House Price Dynamics

Jing Zhang *
The Ohio State University

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Abstract

This paper develops and estimates a model that explains several house price phenomena that bring challenges to housing models. The first phenomenon is a "stationarity-puzzle". The puzzle is that real house prices reject a unit root while real income, one of the most important factors identified in the literature as a determinant of real house prices, contains a unit root. Therefore, the challenge for a coherent house price model is to combine the different stationarity properties of house prices and income. Another phenomenon that has not been well explained in the literature is the short-run positive serial correlation and long-run mean reversion of house price changes. This paper presents a dynamic equilibrium model to explain the above empirical features of real house prices. The model is solved under both rational expectations and adaptive expectations. Separating Metropolitan Statistical Areas into a coastal group and an inland group, we estimate the model's parameters. The model with adaptive expectations fits the empirical features better than the one that assumes rational expectations: it generates a positive one-year autoregressive coefficient, a negative five-year autoregressive coefficient, and rejects a unit root in house prices.

*Department of Economics, The Ohio State University, 410 Arps Hall, 1945 N. High St., Columbus OH 43210, email: zhang.728@osu.edu. I am very grateful to Robert de Jong and Donald Haurin for valuable comments and suggestions.

1 Introduction

The housing market is a large economic sector and house value forms a key component of household assets. In 2010, the value of the primary residence was \$20.7 trillion, representing around 30% of households' total assets, and 62% of the median homeowner's total assets.[1] Therefore, house price movements have a large impact on households' wealth. Also, the performance of the housing market is closely linked to the overall economy. The recent housing bubble triggered a financial crisis and a recession. In light of these facts, understanding house price dynamics is an important issue.

One topic related to house price movements that gained a lot of attention is the relationship between house prices and income. Comparisons between house prices and income are often regarded as an indication of whether the housing market is overvalued or undervalued. Figure 1 presents the log real house prices and log real per capita income at the U.S. national level over 1975-2010.[2] To improve visual clarity, I scale the vertical axis for house prices so that the two series cross each other in 1975. The figure shows that house prices and income behave quite differently. House prices are much more volatile than income. For example, during the recent housing boom, U.S. real house prices rose by 56% over the period 1998-2006, while U.S. real income only rose by 12% during the same period of time. This rapid appreciation in real house prices relative to real income raised the publics' concern that house prices were much higher than the "fundamental value" and would tend to revert to a level that is justified by fundamentals such as income.

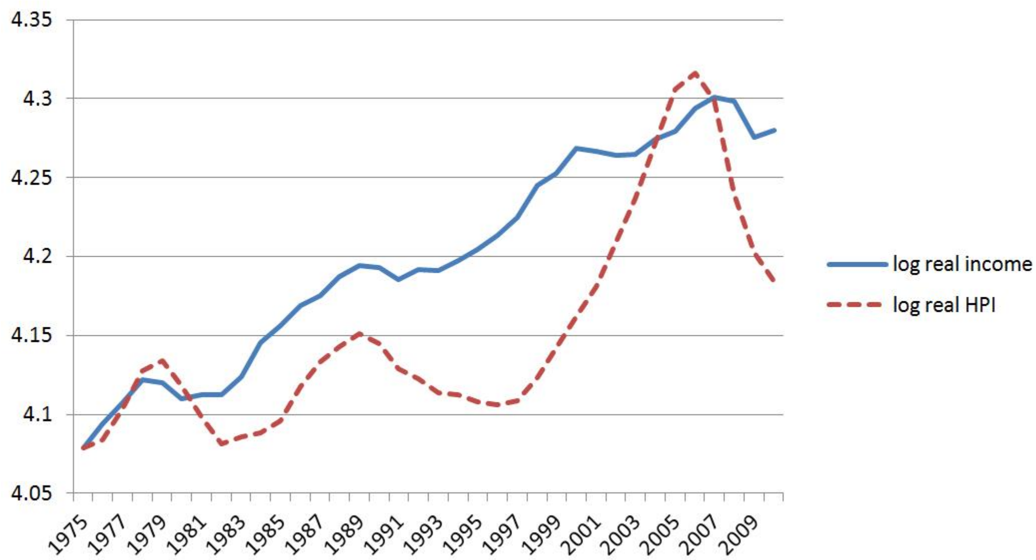
Another widely discussed issue is the inter-temporal evolution of house prices: the short-run momentum and long-run reversion of house price changes. Figure 2 plots the log real house prices for a coastal metropolitan statistical area (MSA) (San Francisco, CA) and an inland MSA (Columbus, OH). Two features are noticeable from the figure. First, house prices are predictable over short periods of time. There are apparent house price cycles. House prices keep rising for several years and then revert back in the following years. Second, the

[1] The numbers are from a National Association of Home Builders study based on Survey of Consumer Finance 2010:
http://www.nahb.org/generic.aspx?sectionID=734&genericContentID=215073&channelID=311#Footnote4

[2] In Figure 1, house price is measured by the Freddie Mac House Price Index (FMHPI), and income is measured by the Bureau of Economic Analysis (BEA) per capita income. We deflate them by CPI.

cycles are much bigger in the coastal city than the inland city: San Francisco experienced three sharp cycles during 1975-2010, while the house price series for Columbus is smoother. These two MSAs are fairly typical of coastal and inland MSAs.

Figure 1. U.S. log real per capita income and log real house price index, 1975-2010



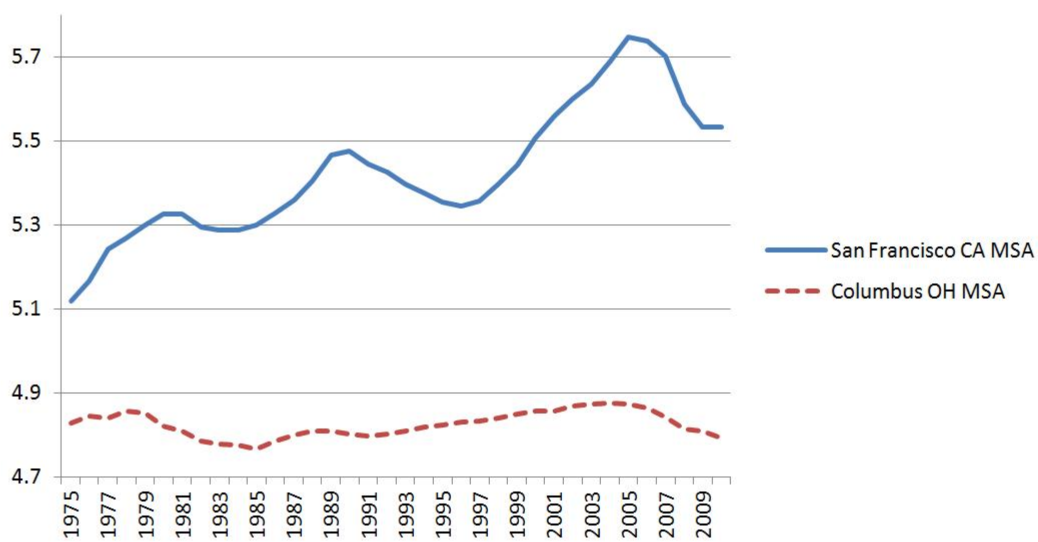
Notes: HPI is from FMHPI, and per captica income is from BEA. The vertical axis are scaled so that the two series cross each other in 1975.

The goal of this paper is to explain several house price phenomena that bring challenges to housing models. The first phenomenon is about a time series property of house prices: are house prices stationary or do they have a unit root.[3] This time series property has important implications for studying the relationship between house prices and income and modeling house prices. The housing literature indicates income as one of the most important factors that determine house prices. For example, Case and Shiller (2003) report that "income alone explains patterns of home price changes since 1985 in all but eight states." Also, the idea of a stable long-run relationship between house prices and income, as in the discussion for Figure 1, is studied in the literature by cointegration analyses. If house prices and income are

[3]When we state "house price" or "income" in the paper, we refer to real house price and real income.

cointegrated, they can drift away from each other temporarily, but they will tend to adjust back to their long-run relationship. Many papers test for a cointegrating relationship between house prices and income (for example, Gallin (2006), Holly, Pesaran, and Yamagata (2010)). Note that for cointegration analyses to be valid, house prices and income should both be unit root processes, which is the common treatment in the housing literature. However, Zhang, de Jong, and Haurin (2013) find that, when examining extended time periods that include the recent full housing cycle, there is strong evidence that house prices are trend stationary while income has a unit root.[4] Therefore, the relationship between house prices and income needs to be carefully reconsidered, and a challenge for a coherent house price model is to combine the different stationarity properties of house prices and income.

Figure 2. Log of real house prices, 1975-2010



Notes: House prices are derived from FMHPI.

Another phenomenon that has not been well explained in the literature is the short-run positive serial correlation and long-run mean reversion of house price changes, as well as their

[4]They examine log of real house price and log of real income. However, we also get similar results using real house price and real income: real house price rejects a unit root and real income does not reject a unit root.

geographic differences, as depicted in Figure 2. This momentum and reversion closely relate to house price cycles. Two questions are raised: what generates the house price fluctuations, and why do coastal cities have much bigger cycles than inland cities. In the literature, empirical studies document that there is short-run inertia in house price changes (Case and Shiller (1989)). If house prices rise this year, they have the tendency to continue this appreciation in the next year. The literature also documents that as house prices rise over a relatively long period, they will tend to revert. For example, Glaeser et al. (2012) report that an increase of house prices in the past five years is associated with a decrease of house prices over the next five-year period. Another way to describe the mean reversion is that when house prices deviate from their "fundamental value" they tend to adjust back, as in error correction models (Abraham and Hendershott (1996), Malpezzi (1999), Capozza, Hendershott, and Mack (2004), etc.). Momentum and reversion generate house price cycles. An equally important issue is the geographic differences of house price cycles. The magnitudes of momentum and reversion are both larger in coastal areas than inland areas (Abraham and Hendershott (1996), Glaeser et al. (2012)). Therefore, understanding the serial correlation and mean reversion of house price changes is useful for explaining how cycles happen, and why house price dynamics differ in different areas.

This paper modifies the dynamic spatial equilibrium model in Glaeser et al. (2012) to explain the above house price issues. Specifically, we study the stationarity property and the autocorrelation of one-year and five-year house price changes. In Glaeser et al.'s model, households' indirect utility of living in a city depends on income, amenities, and housing cost. Hence, differences in house prices across cities reflect the differences in income and amenities across cities. They assume migration among cities is costless and they have a spatial equilibrium condition requiring households to equalize their utility across cities in each period. On the supply side, they assume that the current value of the expected house prices in the next period equals the marginal cost of building a house, and this housing supply condition determines the amount of construction each period and therefore the housing stock in a city. Therefore, house prices are endogenously determined and reflect the local heterogeneity of demand side and supply side factors. They solve and estimate the model under rational expectations.

The main differences between this paper and Glaeser et al.'s are the assumptions about income and expectations. Glaeser et al. assume that income can be expressed as an exogenous productivity variable minus a coefficient times the population. We assume that income is an exogenous unit root process based on results from unit root tests. Our assumption about the exogeneity of income can be interpreted as following Capozza and Helsley (1990)'s urban model assumption. They assume a small open urban area which is affected by external demand shocks through the labor market, and thus household income in that small open city is an exogenous process. The main reason for our assumption of the income process is to study the stationarity puzzle: how a unit root income process is related to a trend stationary house price process. Glaeser et al. do not consider this issue in their model.

The second difference from Glaeser et al. is that they only consider rational expectations, while we solve and estimate the model under both rational and adaptive expectations. Their model under rational expectations failed to explain the short-run momentum of house price changes. Moreover, there are claims, both in the media and in the literature, that people in the housing market are irrational and backward-looking (Case and Shiller (2003), Shiller (2005), Glaeser, Gyourko, and Saiz (2008)). Therefore, we also consider the case where people adaptively adjust their expectations.

