption 5 holds, $\tilde{p}_1^I \leq \hat{p}_1^I$. When parameters are such that an interior solution is optimal, charging a lower price than the equilibrium price would be necessary to sell to the entire market.

the equilibrium first-period sales level satisfies

$$\hat{n}_1^I = \frac{R - c^I}{2t} + \frac{\hat{r}^2}{8t^2} > \frac{R - c^I}{2t} = n_M^I, \tag{16}$$

and the incumbent's equilibrium total profit level satisfies

$$\hat{\Pi}^I = \frac{1}{t}\left(\frac{R - c^I}{2} + \frac{\hat{r}^2}{8t}\right)^2 + \Pi_H^I > \frac{1}{t}\left(\frac{R - c^I}{2}\right)^2 + \Pi_H^I = \Pi_M^I + \Pi_H^I. \tag{17}$$

Proof. The first equality in (15) restates (12), while the inequality follows by noting that $\frac{R-c^I}{2t} < 1$ by Assumption 2 and $\hat{r} \leq t$ by Assumption 4, hence, $\frac{\hat{r}^2}{8t} > \frac{\hat{r}^4}{16t^3}$. The first equality in (16) restates (14), and the inequality follows trivially. The first equality in (17) is a result of substituting (12) and (14) into (11) and simplifying; the inequality follows trivially. Inequality (17) implies that the candidate solution dominates the no-rewards strategy. It remains to show that the candidate solution dominates all corner solutions and to check that $0 \leq \underline{\theta} \leq 1$ and $0 \leq \overline{\theta} \leq 1$ (as this latter point has been ignored in the analysis thus far). The remaining details can be found in Appendix B. □

As might be expected, in light of Proposition 3, the incumbent firm charges a higher price than the no-rewards price in the first period while at the same time increasing first-period sales. Furthermore, this increased first-period profit generated by rewards more than makes up for the loss of second-period profits due to the rewards program, as total profits also rise. The incumbent firm increases its price from the no-rewards level in response to the increased willingness to pay that is generated by the rewards program; however, the incumbent does not increase the price by so much that it merely maintains its no-rewards sales level. The incumbent has an incentive to sell to more consumers for two reasons, as discussed in the previous section. As the first-period sales level rises, the price that the incumbent will charge in the second period increases. While more consumers become eligible for the reward, a smaller portion of these eligible do claim the reward due to the higher price. At the same time, the total second period market share is unaffected, per Proposition 1, so this increase in price increases second-period revenues. The combination of these two effects causes an increase in the first-period sales level above the no-rewards level to have a positive impact on second-period profits as well as first-period profits.

Until this point, the analysis has merely assumed that the market is fully covered in the second period. Given firms' prices, this will only be true provided that the indifferent consumers located at $\underline{\theta}$ and $\overline{\theta}$ realize nonnegative utility upon purchasing from either firm. The following assumption is the necessary and sufficient condition for market coverage, and is derived in Appendix C.

Assumption 6. $R - c^I \geq \frac{3}{2}\hat{r} + \frac{1}{2}\hat{r}\hat{n}_1^I$.

The essence of this assumption is that for the market to be fully covered under the second-period equilibrium prices, the optimal reward amount $\hat{r}$ cannot be too large. The intuition for this lies in that the consumer who did not buy in the first period but is indifferent between firms in the second period, located at $\underline{\theta}$, is hurt *ex post* as a result of the rewards program. Without rewards, she still would not have purchased from the incumbent in the first period, as she must have been located to the right of $\hat{n}_1^I > n_M^I$. In the second period, without rewards, she would have purchased from the incumbent, as $\underline{\theta} < n_H^I$. However, the incumbent's second period equilibrium price is increased from the no-rewards price, by precisely $\hat{r}\hat{n}_1^I$, so this consumer is hurt by the rewards program and may realize negative utility by purchasing if this price markup is too large. Assumption 6 guarantees that this consumer will purchase in the second period, thus ensuring that the period-two market, which is covered under no rewards, is indeed covered under rewards.[19]

It may be of interest to note that Assumption 6 can be relaxed if the parameter R is permitted to increase from period one to period two. To see why, suppose that R does change between periods one and two, so that R_τ is the value of this parameter in period τ. The only time R_2 enters into the analysis, whether rewards are used or not, is to ensure market coverage in the second period. Assumption 3 establishes the necessary condition for the no-rewards case, while Assumption 6 handles the case when rewards are used. As these assumptions are never used elsewhere in the analysis except where they cancel in (8) and (9), all other appearances of R correspond to R_1. That is, as long as the market is covered, then in equilibrium neither firms nor consumers are affected by a change in R_2. Thus, as long as R_2 increases by enough, R_1 remains unrestricted by Assumption 6.

A change in R may occur, for example, if consumers realize higher utility from consumption in period two due to some external event. In the case of automobile markets, the implementation of tax incentives for owning newer, more efficient cars may increase the reservation values of new cars between purchases. Additionally, newer cars have regularly been equipped with more safety features and the like, as well as better gas mileage, than their older counterparts as technology has improved. These changes may also cause an increase in the inherent desirability of the products in question over time. Similarly, if firms are airlines offering different flights on the same departing route out of an airport, this route may become more desirable to consumers due to some event that happens near the other, receiving, end of the route, such as the opening of a vacation resort or tourist attraction.