Our sample consists of 356 MSAs over 1980-2010. We separate the sample into a coastal group and an inland group and estimate the model parameters separately for each group. The estimation results reveal that the coastal group has more persistence in income changes, more inelastically supplied housing, and that the population increase has a smaller effect on amenities in the coastal cities. Based on estimation of the parameters, we simulate the house price series and compare how the predictions from the simulation match the data. The model with adaptive expectations fits the empirical features better than the one that assumes rational expectations: it generates a positive one-year autoregressive coefficient, a negative five-year autoregressive coefficient, and the unit root test rejects a unit root. However, for the coastal group, the model does not generate enough magnitude of the one-year and five-year price change autoregressive coefficients, and for the inland group, it matches closely to the one-year price change, but over generates the magnitude of the five-year price change autocorrelation.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents a model of house prices, solves the solutions separately under rational expectations and adaptive expectations, and provides impulse response analyses. Section 4 describes the data, estimation methodology, and reports the estimation results. Section 5 presents the simulation results showing how the model fits the empirical features of house prices. Section 6 concludes.

2 Related Literature

The discussion of the stationarity puzzle relates to papers that study time series aspects of house prices, especially cointegration analysis and error correction models. House prices and income are usually both treated as unit root processes in the housing literature. This unit root assumption is a prerequisite for applying cointegration analysis and error correction models; otherwise, if either house prices or income does not have a unit root, cointegration tests and error correction specifications would be invalid. Many papers apply cointegration analysis to test for stable long-run equilibrium relationships between house prices and fundamental variables such as income. If such a long-run relationship exists so that house prices and income are cointegrated, then they cannot drift away from each other over the long run. Therefore, a deviation from this long-run equilibrium can be viewed as an indicator of whether house prices are overvalued or undervalued and the tendency to adjust back to the equilibrium relationship, and this is the idea in error correction models. Examples of applying cointegration analysis and error correction models to house prices include Abraham and Hendershott (1996), Malpezzi (1999), Capozza, Hendershott, and Mack (2004), Gallin (2006), Mikhed and Zemcik (2009), and Holly, Pesaran, and Yamagata (2010). In those papers, either they do not formally test for a unit root, or they do not include the recent housing bust period.[5]

However, Zhang, de Jong, and Haurin (2013) find a stationarity puzzle: there is strong

[5]Mikhed and Zemcik (2009) is the only one among those papers that reports unit root tests for real house prices that include the bust period, and over the period 1978-2007 a unit root is rejected at the 5% level. But the authors then choose the sub-periods ending before 2006, and apply cointegration tests when a unit root is not rejected. This is an ad hoc method to treat house prices.

evidence that house prices are trend stationary while income has a unit root. They first study a national log real home price index constructed by Robert Shiller over a 120-year period 1890-2011. The unit root tests results indicate that real house prices are trend stationary with structural breaks. They also apply the Pesaran (2007) CIPS panel unit root test to log real house prices for a metro area panel dataset and a state panel dataset. A unit root is statistically significantly rejected over 1975-2011. They further find that the reason for this disagreement with the literature is because the previous studies do not cover the full housing cycle of 2000-2010. In contrast to log real house prices, a panel unit root test for log real income over 1975-2010 cannot reject a unit root. Therefore, their paper poses a puzzle for housing models: how to explain different stationarity properties between real house price and real income in a typical urban model.

The issues of short-run momentum and long-run reversion of house price changes studied in this paper connect to empirical papers about house price fluctuations. Many papers empirically examine the short-run positive serial correlation and long-run mean reversion of house price changes. Case and Shiller (1989) use repeated sales data from Atlanta, Chicago, Dallas, and San Francisco over period 1970-1986. They find positive autocorrelation over a one year period for both real house price appreciation and after-tax excess housing returns. In a follow-up study (1990), Case and Shiller test a variety of other public economic variables and find that besides one-year lagged real house price appreciation, real income growth, population growth among age 25-40, and the ratio of construction costs to prices also have power to forecast house prices. Abraham and Hendershott (1996) use annual data for 30 metropolitan areas over the period 1977-1992 to investigate the determinants of real house price appreciation. The explanatory variables consist of three parts: the change in fundamental price, the lagged real house price appreciation with its coefficient called the "bubble builder," and the deviation of house price from its equilibrium level in the previous period with its coefficient called the "bubble burster." They find a positive bubble builder coefficient and a negative bubble burster coefficient. Moreover, the absolute values of these coefficients are higher for the coastal city group than for the inland city group.

Motivated by the finding in Abraham and Hendershott (1996) that serial correlation and mean reversion are significantly higher in absolute value for coastal areas than inland areas, Capozza, Hendershott, and Mack (2004) empirically study which variables explain the significant difference in the geographic patterns of the "bubble builder" and the "bubble burster" coefficients found in Abraham and Hendershott's (1996) error correction specification. Using a panel dataset of 62 MSAs from 1979-1995, they find that higher real income growth, higher population growth, and a higher level of real construction costs increase the "bubble builder" coefficient, while higher real income growth, a larger population, and a lower level of real construction costs increase the absolute value of the "bubble burster" coefficient. There is no theoretical model supporting their selection of explanatory variables, but they offer some reasonable justifications such as information and search costs, supply inelasticity, and market euphoria and expectation.

Our model is complementary to housing models that aim to provide theoretical answers to the momentum and reversion of house prices changes found in the above empirical studies. Despite of the large number of empirical papers, theoretical counterparts are relatively limited. The most relevant theoretical study to our paper is Glaeser et al. (2012). They present a dynamic spatial equilibrium model with rational expectations. Housing costs reflect the value of access to an area's income and amenities. Construction is endogenously determined by marginal costs equaling the expected house prices. They divide MSAs into three regions: coastal, sunbelt, and interior, and estimate the parameters separately for each of those regions. When the model is used to fit the data, it has both successes and failures. Notably, it fails to explain the short term serial correlation of house price changes. The model always generates negative autocorrelation at the one year horizon, although the data in their sample reveals strong persistence (over a one year horizon the autocorrelation is 0.59-0.81 when income data is from the Home Mortgage Disclosure Act (HMDA), and 0.53-0.70 when using the Bureau of Economic Analysis (BEA) income data). Their model succeeds qualitatively in explaining the mean reversion of price changes at a five-year horizon, although it often understates the extent of reversion, especially in coastal areas. Moreover, the model generates short term volatility of house price changes and construction, and positive serial correlation of construction quantities that are close to data, but under predicts the volatility over the five-year horizon. The failures and successes of Glaeser et al.'s model suggest that, "a perfectly rational model in which construction is not instantaneous and local income series themselves mean revert" neglects some important elements in explaining short term housing market dynamics, especially the short run persistence in house price changes.

Head, Lloyd-Ellis, and Sun (2013) , use a search and matching model to explain the short-term movements of house prices and construction. In their setting, the demand for buying a house will continue for some periods after a positive demand shock because it takes time for the buyer to find a house through the matching process. Moreover, new houses need time to be built. Therefore, the ratio of buyers to sellers, "market tightness" as they call it, continues to go up for some time, and so the prices will not rise to the maximum immediately and then fall, as in a frictionless market, but keep rising for some periods. As the matching process gradually ends and new houses are built, prices will then begin to fall. In this way, the model can generate short run momentum and long run mean reversion for house price changes. The first-order autocorrelation of house price growth generated by their search model is around half of that in the data, which means that search frictions alone are not sufficient for explaining the short term house price dynamics.

The comparison between rational expectations and adaptive expectations in our paper relates to a number of studies in the housing literature that document the important role of backward-looking expectation and speculation in explaining house price cycles. Malpezzi and Wachter (2005) develop and simulate a model with speculative backward looking expectations. Housing demand increases if house prices rise in the past. They find that speculation in housing demand plays an important role in generating property cycles when supply is inelastic. Their simulation results do not show significant house price cycle patterns if speculation is not incorporated in the model, but when speculation is added, strong boom and bust cycles are generated. But the role of speculation relates to supply conditions: no significant cycle is generated by speculation when supply is very elastic. Their model reveals that backward-looking expectations could be an important contributor to house price dynamics. Glaeser, Gyourko, and Saiz (2008) construct a model with extrapolative expectations and obtain the results that areas with more housing supply inelasticity have greater house price appreciations during housing booms. Their study reveals two important potential factors to explain housing bubbles: backward-looking expectations and supply inelasticity. A group of recent papers adopt heterogeneous expectation models to explain housing cycles. Some of the examples are Burnside, Eichenbaum, and Rebelo (2011), and Dieci and Westerhoff (2012). In these studies, some economic agents form their expectations extrapolatively while others expect house prices to revert to the fundamental values. The relative weights of these views evolve over time. Housing cycles are generated as extrapolative expectations become dominant during booms while more people believe that house price will revert to fundamentals during busts.

3 Model

3.1 Setup

The model relies on three conditions: a spatial equilibrium condition, a housing supply condition, and a condition regarding how people form their expectations.

We assume migration across cities are costless. Therefore, households equalize utility across cities in equilibrium. This equalized utility is assumed to be a national utility level $\overline{U}$. This indifference across space holds for all periods. The no moving costs is a strong assumption that makes the model more tractable. We assume each household consumes one unit of housing, and therefore upsizing or downsizing the house is not considered here. The indirect utility of a household living in city i during period t is assumed to be

$$V_{it} = Income_{it} + Value\ of\ amenities_{it} - Housing\ costs_{it}. \tag{1}$$

This indirect utility expression is the same as in Glaeser et al.

Assume that in city i, the income y_{it} is a unit root process x_{it} plus an intercept and a linear time trend,

$$y_{it} = \overline{D}_i + \mu_i t + x_{it},$$

where

$$x_{it} = x_{i,t-1} + \varepsilon_{it}, \text{ and } \varepsilon_{it} = \rho_i \varepsilon_{i,t-1} + v_{it}. \tag{2}$$

In other words, $\overline{D}_i$ is an area specific fixed effect, $\mu_i t$ is the time trend, and x_{it} is a unit root process with first order autocorrelation in the error terms.

The value of a city's amenities is measured by

$$Amenities_{it} = A_i - \alpha_i N_{it}, \tag{3}$$

where A_i is an area specific fixed effect for amenities such as weather, N_{it} is the housing stock, and α_i is a disamenity coefficient measuring marginal congestion costs. The assumption $\alpha_i > 0$ implies that households prefer less crowded cities. Additional housing stock decreases amenities and decreases households' indirect utility of living in the city.

We focus on homeowners so that housing costs are measured by owner costs. The "owner cost" term[6], used by Hendershott and Slemrod (1983) and Poterba (1984), measures the annual flow of cost for owning a home with the specification,

$$OC_{it} = P_{it}c_{it}, \text{ where } c_{it} = [(r_{it} + \tau_{it}^p)(1 - \tau_{it}^y) + d_i + tc_i/n_i - \pi_{it}]. \tag{4}$$

In this equation, P_{it} is house value, c_{it} is the user cost per unit expense on housing, r_{it} is the mortgage interest rate, τ_{it}^p is the property tax rate, τ_{it}^y is the marginal income tax rate, d_i is the sum of maintenance and depreciation costs, tc_i/n_i is the transaction cost per unit value of the house divided by the expected length of stay, and π_{it} is the expected rate of house price inflation. We assume for simplicity that the interest rate, the property tax rate, the marginal income tax rate, the maintenance and depreciation cost rate, and the transaction cost rate do not change over time. In other words, the term $(r_i + \tau_i^p)(1 - \tau_i^y) + d_i + tc_i/n_i$ does not vary over time, but it can differ across cities. Therefore, we abstract from studying the effects of time-varying interest rates and taxes.[7] Denote

$$\eta_i = (r_i + \tau_i^p)(1 - \tau_i^y) + d_i + tc_i/n_i. \tag{5}$$

[6]Glaeser et al. abstract from income tax, property tax, maintenance costs, and transaction costs by assuming that the expected housing costs equal $P_t^i - \frac{EP_{t+1}^i}{1+r}$, where r is the interest rate.