In some cases, entry may be spurred in the first place by just such an event. Consider the airline example. An increase in R on the departing route in this case will likely apply to other potential routes into and out of the receiving airport. If this happens, the entrant may find establishing itself as a monopolist on previously unoffered routes out of the receiving airport to be profitable. If its cost of entry on the departing route is decreasing in the number of routes it offers into and out of the receiving airport, as very well may be the case, then entering the departing route may also become profitable even if it was not before the increase in R.

[19] Note that this assumption does not violate any other assumptions. For example, $R - c^I \geq 2\hat{r}$ is sufficient for this assumption to hold for any value of $\hat{n}_1^I$. Substituting in the optimal reward amount, this is true if $t \leq \frac{2}{3}\left(c^I - c^E\right)$ by Assumption 3.

5 Conclusion

This paper presented a study of the potential benefits of implementing a coupon loyalty rewards program for a monopolist in the presence of a potential entrant. While the incumbent has no hope of deterring entry, it can increase equilibrium profits by starting a coupon rewards program before entry occurs, even though doing so results in lower profits after entry occurs. In particular, because of the additional expected surplus that coupons generate for consumers, the incumbent is able to increase its first-period sales while charging a higher price.

These results contrast sharply with the standard switching costs models, which permit entry deterrence on the part of the monopolist by means of low pricing in the first period in order to lock in consumers in the second period, thereby preventing the entrant from recouping the cost of entry. In the current model, the incumbent firm is actually hurt by a large established customer base in the second period, because it must pay the reward amount (or charge a lower price) to those who are repeat customers. In standard models of switching costs, it is often the case that firms focus on attracting consumers early and exploiting them later. By contrast, in this model, the focus is on creating surplus early through rewards, which is immediately extracted in order to make up for a loss later on.

Although coupon loyalty rewards programs are indeed a form of price discrimination based on past behavior, the results of this paper are substantially different than those found in other behavior-based price discrimination studies, where firms typically offer higher prices to repeat customers than to new customers. Critically, when consumers preferences vary from period to period, the results of those studies may fail, leaving rewards programs as the optimal price discrimination strategy.

In summary, coupon rewards programs, which are widely observed in real-world environments, have rarely been studied. The current paper is an attempt to better understand the motivation behind such programs, which have not received significant attention thus far. Further work in this area is more than warranted; it is my hope that this paper will make a good starting point.

Appendix

A On the Impossibility of Entry Deterrence

Proposition 1 implies that entry cannot be deterred regardless of the rewards program chosen by the incumbent. That the entrant receives precisely the same profits regardless of the strategy of the incumbent may seem to depend critically on several specific assumptions made. However, this appendix will show that the impossibility of entry deterrence is robust to a number of changes to the model. In particular, Proposition 1 holds under a variety of changes. Moreover, in the event that Proposition 1 may not hold, it is still the case that entry is unavoidable.

I consider several possible adjustments: using quadratic, rather than linear, transportation costs; allowing a portion of consumers to exit the market between the first and second periods; and relaxing the assumption that consumers are redistributed independently of their earlier positions.

A.1 Transportation Costs

I first consider introducing quadratic transportation costs rather than the linear costs of the current model. Consider the modified utility of a consumer located at θ who purchases from the firm located at i at a price of p^i:

$$U(\theta, i) = R - |\theta - i|^2 t - p^i.$$

Although quadratic costs have long been used in linear city models such as this (see, e.g. D'Aspremont *et al.* (1979)), it is easily shown that when firms are located at the endpoints of the unit interval and the market is covered (as is the case in period two), then firms sales levels (given prices) are equivalent under either specification. In other words, firm i's no-rewards level of sales in period two, $n^i_H(p^i_H, p^j_H)$, is identical in each case, so the no-rewards period-two subgame equilibrium is unaffected by this change. Similarly, the cutoff values $\underline{\theta}$ and $\overline{\theta}$ under rewards are the same whether linear or quadratic costs are used, hence the period-two level of sales for each firm under rewards is unaffected as well. Therefore, all of the analysis involving period two holds for quadratic costs as well as linear costs, and Proposition 1 remains valid.

A.2 Overlapping Generations

Next, suppose that a proportion $0 < \gamma \leq 1$ of consumers remain in the market at the end of the first period, while the rest are replaced by an equal measure of consumers. All consumers are then distributed randomly in the second period. When $\gamma = 1$, this model is identical to that in the body of the paper, and the no-rewards analysis is unaffected by allowing for $\gamma < 1$.

Suppose the incumbent's sales in the first period are given by n^I_1. Under rewards, in the second period, there are γn^I_1 consumers eligible for the reward and $(1 - \gamma n^I_1)$ consumers who are not, and both segments are uniformly distributed over the unit interval. In the model presented in the body of the paper, these proportions are n^I_1 and $(1 - n^I_1)$, respectively. Note that replacing n^I_1 with γn^I_1 throughout the period-two analysis yields the correct results for the model with $\gamma < 1$, and the intuition behind the results is identical. Importantly, as neither the period-two equilibrium price for the entrant nor firms' period-two equilibrium market shares depend on n^I_1, these values are unaffected by letting $\gamma < 1$. Thus, Proposition 1 is unaffected as well.

Importantly, all period-two equilibrium values under this specification approach those given in the original model as γ approaches 1. Thus, so long as γ is sufficiently large, allowing for $\gamma < 1$ does not qualitatively affect the rest of the results of the paper.