[7]We try varying values of the parameter η in the estimation, and estimation results do not change much regarding the choice of η.

The annual owner cost can be rewritten as

$$OC_{it} = P_{it}\left(\eta_i - \frac{E_t P_{i,t+1} - P_{it}}{P_{it}}\right) = (1+\eta_i)P_{it} - E_t P_{i,t+1}. \tag{6}$$

Therefore, the utility flow of city i at period t is

$$V_{it} = y_{it} + A_i - \alpha_i N_{it} - (1+\eta_i)P_{it} + E_t P_{i,t+1}. \tag{7}$$

(7) should equal the national utility level in equilibrium:

$$y_{it} + A_i - \alpha_i N_{it} - (1+\eta_i)P_{it} + E_t P_{i,t+1} + w_{it} = \overline{U}, \tag{8}$$

where w_{it} is the error term which captures the disturbances in the national utility level. Substituting (2) into (8), we get

$$D_i + x_{it} + \mu_i t - \alpha_i N_{it} - (1+\eta_i)P_{it} + E_t P_{i,t+1} + w_{it} = \overline{U}, \tag{9}$$

where $D_i = \overline{D}_i + A_i$. Equation (9) is our spatial equilibrium equation. This equation implies that the difference in owner costs of housing between two cities must reflect differences in income or differences in amenities between these two areas.

The second element of the model is housing supply. We assume that the marginal cost of producing a unit of housing is

$$MC_{it} = c_i + c_{1i} I_{it}, \tag{10}$$

where $c_{1i} > 0$, and I_{it} is the amount of new construction in period t. Denote N_{it} as the housing stock at the beginning of period t. Then the change in the housing stock, assuming a depreciation rate of δ_i, satisfies

$$N_{i,t+1} = I_{it} + (1-\delta_i)N_{it}. \tag{11}$$

The condition $c_{1i} > 0$ implies that as construction increases, the marginal cost increases.

This condition can be interpreted as diminishing returns due to scarcity of certain inputs such as land of producing housing.

We assume that the construction market is competitive, and that houses require one period to be built, so houses under construction in this period will be on the market next period, thus the decisions of builders at time t are based on the expected price level for the next period. Therefore, in equilibrium, the costs of producing a unit of housing equal the current value of the expected house price in the next period:

$$E_t P_{i,t+1} = c_i + c_{1i} I_{it}. \tag{12}$$

Substituting (11) into (12), we get

$$E_t P_{i,t+1} = c_i + c_{1i} N_{i,t+1} - (1 - \delta_i) c_{1i} N_{it}. \tag{13}$$

Rearranging (13) and we get an equation in which the next period housing stock depends on the expected house price and the current housing stock:

$$c_{1i} N_{i,t+1} = -c_i + E_t P_{i,t+1} + (1 - \delta_i) c_{1i} N_{it}. \tag{14}$$

We add an error term u_{it} into (14) to capture the effects of factors other than expected house price and current housing stock on the construction and therefore the next period's housing stock. For example, changes in the weather may delay construction. Therefore, we get the housing supply equation:

$$c_{1i} N_{i,t+1} = -c_i + E_t P_{i,t+1} + (1 - \delta_i) c_{1i} N_{it} + u_{it}. \tag{15}$$

A model with only the spatial equilibrium equation (9) and the housing supply equation (15) is not identifiable because house price P and housing stock N are both endogenous variables and the expectation term $E_t P_{i,t+1}$ is unobserved. Therefore, an additional equation regarding the behavior of the expectation $E_t P_{i,t+1}$ is needed. We will consider two cases: rational expectations and adaptive expectations.

The above setup shares many aspects of Glaeser et al. (2012)'s framework. The main differences with their paper are the assumptions about income and about expectations, as mentioned in the introduction and literature review.

Our model treats income as an exogenous unit root process by Equation (2). To justify this assumption, we applied Pesaran (2007)'s CIPS panel unit root test to real per capita income for 363 MSAs over 1975-2010, and found that a unit root cannot be rejected. This finding is robust to varying lag length included.[8] We then regressed the first difference of real income on its one-year lag value and two-year lag value, the coefficient of the former is significant and the coefficient of the latter is insignificant. These results indicate that a unit root process with an AR(1) error is an appropriate assumption for real income.

Table 1. CIPS panel unit root test, 1975-2010

	CADF(1)	CADF(2)	CADF(3)
coastal real house price	-3.131***	-2.684**	-3.076***
inland real house price	-2.825***	-2.823***	-2.702***
coastal real income	-2.060	-1.922	-1.911
inland real income	-1.917	-1.851	-1.856

Notes: Truncated CIPS with an intercept and a trend are reported. 1% 5% and 10% critical values for T=20 and N=100 are -2.70, -2.57, -2.51, for N=200 are -2.65, -2.55, -2.49.

In contrast, Glaeser et al. treat income as endogenous. They use the term "wage" in the model setup, but use income data as the measure of wage in the estimation, thus the variable "wage" is essentially income. They specify wage as $Wage_{it} = W_{it} - d_i N_{it}$, where W_{it} is an exogenous productivity variable and N_{it} is population, and this variable is assumed to equal housing stock. They also assume that amenities change linearly in population: $Amenities_{it} = A_{it} - a_i N_{it}$. Therefore, the parameter α_i in (8) is the combined effects of population on wages and amenities $(d_i + a_i)$ in their model. They make several simplifying assumptions and set $d_i = 0.1$ and $a_i = 0$. Then they construct the productivity variable W_{it}

[8]Table 1 reports Pesaran (2007)'s CIPS panel unit root test for real income and real house prices, for 114 coastal MSAs and 242 inland MSAs, over 1975-2010. Both coastal and inland MSAs reject a unit root for real house prices, but cannot reject a unit root for real income.

from income and housing stock by $W_{it} = Wage_{it} + d_i N_{it}$, assume W_{it} follows an ARMA(1,1) process, and estimate the parameters of the ARMA(1,1) process.

Our approach of treating income is an improvement to Glaeser et al.'s approach for two reasons. First, the choice of the parameter values for d_i and a_i in their approach is somewhat arbitrary. Moreover, they set the same values for d_i and a_i for all areas, thus cannot capture any geographic differences. In contrast, our approach estimates the effect of population on amenities (the disamentity coefficient α_i). Also, the estimation can be implemented for different geographic groups of cities, and hence examine any difference in the disamenity coefficient α_i for different cities. Second, our model examines how the time series properties of income are transmitted to house prices, and studies the stationarity puzzle: how a unit root income series is related to a trend stationary house price process. Glaeser et al. do not consider this issue in their model.

3.2 Trend-adjusted Equations

Before moving on to make any assumptions about expectations and solving the model, we eliminate the deterministic trend from the spatial equilibrium equation (9) and the supply equation (15). Detrending the equations will make the follow-up calculations cleaner. Details of the detrending calculation is in Appendix 1.

The deterministic trends for housing stock and house prices are:

$$\widehat{N}_{it} = \left(\frac{\mu_i}{\eta_i \delta_i c_{1i} + \alpha_i}\right) t + \left(\frac{D_i + \mu_i - \overline{U} - \eta_i c_i}{\eta_i \delta_i c_{1i} + \alpha_i} + \frac{-(\eta_i - \delta_i) c_{1i} - \alpha_i}{(\eta_i \delta_i c_{1i} + \alpha_i)^2} \mu_i\right), \tag{16}$$

and

$$\widehat{P}_{it} = \left(\frac{\mu_i \delta_i c_{1i}}{\eta_i \delta_i c_{1i} + \alpha_i}\right) t + c_i + (1 - \delta_i) c_{1i} \left(\frac{\mu_i}{\eta_i \delta_i c_{1i} + \alpha_i}\right) \tag{17}$$

$$+ \delta_i c_{1i} \left(\frac{D_i + \mu_i - \overline{U} - \eta_i c_i}{\eta_i \delta_i c_{1i} + \alpha_i} + \frac{-(\eta_i - \delta_i) c_{1i} - \alpha_i}{(\eta_i \delta_i c_{1i} + \alpha_i)^2} \mu_i\right). \tag{18}$$

The detrended spatial equilibrium equation and housing supply equation are:

$$(1+\eta_i)p_{it} - E_t p_{i,t+1} = x_{it} - \alpha_i n_{it} + w_{it}, \tag{19}$$

$$c_{1i} n_{i,t+1} = E_t p_{i,t+1} + (1-\delta_i) c_{1i} n_{it} + u_{it}. \tag{20}$$

We will derive the expressions for house prices from the detrended system (19) and (20), under rational expectations and adaptive expectations, respectively.

3.3 Rational Expectations

We apply Blanchard and Kahn (1980) to show that the model has a unique non-explosive solution under rational expectations and we derive this solution. The results are in Proposition 1. Appendix 2 describes Blanchard and Kahn (1980), and Appendix 3 provides the proof for Proposition 1.

Proposition 1: Assume our model of (19) and (20) under rational expectations have the parameter restrictions: $\alpha_i > 0$, $\eta_i > 0$, $1 > \delta_i \geq 0$, and $c_{1i} > 0$. We also require that people do not expect house prices and housing stock to exponentially explode in the future:

$$\forall t,\ \exists \begin{bmatrix} \overline{n}_t \\ \overline{p}_t \end{bmatrix} \in \mathbb{R}^2,\ \sigma_t \in \mathbb{R} \text{ such that}$$

$$-(1+i)^{\sigma_t} \begin{bmatrix} \overline{n}_t \\ \overline{p}_t \end{bmatrix} \leq \begin{bmatrix} E(n_{t+i}|\Omega_t) \\ E(p_{t+i}|\Omega_t) \end{bmatrix} \leq (1+i)^{\sigma_t} \begin{bmatrix} \overline{n}_t \\ \overline{p}_t \end{bmatrix},\ \forall i \geq 0. \tag{21}$$

Then there exists a unique nonexplosive solution under rational expectations. Under this solution, the trend-adjusted house price and housing stock satisfy

$$n_{it} = n_{i0}, \text{ for } t = 0,$$

$$n_{it} = \phi_i n_{i,t-1} + \frac{1}{c_{1i}(\overline{\phi}_i - 1)} x_{i,t-1} + \frac{\rho_i \overline{\phi}_i}{c_{1i}(\overline{\phi}_i - 1)(\overline{\phi}_i - \rho_i)} \varepsilon_{i,t-1} + \frac{1+\eta_i}{c_{1i}\overline{\phi}_i} u_{i,t-1}, \text{ for } t > 0, \quad (22)$$

$$p_{it} = -\frac{\alpha_i + (1-\delta_i - \phi_i)c_{1i}}{1+\eta_i} n_{it} + \frac{\overline{\phi}_i}{(1+\eta_i)(\overline{\phi}_i - 1)} x_{it} + \frac{\rho_i \overline{\phi}_i}{(1+\eta_i)(\overline{\phi}_i - 1)(\overline{\phi}_i - \rho_i)} \varepsilon_{it}$$

$$+ \frac{1}{1+\eta_i} w_{it} + \frac{1+\eta_i - \overline{\phi}_i}{(1+\eta_i)\overline{\phi}_i} u_{it}, \text{ for } t \geq 0. \quad (23)$$

Proposition 1 reveals that house prices are characterized as a first order autoregressive process with errors containing both a stationary component and a nonstationary component. The behavior of the model under rational expectations characterized by Proposition 1 will be studied in a later subsection by an impulse response analysis, to examine the effect of an income shock over time on the path of house prices.