A.3 Correlated Consumer Positions

Until now, the rather strong assumption that consumers' locations in the second period are independent of their positions in the first period has been made. To relax this assumption, I now allow for the possibility that only a proportion $0 < \rho \leq 1$ of consumers change locations, and these consumers do not know whether they will change positions.[20] This introduces (perhaps strong) positive correlation between consumers' first- and second-period locations. A consumer located at θ in period one has a second-period position drawn from a distribution with a mass point at θ: with probability $(1-\rho)$, her period-two location will be θ; with probability ρ, her position will be drawn from a uniform distribution on $[0,1]$.

Note that when $\rho = 1$, this model is identical to that in the body of the paper. Without rewards, the period-two equilibrium is unaffected by allowing for $\rho < 1$. Under rewards, suppose that the incumbent's first-period sales are given by n_1^I. In period two, the cutoff values $\underline{\theta}$ and $\overline{\theta}$ given by (1) and (2) are unaffected; that is, consumers who purchased in the first period still purchase from the incumbent if and only if their position is to the left of $\overline{\theta}$ and consumers who did not purchase in the first period still purchase from the incumbent if and only if their position is to the left of $\underline{\theta}$. The market is still assumed to be covered: if consumers don't purchase from the incumbent, they purchase from the entrant.

It will be helpful to define the profits that each firm derives from the segment of consumers whose location has changed. The ρ consumers whose positions have changed is distributed uniformly over the unit interval, and proportion n_1^I of these are eligible for the reward. Thus, the incumbent's profits from this segment are given by ρ multiplied by the period-two profits from the original model:

$$\Pi_\rho^I = \left(p_2^I - r - c^I\right)\rho n_1^I \overline{\theta} + \left(p_2^I - c^I\right)\rho\left(1 - n_1^I\right)\underline{\theta}.$$

Similarly, the entrant's profits from this segment are given by

$$\Pi_\rho^E = \left(p_2^E - c^E\right)\rho\left(n_1^I\left(1-\overline{\theta}\right) + \left(1-n_1^I\right)\left(1-\underline{\theta}\right)\right).$$

Next, consider the consumers who have not changed location. This segment is of mass $(1-\rho)$ and also distributed uniformly over the unit interval, but all consumers located to the left of n_1^I are eligible for the reward, while those to the right are not. The expressions for firms' profits from this segment vary, depending on the values of n_1^I, $\underline{\theta}$ and $\overline{\theta}$. Noting that $\underline{\theta} < \overline{\theta}$ for all $r > 0$, there are three possible cases in equilibrium:

- Case 1: $\underline{\theta} < \overline{\theta} < n_1^I$

In Case 1, the incumbent's sales as a proportion of this segment will be $\overline{\theta}$. That is, only some of the incumbent's old customers in this segment will purchase again, and no new customers will be attracted. All of the consumers in this segment who do purchase from the incumbent receive the reward.

[20] This guarantees that there is still a unique cutoff n_1^I in period one so that consumers indexed by θ buy if and only if $\theta \leq n_1^I$.

- Case 2: $\underline{\theta} \leq n_1^I \leq \overline{\theta}$

In Case 2, The incumbent's sales as a proportion of this segment are n_1^I. All of the old consumers in this segment are repeat purchasers, and no new consumers are attracted. All of the consumers in this segment who do purchase from the incumbent receive the reward.

- Case 3: $n_1^I < \underline{\theta} < \overline{\theta}$

In Case 3, the incumbent's sales as a proportion of this segment will be $\underline{\theta}$. All old consumers will buy again from the incumbent, but some new consumers are now attracted. The incumbent will pay the reward to n_1^I of the consumers in this segment.

I proceed by showing that Proposition 1 is unaffected for any equilibrium which satisfies Case 1 or Case 3. I then show that even though it may not hold if Case 2 occurs in equilibrium, the incumbent is still powerless to prevent entry. Hence, Corollary 1 is always valid.

A.3.1 Cases 1 and 3

Suppose that an equilibrium exists which satisfies Case 1 or Case 3. The period-two analysis for these cases is similar to that of the model with $\rho = 1$. For these cases, Table 1 displays firms' second-period profit functions, the reaction functions derived from the first-order conditions, the second period equilibrium prices, and the period-two market shares of the incumbent.

The intuition behind the reaction functions and equilibrium prices is identical to that of the model when $\rho = 1$ in either case. In either of these cases, unlike in Case 2, the purchasing decision of marginal consumers in both segments (those who change positions and those who do not) is affected by an incremental change in price by either firm. Thus, the incumbent's reaction function is again affected by the rewards program in two ways: rewards have the same effect on sales as an increase in the entrant's price, and rewards increase the marginal cost of attracting additional consumers. The incumbent raises its price in response to each of these effects. The entrant's reaction is only affected by rewards in a single way: rewards have the same effect on sales as a decrease in the incumbent's price. Again, the increase in the incumbent's reaction function is twice that of the decrease in the entrant's reaction function, so that the entrant's equilibrium price is precisely equal to the no-rewards price.

In both Case 1 and Case 3, evaluating the incumbents market share at the equilibrium prices, i.e.,

$$\underline{\theta} = \frac{1}{2} + \frac{\hat{p}_2^E - \hat{p}_2^I}{2t},$$

yields the result that $n_2^I = n_H^I$. Therefore, Proposition 1 holds for these cases as well. Because the decisions of marginal consumers in both segments are affected by incremental changes in price, subgame perfection again mandates that the incumbent concede the no-rewards market share to the entrant. This is despite the fact that the incumbent could capture a larger share by charging the no-rewards price. The costliness of the reward payout prevents such an action from being profitable, as in the original model.