3.4 Adaptive Expectations

Assume households and home builders form their expectations adaptively for the deviation from the deterministic trend of house prices:

$$E_t p_{i,t+1} = E_{t-1} p_{it} + \lambda_i (p_{it} - E_{t-1} p_{it}) = \lambda_i p_{it} + (1-\lambda_i) E_{t-1} p_{it}, \text{ where } 0 < \lambda_i < 1. \quad (24)$$

The reason for applying the adaptive formula to the trend-adjusted house price rather than the original series P_{it} is to avoid the systematic error made by agents under adaptive expectations when a trend exists. Agents adjust their expectations according to their previous period's expectation error. The parameter λ_i is a measure of the adjustment speed. A small λ_i indicates that agents do not change their expectations much, while a λ_i that is close to 1 indicates that agents' house price expectations for the next period is close to house prices in this period.

The adaptive expectation equation (24), the spatial equilibrium equation (19), and the housing supply equation (20) are the three equations from which we derive the solution.

Combining the supply equation (20) and the adaptive expectation equation (24) yields

$$-\lambda_i p_{it} + c_{1i} n_{i,t+1} - (2 - \lambda_i - \delta_i) c_{1i} n_{it} + (1 - \lambda_i)(1 - \delta_i) c_{1i} n_{i,t-1} = u_{it} - (1 - \lambda_i) u_{i,t-1}. \quad (25)$$

Combining the spatial equilibrium equation (19) and the adaptive expectation equation (24) yields

$$(1+\eta_i-\lambda_i)p_{it}-(1+\eta_i)(1-\lambda_i)p_{i,t-1}+\alpha_i n_{it}-\alpha_i(1-\lambda_i)n_{i,t-1}-x_{it}+(1-\lambda_i)x_{i,t-1} = w_{it}-(1-\lambda_i)w_{i,t-1}. \quad (26)$$

Proposition 2: Assume our model of (19) and (20) under adaptive expectations have the parameter restrictions: $\alpha_i > 0$, $\eta_i > 0$, $1 > \delta_i \geq 0$, $c_{1i} > 0$, and $0 < \lambda_i < 1$. We also assume that the parameters satisfy a stability condition

$$\lambda < \frac{(2 + 2\eta_i)(2 - \delta_i) c_{1i}}{(2 + \eta_i)(2 - \delta_i) c_{1i} + \alpha_i}. \quad (27)$$

Then there is a unique solution under adaptive expectations. Under this solution, the trend-adjusted house price and housing stock satisfy

$$-p_{it} + \frac{c_{1i}}{\lambda_i} n_{i,t+1} - \frac{(2 - \lambda_i - \delta_i) c_{1i}}{\lambda_i} n_{it} + \frac{(1 - \lambda_i)(1 - \delta_i) c_{1i}}{\lambda_i} n_{i,t-1} = \frac{1}{\lambda_i} u_{it} - \frac{1 - \lambda_i}{\lambda_i} u_{i,t-1}. \quad (28)$$

$$(1+\eta_i-\lambda_i)p_{it}-(1+\eta_i)(1-\lambda_i)p_{i,t-1}+\alpha_i n_{it}-\alpha_i(1-\lambda_i)n_{i,t-1}-x_{it}+(1-\lambda_i)x_{i,t-1} = w_{it}-(1-\lambda_i)w_{i,t-1}.$$

(29)

The derivation of the stability condition (27) and the proof for Proposition 2 is in Appendix 4.

Proposition 2 reveals that house prices are characterized as a second order autoregressive process with errors containing both a stationary component and a nonstationary component. The behavior of the model under adaptive expectations characterized by Proposition 2 will be studied in the next subsection by an impulse response analysis, and is compared to the case under rational expectations.

3.5 Impulse Response

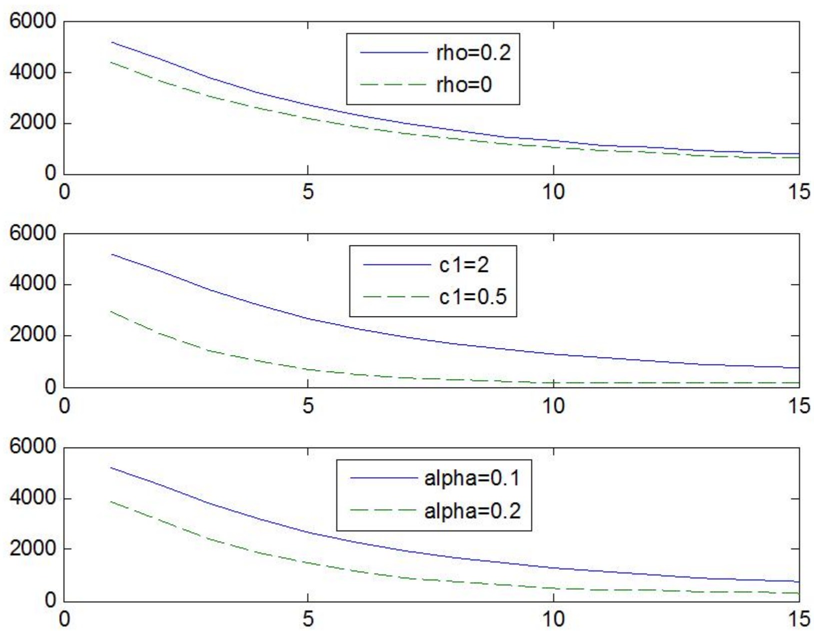
In this subsection, we study the path of trend-adjusted house prices following a one time income shock of 1000 dollars, for both the rational expectations and adaptive expectations cases.

For rational expectations, the impulse responses are graphed in Figure 3. The baseline parameter values are set to $\rho = 0.2$, $c_1 = 2$, and $\alpha = 0.1$, represented by the solid lines.[9] We also vary these values one at a time to examine how the over time effect of income on house prices changes when the parameter changes. Under the baseline parameter values, house prices increase immediately at the time of the income shock, but gradually decline from that point. Intuitively, when at the time of the income shock, house prices will increase to reflect this positive shock. Because of the increase in house prices, new houses will start to be built, and as new constructions occur, house prices decline gradually. The dashed lines in the upper, middle, and lower subgraphs respectively represent cases where there is no persistence in income change ($\rho = 0$), where supply is more elastic ($c_1 = 0.5$), and where the negative effect of housing stock on households' indirect utility is larger ($\alpha = 0.2$). Compared to the baseline case, house prices in all these dashed-line cases increase less and revert faster. Intuitively, when income growth has less persistence, the consequent positive

[9]The parameter values chosen in the impulse response analyses are based on the estimation results in the later section.

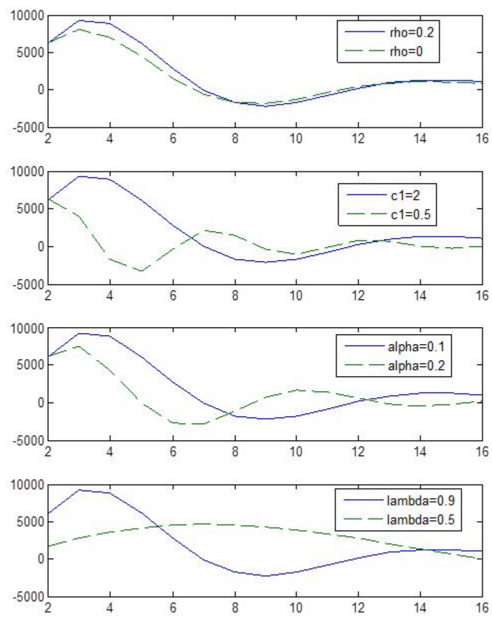
income shock effects on house prices are smaller, which are anticipated by forward-looking households and reflected in the lower house price rise and faster reversion. When housing supply is more elastic, more new homes will be constructed after a positive income shock, and hence moderate the responses of house prices. When the disamenity coefficient is larger, new construction has bigger effects on reducing house prices, resulting in weaker house price responses.

Figure 3. Impulse response to an income shock under rational expectations



Notes: Presented are the paths of de-trended house prices after a one time income shock of 1000 dollars. The solid lines are the baseline case with parameter values being set to $\rho = 0.2$, $c_1 = 2$, and $\alpha = 0.1$.

Figure 4. Impulse response to an income shock under adaptive expectations



Notes: Presented are the paths of de-trended house prices after a one time income shock of 1000 dollars. The solid lines are the baseline case with parameter values being set to $\rho = 0.2$, $c_1 = 2$, $\alpha = 0.1$, and $\lambda = 0.9$.

For adaptive expectations, the impulse responses are depicted in Figure 4. The baseline parameter values are set to $\rho = 0.2$, $c_1 = 2$, $\alpha = 0.1$, and $\lambda = 0.9$, represented by the solid lines. House prices shoot up immediately when hit by an income shock, as in the case of rational expectations. But different from rational expectations where house prices start to decrease right after the shock, houses prices under adaptive expectations continue to increase in the next period, therefore generate momentum in house price appreciation. After one period, house prices begin to decline gradually and thus generate reversion. The dashed lines in the top three subgraphs represent the same parameter changes as for rational expectations. The dashed line in the lowest subgraph represents a case where agents adjust their expectations more slowly ($\lambda = 0.5$). Compared to the baseline case, dashed lines in the top three subgraphs indicate lesser or no momentum and faster reversion, and the dashed line in the lowest subgraph presents a smoother house price path.

3.6 Implications of the Impulse Response Analyses

The impulse response analyses presented in Figure 3 and Figure 4 reveal some information about the behavior of the model related to the issues of stationarity, short-run momentum, and long-run reversion. The figures show that, under both rational expectations and adaptive expectations, house prices rapidly increase when an income shock occurs and gradually decline in the long run as new construction occurs. Therefore, the long-run reversion is generated by the model.

Also, even though the over time effect of the one time shock on income is permanent because income is a unit root process, the effect of the shock on house prices diminishes over time as shown in the figures. Recall that Proposition 1 and Proposition 2 both reveal that house prices contain both a unit root component and a stationary component. The fact that house prices decline over time to a very low level in response to a one time shock indicates that the weighting of the unit root compoment is quite small compare to the stationary component. This provides an intuitive explanation of why a unit root is rejected for house prices.

The short-run momentum is the main disagreement of the model with rational expectations and the one with adaptive expectations. Under rational expectations, since people are forward looking, any future consequence of a positive income shock would be anticipated and incorporated into the initial house price change. Therefore, after a positive income shock, house price immediately rise to its maximum level and then gradually declines and thus short term momentum cannot be generated in this setting. However, under adaptive expectations, people are backward looking instead of forward looking. Therefore, when a positive demand shock occurs, house prices rise in this period and people adjust their expectations to a higher level. In the second period after the shock, the higher expectations will be incorporated into the price level, and price momentum could be generated if the expectation effect is strong enough to outweighs the construction effect.

4 Estimation

4.1 Data

The data we use are annual MSA level income, housing units, and house prices over 1980-2010. Income and house prices are converted to real terms by deflating them by the CPI-U.[10] Levels of the variables rather than logarithms are used in order to be consistent with the model. We obtain the per capita personal income from the Bureau of Economic Analysis (BEA), then multiply it by 2.63 to get household income. House prices are derived from the Freddie Mac House Price Index (FMHPI). It is a constant quality index over time, but not over areas. December 2000 is the base month when house price index in every MSA equals 100. To transform the house price index to a constant quality index both over time and over MSAs, we multiply this index by a MSA level hedonic estimated house value for a national average characteristic house using census 2000 data.[11]

The number of housing units is derived from the housing stock estimated by decadal census and the annual permits data from the Department of Housing and Urban Development (HUD) according to

$$N_{t+j} = N_t + \frac{\sum_{k=0}^{j-1} Permits_{t+k}}{\sum_{k=0}^{9} Permits_{t+k}} (N_{t+10} - N_t). \tag{30}$$

The MSAs in our sample are separated into a coastal group and an inland group. MSAs which contain a county that is within 150km from either the Pacific or Atlantic are defined as coastal. Other MSAs are inland. By this definition, the sample contains 114 coastal MSAs and 242 inland MSAs.

Table 2 presents the summary statistics for real per capita income, real house prices, housing units, and the changes of these three variable. On average, coastal MSAs have

[10] 1984 is the CPI base year.

[11] We regress log house value on a number of independent variable including state dummies, MSA dummies, # bedroom (third-order polynomial), # total rooms (third-order polynomial), categorical dummies of built year (8 categories), and categorical dummies of acreage of house (2 categories).

After obtaining the coefficients, we set the independent variables to their median values, and predict the MSA level log house price for a national average characteristic house. Then, we exponentiate the estimated "log price" and get the price index of a constant-quality house which is comparable cross-sectionally.

higher income, higher house prices, and a larger housing stock. These MSAs also have faster growth on average in income, house price, and the housing stock.

Table 2. Summary statistics

	coastal				inland			
	mean	s.d.	min	max	mean	s.d.	min	max
y	15803	3663	7688	38650	14249	2471	6332	31099
p	98588	55021	35252	557720	56112	16547	21408	172753
n	375421	828067	8257	7500000	191829	355931	11767	3800000
dy	187	463	-4408	5188	164	388	-8670	6457
dp	851	11798	-116216	69486	8	3099	-43321	31304
dn	5280	9061	-188	77787	2840	6713	-515	82627

Notes: Summary statistics for real income, real house prices, housing stock, and their changes.

4.2 Methodology

We set the parameter in the user cost term that represents the after tax per dollar cost of mortgage borrowing , property taxes, depreciation and maintaince, and transaction costs $\eta = 0.06$,[12] set the housing depreciation rate parameter $\delta = 0.02$, and estimate the parameters ρ, μ, c_1,α , and λ. In the model, these parameters can differ for each MSA. But due to limited observations for each MSA (1980-2010 annually), the MSAs are separated into two groups (coastal and inland). We assume that MSAs in the same group have the same parameter values, and we estimate the parameters separately for each group. We allow for city-specific fixed effects within each group. That is, for each group, we estimate a panel data model with fixed effects.

[12] We set the values of the variables in the user cost term as follows: real mortgage rate=0.04 (the average of the conventional conforming 30-year fixed real mortgage rate from Freddie Mac is 4.7% over 1975-2010), marginal income tax rate of the typical home buyer=25%, property tax rate=0.02, annual maintenance and depreciation costs rate=2%, transaction costs are assumed to be zero because our model assumes no moving costs for households. Then we have $\eta_i = (r_i + \tau_i^p)(1 - \tau_i^y) + d_i + tc_i/n_i = 0.065$.

For the rational expectations case, we adopt the sequential two-step GMM estimation methodology following Glaeser et al., but use different moment conditions.[13] The parameters that need to be estimated are divided into a income related vector of parameters denoted by $\theta = (\mu, \rho)$, and a vector which contains the rest of the parameters denoted by $\gamma = (c_1, \alpha)$. Parameters in θ are estimated using moment conditions derived from the income process equation. Given the estimated $\widehat{\theta}$, γ are then estimated using moment conditions derived from equations in Proposition 1. Estimate of $\widehat{\gamma}$ is consistent, but its asymptotic variance must be adjusted to take into account that it depends on the first step estimation. Newey and McFadden (1994) and Hansen (2007) provide the formula for the adjusted asymptotic variance of the second step estimators.

For the adaptive expectations case, the income parameters $\theta = (\mu, \rho)$ are estimated using moment conditions derived from the income process equation. The rest parameters $\gamma = (c_1, \alpha, \lambda)$ are estimated using moment conditions derived from equations in Proposition 2. The moment conditions to estimate γ do not depend on the income parameters θ. Therefore, we implement the normal two-step GMM estimation procedure separately for θ and γ.[14]

Because the variables in the propositions are in trend-adjusted form, we add the deterministic trends back to the equations and use these equations to construct moment conditions.

4.3 Moment Conditions under Rational Expectation

The moment conditions under the rational expectations are based on (22), (23) in Proposition 1, and the exogenous income process (2). The derivation of the moment conditions are in Appendix 5.

[13] We do sequential GMM estimation instead of combining all moment conditions together and estimate the parameters simultaneously because we got more stable estimation results for the income parameters using only the income process equation. The same reasoning applies to the adaptive expectations case below.

[14] The two-step GMM means we take the identity matrix as the weighting matrix in the first step estimation, and in the second step estimation the weighting matrix is computed from the first step estimation.

First stage moment conditions are:

$$m_1(\theta) = \begin{cases} \Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu \\ [\Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu]\Delta y_{i,t-1} \\ [\Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu]\Delta y_{i,t-2} \end{cases} . \tag{31}$$

Second stage moment conditions are:

$$m_R(\theta) = \begin{cases} \psi_{it} \\ \psi_{it}\Delta y_{i,t-4} \\ \psi_{it}\Delta P_{i,t-4} \\ \psi_{it}\Delta N_{i,t-4} \\ \zeta_{it} \\ \zeta_{it}\Delta y_{i,t-3} \\ \zeta_{it}\Delta P_{i,t-3} \\ \zeta_{it}\Delta N_{i,t-3} \end{cases} , \tag{32}$$

where ψ_{it} and ζ_{it} are defined as

$$\psi_{it} = -(1-\rho)(1-\phi)K_1 + \Delta N_{it} - (\rho+\phi)\Delta N_{i,t-1} + \rho\phi\Delta N_{i,t-2}, \tag{33}$$

and

$$\zeta_{it} = -(1-\rho)[S_1 + \frac{\alpha + (1-\delta-\phi)c_1}{1+\eta}K_1] + \Delta P_{it} - \rho\Delta P_{i,t-1} + \frac{\alpha + (1-\delta-\phi)c_1}{1+\eta}(\Delta N_{it} - \rho\Delta N_{i,t-1}). \tag{34}$$

4.4 Moment Conditions under Adaptive Expectations

The moment conditions under the adaptive expectations are based on (28), (29) in Proposition 2, and the exogenous income process (2). The derivation of the moment conditions are in Appendix 6.

First stage moment conditions are:

$$m_1(\theta) = \begin{cases} \Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu \\ [\Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu]\Delta y_{i,t-1} \\ [\Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu]\Delta y_{i,t-2} \end{cases} . \tag{35}$$

Second stage moment conditions are:

$$m_A(\theta) = \begin{cases} \chi_{it} \\ \chi_{it}\Delta y_{i,t-4} \\ \chi_{it}\Delta P_{i,t-4} \\ \chi_{it}\Delta N_{i,t-4} \\ \kappa_{it} \\ \kappa_{it}\Delta y_{i,t-3} \\ \kappa_{it}\Delta P_{i,t-3} \\ \kappa_{it}\Delta N_{i,t-3} \end{cases} , \tag{36}$$

where χ_{it} is defined as

$$\chi_{it} = -\Delta P_{i,t-1} + \frac{c_1}{\lambda}\Delta N_{it} - \frac{(2-\lambda-\delta)c_1}{\lambda}\Delta N_{i,t-1} + \frac{(1-\lambda)(1-\delta)c_1}{\lambda}\Delta N_{i,t-2}, \tag{37}$$

and κ_{it} is defined as

$$\kappa_{it} = (1+\eta-\lambda)\Delta P_{it} - (1+\eta)(1-\lambda)\Delta P_{i,t-1} + \alpha \Delta N_{it} - \alpha(1-\lambda)\Delta N_{i,t-1} - \Delta y_{it} + (1-\lambda)\Delta y_{i,t-1}. \tag{38}$$

4.5 Estimation Results

Table 3 reports the estimates of the income trend parameter μ and the persistence coefficient of income changes ρ. The coastal group has a higher income trend than the inland group. The persistence in income changes are also higher for the coastal group: the persistence coefficient is 0.24 for the coastal group while it is not significantly different from zero for the inland group.

Table 3. Income parameters

	μ	ρ
Coastal	488.78	0.24
	(26.349)	(0.051)
Inland	422.49	-0.03
	(11.455)	(0.060)

Notes: Standard errors are in the parentheses.

Table 4. Rational expectations

	c_1	α
Coastal	1.487	0.111
	(0.215)	(0.010)
Inland	0.567	0.172
	(0.032)	(0.014)

Notes: Standard errors are in the parentheses.

Table 5. Adaptive expectations

	c_1	α	λ
Coastal	3.557	0.041	0.969
	(0.585)	(0.004)	(0.008)
Inland	0.428	0.065	0.661
	(0.188)	(0.008)	(0.029)

Notes: Standard errors are in the parentheses.

Table 4 and Table 5 report the estimates of the rest of the parameters for the rational expectation case and the adaptive expectation case. For both cases, the coastal group has a bigger c_1 and a smaller α than the inland group. The parameter c_1 is inversely related to the responsiveness of construction to an expected house price change. Under rational expectations, the estimate of c_1 equals 1.5 for the coastal group and is 0.6 for the inland group. Under adaptive expectations, c_1 is 3.6 for the coastal group and is 0.4 for the inland group. Thus, the estimation results indicate that the coastal group has more inelastically supplied housing. This finding is consistent with the literature that housing supply is more inelastic in coastal cities because coastal areas have less developable land and more stringent land regulation. The parameter α is the disamenity coefficient. The estimate of α is 0.1 for the coastal group and 0.2 for the inland group under rational expectations, and is 0.04 for the coastal group and 0.07 for the inland group under adaptive expectations. The estimation results of α indicate that as housing units grow in number, the amenities decrease faster in inland cities than in coastal cities. The city expansion can have negative amenity effects due to congestion and pollution, and it may also have positive amenity effects due to agglomeration. The city expansion may have a larger effect in both congestion and agglomeration for a coastal area than for an inland area, resulting in a smaller net disamenity effect for coastal cities.

The adaptive speed parameter λ is bigger in the coastal group, suggesting that households in the coastal areas adjust their expectations of house price faster, possibly because households living in places with more volatile house price cycles pay more attention to house price changes and therefore adjust their expectations faster.

5 Simulation

To examine how well the model can match the empirical features of house prices raised in the beginning of this paper: rejecting a unit root, positive autocorrelation for one-year price change, and negative autocorrelation for five-year price change, we simulate house price series for the coastal group and the inland group under both rational expectations case and the adaptive expectations case.

The estimates of the variance for the errors (v, u, and w) are derived from (77), (81), and (83) for the rational expectations case, and (77), (88), and (89) for the adaptive expectations case. After obtaining the estimates for the parameters and the variance of the errors, house price series are simulated for 100 periods. We then calculate the first-order autoregressive coefficients for one-year house price change and five-year house price change, as well as the t-statistic value for the Augmented Dickey-Fuller (ADF) unit root test with the number of lags ranging from 0 through 3. The simulations are repeated 10,000 times, and we calculate the mean of the autoregressive coefficients and unit root test statistics.