	Case 1	Case 3
	Profit Functions	
Π_2^I Π_2^E	$\Pi_\rho^I + \left(p_2^I - r - c^I\right)(1-\rho)\overline{\theta}$ $\Pi_\rho^E + \left(p_2^E - c^E\right)(1-\rho)\left(1-\overline{\theta}\right)$	$\Pi_\rho^I + \left(p_2^I - c^I\right)(1-\rho)\underline{\theta} - r(1-\rho)n_1^I$ $\Pi_\rho^E + (1-\rho)\left(p_2^E - c^E\right)(1-\underline{\theta})$
	Reaction Functions	
p_2^I p_2^E	$\frac{1}{2}\left(c^I + t + p_2^E + 2\left(\rho r n_1^I + (1-\rho)r\right)\right)$ $\frac{1}{2}\left(c^E + t + p_2^I - \left(\rho r n_1^I + (1-\rho)r\right)\right)$	$\frac{1}{2}\left(c^I + t + p_2^E + 2\rho r n_1^I\right)$ $\frac{1}{2}\left(c^E + t + p_2^I - \rho r n_1^I\right)$
	Equilibrium Prices	
$\hat{p}_2^I$ $\hat{p}_2^E$	$c^I - \frac{1}{3}\left(c^I - c^E\right) + t + \left(\rho r n_1^I + (1-p)r\right)$ $c^E + \frac{1}{3}\left(c^I - c^E\right) + t$	$c^I - \frac{1}{3}\left(c^I - c^E\right) + t + \rho r n_1^I$ $c^E + \frac{1}{3}\left(c^I - c^E\right) + t$
	Incumbent's Market Share	
n_2^I	$\underline{\theta} + \frac{1}{2t}\left(\rho r n_1^I + (1-\rho)r\right)$	$\underline{\theta} + \frac{1}{2t}\rho r n_1^I$

Table 1: Analysis of Cases 1 and 3

A.3.2 Case 2

Case 2 is qualitatively different from Cases 1 or 3 in that, of those consumers who do not change locations, no old consumers are lost by the incumbent (unlike in Case 1) nor are any new consumers attracted (unlike in Case 3). Instead, the incumbent's sales to this segment are precisely equal in both periods, and the exact same consumers are served in each period. In such an equilibrium, the purchasing decision of the marginal consumers in the segment which has not changed location is unaffected by an incremental price change.

Assume that an equilibrium satisfying Case 2 exists. In this case, firms' second-period profit functions are

$$\begin{aligned}\Pi_2^I &= \Pi_\rho^I + \left(p_2^I - r - c^I\right)(1-\rho)\,n_1^I \\ \Pi_2^E &= \Pi_\rho^E + \left(p_2^E - c^E\right)(1-\rho)\left(1-n_1^I\right).\end{aligned}$$

Firms' reaction functions are then

$$\begin{aligned}p_2^I &= \frac{1}{2}\left(c^I + t + p_2^E + 2rn_1^I + 2t\frac{1-\rho}{\rho}n_1^I\right) \\ p_2^E &= \frac{1}{2}\left(c^E + t + p_2^I - rn_1^I + 2t\frac{1-\rho}{\rho}\left(1-n_1^I\right)\right),\end{aligned}$$

and the second-period equilibrium prices are

$$\hat{p}_2^I = c^I - \frac{1}{3}\left(c^I - c^E\right) + t + rn_1^I + \frac{2}{3}\frac{1-\rho}{\rho}\left(1 + n_1^I\right)t$$
$$\hat{p}_2^E = c^E + \frac{1}{3}\left(c^I - c^E\right) + t + \frac{2}{3}\frac{1-\rho}{\rho}\left(2 - n_1^I\right)t.$$

Clearly, the entrant's price is now affected by the incumbent's first-period sales level. However, it is now always strictly greater than the no-rewards price. The entrant's equilibrium market share is given by

$$\begin{aligned} n_2^E &= \rho\left((1-\underline{\theta}) - \frac{r}{2t}n_1^I\right) + (1-\rho)\left(1 - n_1^I\right) \\ &= \rho\left(\frac{1}{2} + \frac{1}{6t}\left(c^I - c^E\right)\right) + \frac{1}{3}(1-\rho)\left(2 - n_1^I\right), \end{aligned}$$

and the entrant's equilibrium profit level is

$$\Pi_2^E = \frac{\rho}{2t}\left[t + \frac{1}{3}\left(c^I - c^E\right) + \frac{2}{3}\frac{1-\rho}{\rho}\left(2 - n_1^I\right)t\right]^2.$$

Note that when $\rho = 1$, this is precisely equal to the no-rewards profit level. However, if $\rho < 1$, Proposition 1 may not hold. I now show that the more important result, Corollary 1, cannot fail even if Proposition 1 does not hold.

Differentiating the entrant's equilibrium profits with respect to ρ yields

$$\frac{1}{18\rho^2 t}\left[\rho\left(c^I - c^E\right) + 2t\left(2 - n_1^I\right) + \rho t\left(2n_1^I - 1\right)\right]\left[\rho\left(c^I - c^E\right) - 2t\left(2 - n_1^I\right) + \rho t\left(2n_1^I - 1\right)\right].$$

Now,

$$\begin{aligned} &\rho\left(c^I - c^E\right) + 2t\left(2 - n_1^I\right) + \rho t\left(2n_1^I - 1\right) \\ &\qquad > \rho\left(c^I - c^E\right) + 2\rho t\left(2 - n_1^I\right) + \rho t\left(2n_1^I - 1\right) \\ &\qquad = \rho\left(c^I - c^E\right) + 3\rho t \\ &\qquad > 0, \end{aligned}$$

so the entrant's profits are (at least weakly) decreasing in ρ if

$$\rho\left(c^I - c^E\right) - 2t\left(2 - n_1^I\right) + \rho t\left(2n_1^I - 1\right) \leq 0. \tag{18}$$

Recall that when $\rho = 1$, the entrant's profits are equal to the no-rewards profit level. Thus, if (18) is satisfied, then implementing a rewards program when $\rho < 1$ weakly increases the profit level of the entrant, and Corollary 1 holds.

Finally, I show that (18) cannot fail in an equilibrium that satisfies Case 2. This means that there is no equilibrium where entry is deterred, regardless of the value of ρ. Suppose that (18) fails. Then

$$\rho\left(\left(c^I - c^E\right) + t\left(2n_1^I - 1\right)\right) > 2t\left(2 - n_1^I\right),$$

which implies

$$\left(c^I - c^E\right) + t\left(2n_1^I - 1\right) > 2t\left(2 - n_1^I\right),$$

or, equivalently,

$$\left(c^I - c^E\right) > 5t - 4n_1^I t. \tag{19}$$

This also implies that

$$n_1^I > \frac{5}{4} - \frac{1}{4t}\left(c^I - c^E\right) > \frac{1}{2}, \tag{20}$$

where the second inequality follows by Assumption 4. Aditionally, note that $\frac{r}{2t} \leq 1$ in equilibrium, otherwise $\overline{\theta} = \underline{\theta} + \frac{r}{2t} > 1$, which cannot be optimal for the incumbent.

Now, for Case 2 to be satisfied in equilibrium, it must be true that $\overline{\theta} \geq n_1^I$. Substituting the equilibrium prices into $\overline{\theta}$ yields

$$\begin{aligned}
\overline{\theta} &= \frac{1}{2} - \frac{1}{6t}\left(c^I - c^E\right) + \frac{1}{3}\frac{1-\rho}{\rho}\left(1 - 2n_1^I\right) + \frac{r}{2t}\left(1 - n_1^I\right) \\
&< \frac{1}{2} - \frac{1}{6t}\left(c^I - c^E\right) + 1 - n_1^I \qquad (21)\\
&< \frac{1}{2} - \frac{1}{6t}\left(5 - 4n_1^I\right)t + 1 - n_1^I \qquad (22)\\
&= \frac{1}{3}\left(2n_1^I - 1\right) + 1 - n_1^I, \qquad (23)
\end{aligned}$$

where (21) follows by (20) and the fact that $\frac{r}{2t} \leq 1$, and (22) follows by (19). Finally, note that line (23) is less than n_1^I if and only if

$$\frac{1}{3}\left(2n_1^I - 1\right) < 2n_1^1 - 1,$$

which is clearly true by (20). Therefore, Case 2 cannot be satisfied in equilibrium if (18) fails. This means that in any equilibrium satisfying Case 2, the entrant's profits are decreasing in ρ. Because $\rho = 1$ corresponds to the original model, the entrant's profits in such an equilibrium can only increase as a result of a rewards program.

The intuition for this result is as follows. In Case 2, the consumers who have changed locations become locked in for the incumbent, while potential new consumers in this segment are locked out. Recall that these consumers are not sensitive to an incremental price change. This provides the incumbent with an additional incentive to increase its price to "milk" these consumers. Similarly, the entrant has its own segment of consumers who are insensitive to a price change. Meanwhile, as both firms are incentivized to increase their prices on account of the non-moving segment, competition in the moving segment is dampened. As the non-moving segment grows larger (that is, as ρ becomes smaller), these effects are amplified. Hence, the entrant's profits increase as ρ falls.

Note that all period-two equilibrium values approach those given in the original model as ρ approaches 1, regardless of which of these three cases arises in equilibrium. Thus, so long as ρ is sufficiently large, allowing for $\rho < 1$ does not qualitatively affect the rest of the results of the paper, despite significantly complicating the analysis.

B Period One Analysis

This appendix presents the derivation of the optimal first-period price and reward amount for the incumbent, which are announced at the beginning of period one. First, the unique candidate optimal interior solution of the incumbent is derived. Next, the optimal corner solution is identified. Finally, it is shown that the candidate interior solution, if indeed a valid solution, dominates the corner solution and hence is the optimal strategy for the incumbent. The final section outlines the situation where the interior solution fails to be interior; in this case, the corner solution is the unique equilibrium.