Table 6 reports the simulation results. The actual autoregressive coefficients for one-year and five-year house price changes are 0.66 and -0.61 for the coastal group and 0.44 and -0.11 for the inland group. The model under rational expectations does not match the data well. The variance of the error u is negative for the coastal group and thus a house price series cannot be generated. This failure suggests that the model with rational expectations is not a good setting for coastal cities, or that the assumption that the errors are independent is violated for coastal cities under rational expectations. For the inland group under rational expectations, the ADF unit root test significantly rejects a unit root. A negative autoregressive coefficient of -0.48 is generated for the five-year price changes, which over predicts the magnitude of the actual coefficient. The model under rational expectations fails to generate the positive autocorrelation for the one-year price changes.

Table 6. Simulation: autocorrelation of house price changes over 1-year and 5-year and unit root test for house prices

		1-year	5-year	ADF(0)	ADF(1)	ADF(2)	ADF(3)
Rational	Coastal						
	Inland	-0.395	-0.476	-6.910	-5.074	-4.290	-3.844
Adaptive	Coastal	0.105	-0.315	-2.276	-2.671	-2.672	-2.555
	Inland	0.373	-0.754	-3.188	-5.581	-6.904	-6.587

Notes: The 5% and 1% critical values for the ADF unit root test are -1.95 and -2.60 (no intercept, no trend, T=100). The autoregressive coefficients for 1-year and 5-year house price changes from the data are 0.66 and -0.61 for the coastal group, and 0.44 and -0.11 for the inland group, over the period 1975-2010.

The model under adaptive expectations fits the empirical features better. For both the coastal group and the inland group, the ADF unit root test rejects a unit root, and the model generates positive autocorrelation for one-year price changes and negative autocorrelation for five-year price changes. But for the coastal group, the model does not generate enough magnitude of the one-year and five-year autoregressive coefficients. The one-year price change autoregressive coefficient predicted by the model is 0.1 which is smaller than the actual value of 0.66, and the five-year price change autoregressive coefficient is -0.32 from the model, which is smaller in absolute value than the actual coefficient of -0.61 from the data. For the inland group, the model matches relatively closely with the data for the one-year price change autoregressive coefficient (0.37 predicted by the model and 0.44 from the data), but over generates the magnitude of the five-year change autoregressive coefficient (-0.75 from the model and -0.11 from the data).

The above comparisons suggest that the model under rational expectations captures the negative autocorrelation of house price changes over the five-year horizon and the rejection of a unit root in house prices for the inland cities, but it completely fails in explaining the shot run momentum of house price changes, which is consistent with the finding in Glaeser et al. When we change the assumption of expectations from rational to adaptive, the model's ability of matching data improves. For both coastal and inland groups, the model rejects a unit root and generates a negative five-year autocorrelation. Moreover, the model succeeds in predicting a positive autocorrelation of house price changes over the one-year horizon.

6 Conclusion

This paper modifies the dynamic spatial equilibrium model in Glaeser et al. (2012) to study the stationarity of house prices, and the short-run momentum and the long-run reversion of house price changes. The model is solved and estimated separately under rational expectations and under adaptive expectations, and then predictions of the model are used to fit the data. The estimation results reveal that, under both rational expectations and adaptive expectations, coastal areas have higher persistence in income growth, more housing supply inelasticity, and a smaller disamenity coefficient. Moreover, in the case of adaptive expectations, the coastal city group has a higher adjustment speed in the expectation formation.

The model under adaptive expectations matches the data better than the one with rational expectations. When expectations are assumed to be formed adaptively, the model succeeds in accounting for the rejection of a unit root and the positive one-year autocorrelation and negative five-year autocorrelation of house price changes. But the model underpredicts the magnitude of autocorrelations over both the one-year and five-year horizons for the coastal areas, and overpredicts the magnitude of five-year autocorrelation for the inland city group.

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Appendix 1: Deterministic trends and trend-adjusted equations

In this appendix, we derive the deterministic trends for house price and housing stock, and derive the trend-adjusted spatial equilibrium equation and housing supply equation.

Detrending is done by solving the equations without randomness for house price and housing stock.

Therefore, we set $x_{it} = w_{it} = u_{it} = 0$, implying that $E_t P_{i,t+1} = P_{i,t+1}$ because there is no randomness. Combining equations (9) and (15), we get

$$c_{1i}\widehat{N}_{i,t+1} - [\alpha_i + (2 + \eta_i - \delta_i)c_{1i}]\widehat{N}_{it} + (1 + \eta_i)(1 - \delta_i)c_{1i}\widehat{N}_{i,t-1} = \overline{U} - D_i + \eta_i c_i - \mu_i t \quad (39)$$

The solution to (39) that is linear in time is given by

$$\widehat{N}_{it} = \left(\frac{\mu_i}{\eta_i \delta_i c_{1i} + \alpha_i}\right) t$$

$$+ \left(\frac{D_i + \mu_i - \overline{U} - \eta_i c_i}{\eta_i \delta_i c_{1i} + \alpha_i} + \frac{-(\eta_i - \delta_i)c_{1i} - \alpha_i}{(\eta_i \delta_i c_{1i} + \alpha_i)^2}\mu_i\right). \quad (40)$$

Substituting (40) into (15) with $E_t P_{i,t+1}$ replaced by $P_{i,t+1}$, we find that the deterministic component of house price equals

$$\widehat{P}_{it} = \left(\frac{\mu_i \delta_i c_{1i}}{\eta_i \delta_i c_{1i} + \alpha_i}\right) t$$

$$+ c_i + (1 - \delta_i)c_{1i}\left(\frac{\mu_i}{\eta_i \delta_i c_{1i} + \alpha_i}\right)$$

$$+ \delta_i c_{1i}\left(\frac{D_i + \mu_i - \overline{U} - \eta_i c_i}{\eta_i \delta_i c_{1i} + \alpha_i} + \frac{-(\eta_i - \delta_i)c_{1i} - \alpha_i}{(\eta_i \delta_i c_{1i} + \alpha_i)^2}\mu_i\right). \quad (41)$$

Now define the detrended house price and detrended house stock as

$$p_{it} = P_{it} - \widehat{P}_{it}, \text{ and } n_{it} = N_{it} - \widehat{N}_{it}. \tag{42}$$

Substituting (42) into (9) and (15) to substitute for P_{it} and N_{it}, we find

$$D_i + \mu_i t - \alpha_i \widehat{N}_{it} - (1+\eta_i)\widehat{P}_{it} + \widehat{P}_{i,t+1} + [x_{it} - \alpha_i n_{it} - (1+\eta_i)p_{it} + E_t h_{i,t+1}] = \overline{U}, \tag{43}$$

$$E_t p_{i,t+1} + \widehat{P}_{i,t+1} = c_i + c_{1i}\widehat{N}_{i,t+1} - (1-\delta_i)c_{1i}\widehat{N}_{it} + [c_{1i}n_{i,t+1} - (1-\delta_i)c_{1i}n_{it}]. \tag{44}$$

Since $\widehat{P}_{it}$ and $\widehat{N}_{it}$ by definition satisfy

$$D_i + \mu_i t - \alpha_i \widehat{N}_{it} - (1+\eta_i)\widehat{P}_{it} + \widehat{P}_{i,t+1} = \overline{U},$$

$$\widehat{P}_{i,t+1} = c_i + c_{1i}\widehat{N}_{i,t+1} - (1-\delta_i)c_{1i}\widehat{N}_{it}, \tag{45}$$

(43) and (44) simplify to

$$(1+\eta_i)p_{it} - E_t p_{i,t+1} = x_{it} - \alpha_i n_{it} + w_{it}, \tag{46}$$

$$c_{1i}n_{i,t+1} = E_t p_{i,t+1} + (1-\delta_i)c_{1i}n_{it} + u_{it}. \tag{47}$$

Appendix 2: Blanchard and Kahn (1980)

The model in Blanchard and Kahn is:

$$\begin{bmatrix} X_{t+1} \\ {}_tP_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ P_t \end{bmatrix} + \gamma Z_t,\ X_{t=0} = X_0, \tag{48}$$

where X_t is an $(n \times 1)$ vector of predetermined variables at t; P_t is an $(m \times 1)$ vector of non-predetermined variables at t; Z_t is a $(k \times 1)$ vector of exogenous variables; ${}_tP_{t+1}$ is the agents' expectation of P_{t+1} held at t; A, γ are $(n+m) \times (n+m)$ and $(n+m) \times k$ matrices.

The rational expectation defined in Blanchard and Kahn is

$${}_tP_{t+1} = E(P_{t+1}|\Omega_t), \tag{49}$$

where $E(\cdot)$ is the mathematical expectation operator; Ω_t is the information set at t; $\Omega_t \supseteq \Omega_{t-1}$.

The non-explosion assumption in their paper is

$\forall t,\ \exists \overline{Z}_t \in R^k,\ \theta_t \in R$ such that

$$-(1+i)^{\theta_t}\overline{Z}_t \leq E(Z_{t+i}|\Omega_t) \leq (1+i)^{\theta_t}\overline{Z}_t,\ \forall i \geq 0. \tag{50}$$

Also, they require that the expectations of X_t and P_t do not explode:

$\forall t,\ \exists \begin{bmatrix} \overline{X}_t \\ \overline{P}_t \end{bmatrix} \in R^{n+m},\ \sigma_t \in R$ such that

$$-(1+i)^{\sigma_t} \begin{bmatrix} \overline{X}_t \\ \overline{P}_t \end{bmatrix} \leq \begin{bmatrix} E(X_{t+i}|\Omega_t) \\ E(P_{t+i}|\Omega_t) \end{bmatrix} \leq (1+i)^{\sigma_t} \begin{bmatrix} \overline{X}_t \\ \overline{P}_t \end{bmatrix},\ \forall i \geq 0. \tag{51}$$

The non-explosion condition rules out exponential growth of expectations held at time t.

Under the above settings, Blanchard and Kahn's Proposition 1 gives the condition for the existence of a unique solution for (48): if the number of eigenvalues of A in (48) outside the unit circle is equal to the number of non-predetermined variables, then there exists a unique solution.

Their paper also provides the expression of this solution for the general case, as well as the case with one predetermined and one non-predetermined variable, which is described below. Let

$$\underset{(2\times 2)}{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \underset{(2\times k)}{\gamma} = \begin{bmatrix} \underset{(1\times k)}{\gamma_1} & \underset{(1\times k)}{\gamma_2} \end{bmatrix},$$

let λ_1, λ_2 be the eigenvalues of A, $|\lambda_1| < 1$, $|\lambda_2| > 1$, and define $\mu \equiv (\lambda_1 - a_{11})\gamma_1 - a_{12}\gamma_2$.[15] Then, there exists a unique solution and is given by

$$\begin{aligned} X_t &= X_0, \text{ for } t = 0, \\ &= \lambda_1 X_{t-1} + \gamma_1 Z_{t-1} + \mu \sum_{i=0}^{\infty} \lambda_2^{-i-1} E(Z_{t+i-1}|\Omega_{t-1}), \text{ for } t > 0, \end{aligned} \tag{52}$$

$$P_t = a_{12}^{-1}[(\lambda_1 - a_{11})X_t + \mu \sum_{i=0}^{\infty} \lambda_2^{-i-1} E(Z_{t+i}|\Omega_t)], \text{ for } t \geq 0. \tag{53}$$

[15]Their paper writes $\mu \equiv (\lambda_1 - a_{11})\lambda_1 - a_{12}\lambda_2$, and we believe it is a typo.