B.1 The Candidate Interior Solution

First, differentiating the incumbent's profit function, given by (11), with respect to p_1^I, and setting the resulting partial derivative equal to zero yields the first-order condition for the optimal price:

$$\frac{\partial \Pi^I}{\partial p_1^I} = \frac{\partial n_1^I}{\partial p_1^I}\left(p_1^I - c^I\right) + n_1^I - \frac{\partial n_1^I}{\partial p_1^I}\left(1 - 2n_1^I\right)\frac{r^2}{2t} = 0. \tag{24}$$

From (10),

$$\frac{\partial n_1^I}{\partial p_1^I} = -\frac{2t}{r^2 + 2t^2}.$$

Substituting this and the expression for n_1^I, given by (10), into (24) allows the first-order condition to be solved for the price in terms of the reward amount:

$$p_1^I(r) = \frac{R + c^I}{2} + \frac{\left(2t^2 - r^2\right)\left[2r\left(3t - \left(c^I - c^E\right)\right) - 3r^2\right] - 12r^2 t\left(R - c^I\right) + 24r^2t^2}{48t^3} \tag{25}$$

Differentiating the profit function (11) with respect to r, the reward amount, and setting the resulting partial derivative equal to zero yields the first-order condition for the reward amount:

$$\frac{\partial \Pi^I}{\partial r} = \frac{\partial n_1^I}{\partial r}\left(p_1^I - c^I\right) - \frac{\partial n_1^I}{\partial r}\left(1 - 2n_1^I\right)\frac{r^2}{2t} - \frac{2r}{2t}n_1^I\left(1 - n_1^I\right) = 0. \tag{26}$$

At any critical point, both first-order conditions must hold simultaneously. Next is to show that there can be at most one interior critical point; that is, one critical point $\left(p_1^I, r\right)$ such that $0 < n_1^I < 1$. To do so, I evaluate each of the terms in (26) at $p_1^I = p_1^I(r)$ and solve for r. The partial derivative of the first-period sales level (10) is

$$\frac{\partial n_1^I}{\partial r} = \frac{\left(2t^2 - r^2\right)\left(3t - \left(c^I - c^E\right)\right) - 12rt\left(R - p_1^I\right) + 6rt^2}{3\left(r^2 + 2t^2\right)^2},$$

and evaluating this derivative at $p_1^I = p_1^I(r)$ yields

$$\left.\frac{\partial n_1^I}{\partial r}\right|_{p_1^I = p_1^I(r)} = \frac{2\left(2t^2 - r^2\right)\left(3t - \left(c^I - c^E\right)\right) - 12rt\left(R - c^I\right) + 12rt^2 + 3r^3}{12t^2\left(r^2 + 2t^2\right)}.$$

Evaluating the sales level at $p_1^I = p_1^I(r)$ yields

$$n_1^I\left(p_1^I(r), r\right) = \frac{2r\left(3t - \left(c^I - c^E\right)\right) + 12t\left(R - c^I\right) - 3r^2}{24t^2}. \tag{27}$$

Finally,

$$p_1^I(r) - c^I = \frac{\left(2t^2 - r^2\right)\left[2r\left(3t - \left(c^I - c^E\right)\right) + 12t\left(R - c^I\right) - 3r^2\right] + 24r^2t^2}{48t^3}.$$

Substituting each of the last three expressions into (26) and rearranging yields the following condition:

$$\frac{\left[3t - \left(c^I - c^E\right) - 3r\right]\left[2r\left(3t - \left(c^I - c^E\right)\right) + 12t\left(R - c^I\right) - 3r^2\right]}{144t^3} = 0$$

The left-hand side of this equation is cubic in r. However, by looking at (27), it can be seen that the roots which are defined by

$$2r\left(3t - \left(c^I - c^E\right)\right) + 12t\left(R - c^I\right) - 3r^2 = 0$$

correspond to $n_1^I = 0$. This is not an interior solution, nor can it be optimal. $n_1^I = 0$ implies a first-period profit of zero, and a second-period profit of Π_H^I. Clearly, the incumbent does better by setting $r = 0$, which results in total profits $\Pi_M^I + \Pi_H^I$. The unique candidate for an interior solution is thus defined by

$$3t - \left(c^I - c^E\right) - 3r = 0,$$

or, equivalently,

$$r = t - \frac{1}{3}\left(c^I - c^E\right).$$

This is the expression given by (13). Substituting this into (25) yields (12).

B.2 The Corner Solution

As shown above, any solution resulting in $n_1^I = 0$ cannot be optimal. Thus, an optimal corner solution must satisfy $n_1^I = 1$. From (10), such a solution must then satisfy $1 \leq n_1^I(p_1^I, r)$. From (10), this is equivalent to

$$2\left(r^2 + 2t^2\right) \leq 4t(R - p_1^I) + 2r\left(t - \frac{1}{3}\left(c^I - c^E\right)\right) + r^2,$$

which is equivalent to

$$p_1^I \leq R - t + r\left(\frac{1}{2} - \frac{1}{6t}\left(c^I - c^E\right)\right) - \frac{r^2}{4t}. \tag{28}$$

With $n_1^I = 1$, the profit function (11) can be rewritten as

$$\Pi^I = p_1^I - c^I + \Pi_H^I. \tag{29}$$

Thus, (28) must hold as an equality. Substituting into the profit function yields

$$\Pi^I = R - c^I - t + r\left(\frac{1}{2} - \frac{1}{6t}\left(c^I - c^E\right)\right) - \frac{r^2}{4t} + \Pi_H^I,$$

which is strictly concave in r. The first-order condition yields the optimal corner solution reward:

$$\tilde{r} = t - \frac{1}{3}\left(c^I - c^E\right) = \hat{r}.$$

Thus, the optimal reward, regardless of whether the solution to the incumbent's optimization problem is interior or not, is equal to the reward given by (13). Substituting this reward into (28) (where the inequality holds as an equality) yields the optimal corner solution price:

$$\tilde{p}_1^I = R - t + \frac{\hat{r}^2}{4t}.$$

Substituting this price into (29), the incumbent's profits when the optimal corner solution is chosen are equal to

$$\tilde{\Pi}^I = R - c^I - t + \frac{\hat{r}^2}{4t} + \Pi_H^I. \tag{30}$$

Next, I show that if the candidate interior solution is, in fact, an interior point, then it dominates the corner solution. This is one of the statements made by Proposition 4, the proof of which is also completed shortly.

B.3 Completing the Proof of Proposition 4

The trivial portions of the proof of Proposition 4 are given in the body of the paper. Now, I show that $\left(\hat{p}_1^I, \hat{r}\right)$ dominates $\left(\tilde{p}_1^I, \hat{r}\right)$ if Assumption 5 holds; that is, when the candidate interior solution is, in fact, interior, then it dominates the corner solution. The task at hand is to show that $\hat{\Pi}^I - \tilde{\Pi}^I \geq 0$. Define the function $f\left(\hat{r}^2\right)$ as this difference, which can be written as

$$f\left(\hat{r}^2\right) = \hat{\Pi}^I - \tilde{\Pi}^I = \frac{\left(R - c^I\right)^2}{4t} + \frac{\hat{r}^2}{4t}\left(\frac{\left(R - c^I\right)}{2t} - 1\right) - \left(R - c^I\right) + t + \frac{\left(\hat{r}^2\right)^2}{64t^3}.$$

It is simple to show that when Assumption 5 holds as an equality, i.e, $\hat{r}^2 = 4t\left(2t - \left(R - c^I\right)\right)$, then $f\left(\hat{r}^2\right) = 0$. If this is the case, then the candidate interior solution is not strictly interior; in fact, it coincides with the corner solution. It remains to show that $f\left(\hat{r}^2\right) \geq 0$ for all $\hat{r}^2 < 4t\left(2t - \left(R - c^I\right)\right)$. It suffices to show that $f'\left(\hat{r}^2\right) < 0$ for all $\hat{r}^2 < 4t\left(2t - \left(R - c^I\right)\right)$. Taking the derivative of f yields

$$f'\left(\hat{r}^2\right) = \frac{1}{4t}\left(\frac{R - c^I}{2t} - 1\right) + \frac{1}{4t}\left(\frac{\hat{r}^2}{8t^2}\right).$$

So, when $\hat{r}^2 < 4t\left(2t - \left(R - c^I\right)\right)$,

$$f'\left(\hat{r}^2\right) < \frac{1}{4t}\left(\frac{R - c^I}{2t} - 1\right) + \frac{1}{4t}\left(\frac{2t - \left(R - c^I\right)}{2t}\right) = 0.$$

This is the desired result. To complete the proof, it remains only to check that $0 \le \underline{\theta} \le \overline{\theta} \le 1$. If either of these values lies outside of the unit interval, then the analysis used to derive the equilibrium is invalid. Substituting the second-period equilibrium prices, given by (5) and (6), into (1) and (2) yields the following:

$$\underline{\theta} = \frac{\hat{r}\left(1 - \hat{n}_1^I\right)}{2t} \tag{31}$$

$$\overline{\theta} = \frac{\hat{r} + \hat{r}\left(1 - \hat{n}_1^I\right)}{2t}. \tag{32}$$

Clearly, $0 < \underline{\theta} < \overline{\theta} < \frac{2\hat{r}}{2t} \le 1$, as $\hat{r} \le t$. This completes the proof.

B.4 When Period-One Market Coverage is Optimal

This appendix contains the proof that the optimal corner solution becomes the equilibrium strategy when Assumption 5 fails. If the assumption fails, the only possible equilibrium strategy when a rewards program is used is the optimal corner solution $\left(\tilde{p}_1^I, \hat{r}\right)$, as there is no interior critical point.

Substituting the reward $\hat{r}$ into the period-two price for the incumbent, and noting that $n_1^I = 1$, shows that all consumers effectively face the no-rewards prices in period two if this strategy is used. Therefore, the market is fully covered in period two (because it is covered when rewards aren't used), $0 < \overline{\theta} = n_H^I < 1$, and, from (31), $\underline{\theta} = 0$. This implies that if $\tilde{\Pi}^I \ge \Pi_M^I + \Pi_H^I$ (that is, if the corner solution dominates the no-rewards strategy) then this strategy does, in fact, constitute the unique equilibrium strategy. I now prove that the required inequality holds.

While the number $\hat{\Pi}^I$ does not represent a profit level in this case ($\hat{n}_1^I > 1$, so the sales level used to derive it no longer corresponds to a sales level), the value of $\hat{\Pi}^I$ still will be helpful. In particular, it is still the case that $\hat{\Pi}^I > \Pi_M^I + \Pi_H^I$, as this result did not depend on Assumption 5. By an argument identical to that used to show that $\hat{\Pi}^I - \tilde{\Pi}^I \ge 0$ when Assumption 5 holds, it can be shown that $\hat{\Pi}^I - \tilde{\Pi}^I < 0$ when the assumption fails. Therefore, when the assumption fails, $\tilde{\Pi}^I > \hat{\Pi}^I > \Pi_M^I + \Pi_H^I$, so the corner solution does indeed dominate the no-rewards strategy. From (29) and (11), $\tilde{\Pi}^I > \hat{\Pi}^I$ implies that

$$\begin{aligned}
\tilde{p}_1^I - c^I &> \left(\hat{p}_1^I - c^I\right)\hat{n}_1^I - \frac{\hat{r}^2}{2t}\hat{n}_1^I\left(1 - \hat{n}_1^I\right) \\
&> \left(\hat{p}_1^I - c^I\right)\hat{n}_1^I \\
&> \hat{p}_1^I - c^I,
\end{aligned}$$

as $\hat{n}_1^I > 1$ and $1 - \hat{n}_1^I < 0$. This, in turn, implies that $\tilde{p}_1^I > \hat{p}_1^I > p_M^I$, as the result given by (15) also did not depend on Assumption 5. Trivially, the equilibrium first-period sales level is given by $\tilde{n}_1^I = 1 > n_M^I$.