Appendix 3: Proof of Proposition 1

To apply Blanchard and Kahn to our model, substitute (20) into (19) and get

$$c_{1i}n_{i,t+1} = (1+\eta_i)p_{it} + [\alpha_i + (1-\delta_i)c_{1i}]n_{it} - x_{it} - w_{it} + u_{it}. \tag{54}$$

Equations (19) and (54) can then be rewritten as

$$\begin{bmatrix} n_{i,t+1} \\ E_t p_{i,t+1} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_i+(1-\delta_i)c_{1i}}{c_{1i}} & \frac{1+\eta_i}{c_{1i}} \\ \alpha_i & 1+\eta_i \end{bmatrix} \begin{bmatrix} n_{it} \\ p_{it} \end{bmatrix} + \begin{bmatrix} \frac{1}{c_{1i}}(-x_{it} - w_{it} + u_{it}) \\ -x_{it} - w_{it} \end{bmatrix}, \tag{55}$$

where n_t is predetermined at time t, p_t is non-predetermined at time t, and x_t, w_{it}, and u_{it} are exogenous variables. This is the form of (48).

The corresponding non-explosion assumption in our model is

$$\forall t,\ \exists \begin{bmatrix} \overline{n}_t \\ \overline{p}_t \end{bmatrix} \in R^2,\ \sigma_t \in R \text{ such that}$$

$$-(1+i)^{\sigma_t} \begin{bmatrix} \overline{n}_t \\ \overline{p}_t \end{bmatrix} \leq \begin{bmatrix} E(n_{t+i}|\Omega_t) \\ E(p_{t+i}|\Omega_t) \end{bmatrix} \leq (1+i)^{\sigma_t} \begin{bmatrix} \overline{n}_t \\ \overline{p}_t \end{bmatrix},\ \forall i \geq 0. \tag{56}$$

This means that people do not expect house price and house stock to exponentially explode in the future.

Denote

$$A = \begin{bmatrix} \frac{\alpha_i+(1-\delta_i)c_{1i}}{c_{1i}} & \frac{1+\eta_i}{c_{1i}} \\ \alpha_i & 1+\eta_i \end{bmatrix} \tag{57}$$

then the eigenvalues of A can be found by solving the equation below:

$$0 = \det(A - \lambda I)$$

$$= \lambda^2 - \frac{\alpha_i + (2+\eta_i-\delta_i)c_{1i}}{c_{1i}}\lambda + \frac{(1+\eta_i)(1-\delta_i)c_{1i}}{c_{1i}} = f(\lambda). \tag{58}$$

Assume $\alpha_i > 0$, $\eta_i > 0$, $1 > \delta_i \geq 0$, and $c_{1i} > 0$, then it can be verified that

$$f(0) = (1+\eta_i)(1-\delta_i) > 0,\ f(1) = -\frac{\delta_i\eta_i c_{1i} + \alpha_i}{c_{1i}} < 0. \tag{59}$$

Therefore, the matrix A has two real eigenvalues, one of them is between 0 and 1, and the other is larger than 1. Denote them as $0 < \phi < 1$ and $\overline{\phi} > 1$. Thus, the number of eigenvalues of A outside the unit circle is equal to the number of non-predetermined variable p_{it} (both equal one). Applying Proposition 1 in Blanchard and Kahn, our model has a unique solution.

Next, we derive the solution.

The a_{11}, a_{12}, γ_1, γ_2, λ_1, λ_2 and Z_{t+i} in (52) and (53) correspond to our model as $\frac{\alpha_i+(1-\delta_i)c_{1i}-c_{2i}}{c_{1i}}$, $\frac{1+\eta_i}{c_{1i}}$, (1, 0), (0, 1), ϕ_i, $\overline{\phi}_i$ and $\begin{bmatrix} \frac{1}{c_{1i}}(-x_{it}-w_{it}+u_{it}) \\ -x_{it}-w_{it} \end{bmatrix}$. Therefore,

$$\mu = (-\frac{\alpha + (1-\delta-\phi)c_1 - c_2}{c_1}, -\frac{1+\eta}{c_1}).$$

Hence,

$$\begin{aligned} \mu E(Z_{t+i}|\Omega_t) &= \frac{\alpha + (1-\delta-\phi)c_1 - c_2}{c_1^2} E_t(x_{t+i} + w_{t+i} - u_{t+i}) + \frac{1+\eta}{c_1} E_t(x_{t+i} + w_{t+i}) \\ &= \frac{\alpha + (2+\eta-\delta-\phi)c_1 - c_2}{c_1^2} E_t(x_{t+i} + w_{t+i}) - \frac{\alpha + (1-\delta-\phi)c_1 - c_2}{c_1^2} E_t u_{t+i} \\ &= \frac{\overline{\phi}}{c_1} E_t(x_{t+i} + w_{t+i}) + \frac{1+\eta-\overline{\phi}}{c_1} E_t u_{t+i}. \end{aligned}$$

We have

$$E_t w_{t+i} = E_t u_{t+i} = 0, \text{ for } i > 0,$$

and

$$E_t x_{t+i} = x_t + \frac{\rho(1-\rho^i)}{1-\rho}\varepsilon_t, \text{ for } i > 0.$$

Thus,

$$\begin{aligned}
\mu \sum_{i=0}^{\infty} \overline{\phi}^{-i-1} E(Z_{t+i}|\Omega_t) &= \overline{\phi}^{-1}\frac{\overline{\phi}}{c_1}(x_t + w_t) + \overline{\phi}^{-1}\frac{1+\eta-\overline{\phi}}{c_1}u_t + \sum_{i=1}^{\infty}\overline{\phi}^{-i-1}\frac{\overline{\phi}}{c_1}(x_t + \frac{\rho(1-\rho^i)}{1-\rho}\varepsilon_t) \\
&= \frac{1}{c_1}x_t + \frac{1}{c_1}w_t + \frac{1+\eta-\overline{\phi}}{c_1\overline{\phi}}u_t + \frac{1}{c_1(\overline{\phi}-1)}x_t + \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_t.
\end{aligned}$$

Therefore, (52) becomes

$$\begin{aligned}
n_t &= \phi n_{t-1} + \frac{1}{c_1}(-x_{t-1} - w_{t-1} + u_{t-1}) \\
&\quad + \frac{1}{c_1}x_{t-1} + \frac{1}{c_1}w_{t-1} + \frac{1+\eta-\overline{\phi}}{c_1\overline{\phi}}u_{t-1} + \frac{1}{c_1(\overline{\phi}-1)}x_{t-1} + \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_{t-1} \\
&= \phi n_{t-1} + \frac{1}{c_1(\overline{\phi}-1)}x_{t-1} + \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_{t-1} + \frac{1+\eta}{c_1\overline{\phi}}u_{t-1}, \text{ for } t > 0,
\end{aligned}$$

and (53) becomes

$$\begin{aligned}
p_t &= \frac{c_1}{1+\eta}\left[\begin{array}{c} (\phi - \frac{\alpha+(1-\delta)c_1-c_2}{c_1})n_t \\ +\frac{1}{c_1}x_t + \frac{1}{c_1}w_t + \frac{1+\eta-\overline{\phi}}{c_1\overline{\phi}}u_t + \frac{1}{c_1(\overline{\phi}-1)}x_t + \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_t \end{array}\right] \\
&= -\frac{\alpha+(1-\delta-\phi)c_1-c_2}{1+\eta}n_t \\
&\quad + \frac{\overline{\phi}}{(1+\eta)(\overline{\phi}-1)}x_t + \frac{\rho\overline{\phi}}{(1+\eta)(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_t + \frac{1}{1+\eta}w_t + \frac{1+\eta-\overline{\phi}}{(1+\eta)\overline{\phi}}u_t.
\end{aligned}$$

Q.E.D.

Appendix 4: Proof of Proposition 2 and derivation of the stability condition

(19) and (20) imply that (the same as (54))

$$c_{1i}n_{i,t+1} = (1+\eta_i)p_{it} + [\alpha_i + (1-\delta_i)c_{1i} - c_{2i}]n_{it} - x_{it} - w_{it} + u_{it}. \tag{60}$$

Plug (60) into (25), we get

$$(1+\eta_i-\lambda_i)c_{1i}n_{i,t+1} + \{\lambda_i\alpha_i - (1+\eta_i-\lambda_i)[(1-\delta_i)c_{1i} - c_{2i}] - (1+\eta_i)(1-\lambda_i)c_{1i}\}\, n_{it}$$

$$+(1+\eta_i)(1-\lambda_i)[(1-\delta_i)c_{1i}-c_{2i}]n_{i,t-1} = \lambda_i x_{it} + \lambda_i w_{it} + (1+\eta_i-\lambda_i)u_{it} - (1+\eta_i)(1-\lambda_i)u_{i,t-1}. \tag{61}$$

Using lag operator L, the above equation can be rewritten as

$$[(1+\eta_i-\lambda_i)c_{1i} + \{\lambda_i\alpha_i - (1+\eta_i-\lambda_i)[(1-\delta_i)c_{1i} - c_{2i}] - (1+\eta_i)(1-\lambda_i)c_{1i}\}\, L$$

$$+(1+\eta_i)(1-\lambda_i)[(1-\delta_i)c_{1i}-c_{2i}]L^2]n_{i,t+1} = \lambda_i x_{it} + \lambda_i w_{it} + (1+\eta_i-\lambda_i)u_{it} - (1+\eta_i)(1-\lambda_i)u_{i,t-1}. \tag{62}$$

Under the stability condition (27), the roots of the polynomial $f(L)$ lie outside the unit circle, where

$$f(L) = (1+\eta-\lambda)c_1 + \{\lambda\alpha - (1+\eta-\lambda)[(1-\delta)c_1 - c_2] - (1+\eta)(1-\lambda)c_1\}\, L$$

$$+(1+\eta)(1-\lambda)[(1-\delta)c_1 - c_2]L^2. \tag{63}$$

Then the polynomial $f(L)$ in (62) is invertable, and this guarantees the existence of a unique particular solution for (61).

We derive the stability condition (27) under the adaptive expectations.

To ensure that (61) do not explode, we need to require the roots of

$$g(y) = y^2 + \frac{\lambda\alpha - (1+\eta-\lambda)[(1-\delta)c_1 - c_2] - (1+\eta)(1-\lambda)c_1}{(1+\eta-\lambda)c_1} y + \frac{(1+\eta)(1-\lambda)[(1-\delta)c_1 - c_2]}{(1+\eta-\lambda)c_1} \tag{64}$$

to lie within the unit circle.

Enders (2010)[16] provides the stability conditions for a quadratic equation of

$$y^2 - a_1 y - a_2 = 0. \tag{65}$$

There are three cases depending on the discriminant

$$d = (a_1)^2 + 4a_2. \tag{66}$$

In case 1, $(a_1)^2 + 4a_2 > 0$, there are two distinct real roots, and the conditions for these two roots to lie within the unit circle are $a_1 + a_2 < 1$ and $a_2 - a_1 < 1$. In case 2, $(a_1)^2 + 4a_2 = 0$, there are repeated real roots, and the condition is $|a_1| < 2$. In case 3, $(a_1)^2 + 4a_2 < 0$, the roots are imaginary, and the condition is $-a_2 < 1$ $(a_2 < 0)$.

We apply these results to (64).

$$a_1 = -\frac{\lambda\alpha - (1+\eta-\lambda)[(1-\delta)c_1 - c_2] - (1+\eta)(1-\lambda)c_1}{(1+\eta-\lambda)c_1}, \tag{67}$$

$$a_2 = -\frac{(1+\eta)(1-\lambda)[(1-\delta)c_1 - c_2]}{(1+\eta-\lambda)c_1}, \tag{68}$$

[16] Applied Econometric Time Series, 3rd Edition, page 28.

$$d = \frac{\{\lambda\alpha - (1+\eta-\lambda)[(1-\delta)c_1 - c_2] - (1+\eta)(1-\lambda)c_1\}^2 - 4(1+\eta)(1-\lambda)(1+\eta-\lambda)c_1[(1-\delta)c_1 - c_2]}{(1+\eta-\lambda)^2 c_1^2}. \tag{69}$$

Case 3.

$$\begin{aligned} -a_2 - 1 &= \frac{(1+\eta)(1-\lambda)[(1-\delta)c_1 - c_2] - (1+\eta-\lambda)c_1}{(1+\eta-\lambda)c_1} \\ &= \frac{-\eta\lambda c_1 - (1+\eta)(1-\lambda)(\delta c_1 + c_2)}{(1+\eta-\lambda)c_1} < 0, \end{aligned}$$

so in case 3, $-a_2 < 1$ and therefore the roots are inside the unit circle.