C Market Coverage

This appendix derives the necessary and sufficient condition for the market to be fully covered in the second period, which is given as Assumption 6 in the body of the paper. The analysis used to derive the equilibrium assumed that consumers buy in the second period from the firm which yields them higher utility. By Assumption 3, the market is covered when rewards are not used. However, it is easily seen from (5) that the incumbent's second-period price increases when the incumbent uses a rewards program, so without additional restrictions on the parameter values, it is possible that some consumers would realize negative utility by purchasing in the second period. Therefore, it is necessary to impose that the indifferent consumers located at $\underline{\theta}$ and $\overline{\theta}$ do indeed maximize their utility by purchasing in period two given the equilibrium prices. This is also sufficient, as consumers who are not indifferent necessarily realize higher utility than the indifferent consumers by purchasing from their preferred firms. From (5) and (31), the utility that the indifferent consumer who did not purchase in the first period realizes by purchasing in period two is given by

$$\begin{aligned} R - \underline{\theta}t - \hat{p}_2^I &= R - \frac{\hat{r}\left(1 - \hat{n}_1^I\right)}{2} - c^I + \frac{1}{3}\left(c^I - c^E\right) - t - \hat{r}\hat{n}_1^I \\ &= R - c^I - \frac{3}{2}\hat{r} - \frac{1}{2}\hat{r}\hat{n}_1^I. \end{aligned}$$

Similarly, from (32) and (5), the second-period utility that the indifferent consumer who did purchase in period one realizes by purchasing is given by

$$\begin{aligned} R - \overline{\theta}t - \hat{p}_2^I + \hat{r} &= R - \frac{\hat{r} + \hat{r}\left(1 - \hat{n}_1^I\right)}{2} - c^I + \frac{1}{3}\left(c^I - c^E\right) - t - \hat{r}\hat{n}_1^I + \hat{r} \\ &= R - c^I - \hat{r} - \frac{1}{2}\hat{r}\hat{n}_1^I. \end{aligned}$$

It is easily seen that the indifferent consumer who purchased in the first period is strictly better off than the indifferent consumer who did not. Therefore, it is sufficient that the indifferent consumer who did not purchase realizes nonnegative utility by purchasing in period two, i.e.,

$$R - c^I - \frac{3}{2}\hat{r} - \frac{1}{2}\hat{r}\hat{n}_1^I \geq 0.$$

This is equivalent to Assumption 6.

References

[1] Banerjee, A. and L. Summers (1987). "On Frequent Flyer Programs and Other Loyalty Arrangements." Working Paper 1337, Harvard University.

[2] Beggs, A. and P. Klemperer. (1989). "Multiperiod Competition with Switching Costs." Discussion Paper 45. Nuffield College, Oxford University.

[3] Caminal, R. and C. Matutes (1990). "Endogenous Switching Costs in a Duopoloy Model." *International Journal of Industrial Organization*, Vol. 8, pp. 353-373.

[4] D'Aspremont, C., J. Gabszewicz and Thisse, J.-F. (1979). "On Hotelling's Stability in Competition." *Econometrica*, Vol. 47, pp. 1145-1150.

[5] Farrell, J. and P. Klemperer. (2007). "Coordination and Lock-In: Competition with Switching Costs and Network Externalities." In: M. Armstrong and R. Porter (eds.), *Handbook of Industrial Organization*, Vol. 3, pp. 1970-2072. North-Holland.

[6] —–, and C. Shapiro. (1988). "Dynamic Competition with Switching Costs." *Rand Journal of Economics*, Vol. 19, pp. 123-137.

[7] Fudenberg, D. and J. Tirole. (2000). "Customer Poaching and Brand Switching." *The Rand Journal of Economics*, Vol. 31, pp. 634-657.

[8] Hotelling, H. (1929). "Stability in Competition." *Economic Journal*, Vol. 39, pp. 41-57.

[9] Kim, J. and D. Koh (2002). "Attracting the Rival's Customers with Switching Costs." *Japanese Economic Review*, Vol. 53, pp. 134-139.

[10] Klemperer, P. (1987a). "Entry Deterrence with Consumer Switching Costs." *The Economic Journal*, Vol. 97, pp. 99-117.

[11] —–. (1987b). "The Competitiveness of Markets with Switching Costs." *The Rand Journal of Economics*, Vol. 18, pp. 138-150.

[12] Padilla, A. (1995). "Revisiting Dynamic Duopoly with Consumer Switching Costs." *Journal of Economic Theory*, Vol. 67, pp. 520-530.

[13] Taylor, C. (2003). "Supplier Surfing: Competition and Consumer Behavior in Subscription Markets." *The Rand Journal of Economics*, Vol. 34, pp. 223-246.

[14] Villas-Boas, J. (1999). "Dynamic Competition with Customer Recognition." *The Rand Journal of Economics*, Vol. 30, pp. 604-631.

[15] von Weizäcker, C. (1984). "The Costs of Substitution." *Econometrica*, Vol. 52, pp. 1085-1116.