Case 2. The stability condition is $-2 < a_1 < 2$.

$$a_1 - 2 = \frac{-\lambda\alpha - \eta\lambda c_1 - (1+\eta-\lambda)(\delta c_1 + c_2)}{(1+\eta-\lambda)c_1} < 0.$$

Therefore it remains to verify that $a_1 + 2 > 0$. From (67) we need to verify that

$$a_1 + 2 = \frac{-\lambda\alpha + (1+\eta-\lambda)[(1-\delta)c_1 - c_2] + (1+\eta)(1-\lambda)c_1 + 2(1+\eta-\lambda)c_1}{(1+\eta-\lambda)c_1} > 0. \tag{70}$$

If $a_1 \geq 0$, the condition $a_1 + 2 > 0$ is satisfied. If $a_1 < 0$, because $d = (a_1)^2 + 4a_2 = 0$, we have $|a_1| = \sqrt{-4a_2}$, that is $-a_1 = \sqrt{-4a_2}$, hence $a_1 = -2\sqrt{-a_2}$. Therefore,

$$-\frac{\lambda\alpha - (1+\eta-\lambda)[(1-\delta)c_1 - c_2] - (1+\eta)(1-\lambda)c_1}{(1+\eta-\lambda)c_1} = -2\sqrt{\frac{(1+\eta)(1-\lambda)[(1-\delta)c_1 - c_2]}{(1+\eta-\lambda)c_1}},$$

rearranging,

$$-\lambda\alpha + (1+\eta-\lambda)[(1-\delta)c_1 - c_2] + (1+\eta)(1-\lambda)c_1 = -2\sqrt{(1+\eta)(1-\lambda)[(1-\delta)c_1 - c_2](1+\eta-\lambda)c_1}.$$

Plug the above equation into the numerator of (70), we get the following inequality that needs to be verified

$$-2\sqrt{(1+\eta)(1-\lambda)[(1-\delta)c_1-c_2](1+\eta-\lambda)c_1}+2(1+\eta-\lambda)c_1>0,$$

which is equivalent to

$$(1+\eta-\lambda)c_1>(1+\eta)(1-\lambda)[(1-\delta)c_1-c_2],$$

which is equivalent to

$$\eta\lambda c_1+(1+\eta)(1-\lambda)(\delta c_1+c_2)>0,$$

which satisfies. Therefore, in case 2 we have $-2<a_1<2$, and thus the roots are inside the unit circle.

Case 1. We need to verify that $a_1+a_2<1$ and $a_2-a_1<1$.

$$\begin{aligned} a_1+a_2-1 &= \frac{-\lambda\alpha+(1+\eta-\lambda)[(1-\delta)c_1-c_2]+(1+\eta)(1-\lambda)c_1-(1+\eta)(1-\lambda)[(1-\delta)c_1-c_2]}{(1+\eta-\lambda)c_1}-1 \\ &= \frac{-\lambda\alpha+[(1+\eta-\lambda)-(1+\eta)(1-\lambda)][(1-\delta)c_1-c_2-c_1]}{(1+\eta-\lambda)c_1} \\ &= \frac{-\lambda\alpha-\eta\lambda(\delta c_1+c_2)}{(1+\eta-\lambda)c_1}<0. \end{aligned}$$

Hence it remains to verify that $a_2-a_1<1$.

$$\begin{aligned} a_2-a_1-1 &= \frac{-(1+\eta)(1-\lambda)[(1-\delta)c_1-c_2]+\lambda\alpha-(1+\eta-\lambda)[(1-\delta)c_1-c_2]-(1+\eta)(1-\lambda)c_1}{(1+\eta-\lambda)c_1}-1 \\ &= \frac{\lambda\alpha-[(1+\eta)(1-\lambda)+(1+\eta-\lambda)][(1-\delta)c_1-c_2+c_1]}{(1+\eta-\lambda)c_1} \\ &= \frac{\lambda\alpha-[(2+2\eta)-(2+\eta)\lambda][(2-\delta)c_1-c_2]}{(1+\eta-\lambda)c_1} \\ &= \frac{\{(2+\eta)[(2-\delta)c_1-c_2]+\alpha\}\lambda-(2+2\eta)[(2-\delta)c_1-c_2]}{(1+\eta-\lambda)c_1}. \end{aligned}$$

Hence, to require $a_2 - a_1 < 1$ is equivalent to require

$$\{(2+\eta)[(2-\delta)c_1 - c_2] + \alpha\}\lambda - (2+2\eta)[(2-\delta)c_1 - c_2] < 0,$$

which is equivalent to

$$\lambda < \frac{(2+2\eta)[(2-\delta)c_1 - c_2]}{(2+\eta)[(2-\delta)c_1 - c_2] + \alpha} \equiv \overline{\lambda}. \tag{71}$$

Thus, in case 1, for the roots to be inside the unit circle, we require that $\lambda < \overline{\lambda}$.

In sum, when $d \leq 0$, the roots would be inside the unit circle; when $d > 0$, a condition of $\lambda < \overline{\lambda}$ is required for the roots to lie inside the unit circle.

Appendix 5: Moment conditions for rational expectations

Simplify the notation of (16) and (18) as

$$\widehat{N}_{it} = K_{0i} + K_1 t \text{ and } \widehat{P}_{it} = S_{0i} + S_1 t. \tag{72}$$

The intercepts have subscript i because we allow for city fixed effects. The time trend coefficients do not have subscript i because cities within the same group are assumed to have the same parameters. Substitute $n_{it} = N_{it} - \widehat{N}_{it}$, $p_{it} = P_{it} - \widehat{P}_{it}$ and $y_{it} = \overline{D}_i + \mu t + x_{it}$ into (22) and (23), and together with (2), we have the following three equations based on which the moment conditions will be derived:

$$Z_i - [(1-\phi)K_1 - \frac{\mu}{c_1(\overline{\phi}-1)}]t + N_{it} - \phi N_{i,t-1} - \frac{1}{c_1(\overline{\phi}-1)} y_{i,t-1} = \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_{i,t-1} + \frac{1+\eta}{c_1\overline{\phi}} u_{i,t-1}, \tag{73}$$

$$Q_i - [S_1 + \frac{\alpha + (1-\delta-\phi)c_1}{1+\eta} K_1 - \frac{\overline{\phi}\mu}{(1+\eta)(\overline{\phi}-1)}]t + P_{it} + \frac{\alpha + (1-\delta-\phi)c_1}{1+\eta} N_{it} - \frac{\overline{\phi}}{(1+\eta)(\overline{\phi}-1)} y_{it} \tag{74}$$

$$= \frac{\rho\overline{\phi}}{(1+\eta)(\overline{\phi}-1)(\overline{\phi}-\rho)}\varepsilon_{it} + \frac{1}{1+\eta} w_{it} + \frac{1+\eta-\overline{\phi}}{(1+\eta)\overline{\phi}} u_{it},$$

$$y_{it} = \overline{D}_i + \mu t + x_{it},\ x_{it} = x_{i,t-1} + \varepsilon_{it},\ \varepsilon_{it} = \rho\varepsilon_{i,t-1} + v_{it}. \tag{75}$$

where Q_i and Z_i are fixed effects.

Take first difference of $y_{it} = \overline{D}_i + \mu t + x_{it}$, we eliminate the fixed effects and convert the unit root series into a stationary series:

$$\Delta y_{it} = \mu + \varepsilon_{it}. \tag{76}$$

We have

$$\Delta y_{it} - \rho \Delta y_{i,t-1} - (1-\rho)\mu = v_{it}. \tag{77}$$

Take the first difference of (73),

$$-[(1-\phi)K_1 - \frac{\mu}{c_1(\overline{\phi}-1)}] + \Delta N_{it} - \phi \Delta N_{i,t-1} - \frac{1}{c_1(\overline{\phi}-1)}\Delta y_{i,t-1} = \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\Delta\varepsilon_{i,t-1} + \frac{1+\eta}{c_1\overline{\phi}}\Delta u_{i,t-1}. \tag{78}$$

Because

$$\Delta\varepsilon_{it} = \rho\Delta\varepsilon_{i,t-1} + \Delta v_{it}, \tag{79}$$

thus applying the Cochrane-Orcutt transformation yields

$$\begin{aligned} &-(1-\rho)\left[(1-\phi)K_1 - \frac{\mu}{c_1(\overline{\phi}-1)}\right] + \Delta N_{it} - \rho\Delta N_{i,t-1} - \phi(\Delta N_{i,t-1} - \rho\Delta N_{i,t-2}) \\ &-\frac{1}{c_1(\overline{\phi}-1)}(\Delta y_{i,t-1} - \rho\Delta y_{i,t-2}) = \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\Delta v_{i,t-1} + \frac{1+\eta}{c_1\overline{\phi}}(\Delta u_{i,t-1} - \rho\Delta u_{i,t-2}). \end{aligned} \tag{80}$$

Rearrange and substitute (77) in, we get

$$\begin{aligned} & -(1-\rho)(1-\phi)K_1 + \Delta N_{it} - (\rho+\phi)\Delta N_{i,t-1} + \rho\phi\Delta N_{i,t-2} \\ = \; & \frac{v_{i,t-1}}{c_1(\overline{\phi}-1)} + \frac{\rho\overline{\phi}}{c_1(\overline{\phi}-1)(\overline{\phi}-\rho)}\Delta v_{i,t-1} + \frac{1+\eta}{c_1\overline{\phi}}(\Delta u_{i,t-1} - \rho\Delta u_{i,t-2}). \end{aligned} \tag{81}$$

Take the first difference of (74),

$$-[S_1+\frac{\alpha+(1-\delta-\phi)c_1}{1+\eta}K_1-\frac{\overline{\phi}\mu}{(1+\eta)(\overline{\phi}-1)}]+\Delta P_{it}+\frac{\alpha+(1-\delta-\phi)c_1}{1+\eta}\Delta N_{it}-\frac{\overline{\phi}}{(1+\eta)(\overline{\phi}-1)}\Delta y_{it}$$

$$=\frac{\rho\overline{\phi}}{(1+\eta)(\overline{\phi}-1)(\overline{\phi}-\rho)}\Delta\varepsilon_{it}+\frac{1}{1+\eta}\Delta w_{it}+\frac{1+\eta-\overline{\phi}}{(1+\eta)\overline{\phi}}\Delta u_{it}. \tag{82}$$

Then apply the Cochrane-Orcutt transformation and plug (77) in, we get

$$-(1-\rho)[S_1+\frac{\alpha+(1-\delta-\phi)c_1}{1+\eta}K_1]+\Delta P_{it}-\rho\Delta P_{i,t-1}+\frac{\alpha+(1-\delta-\phi)c_1}{1+\eta}(\Delta N_{it}-\rho\Delta N_{i,t-1}) \tag{83}$$

$$=\frac{\overline{\phi}v_{it}}{(1+\eta)(\overline{\phi}-1)}+\frac{\rho\overline{\phi}}{(1+\eta)(\overline{\phi}-1)(\overline{\phi}-\rho)}\Delta v_{it}+\frac{1}{1+\eta}(\Delta w_{it}-\rho\Delta w_{i,t-1})+\frac{1+\eta-\overline{\phi}}{(1+\eta)\overline{\phi}}(\Delta u_{it}-\rho\Delta u_{i,t-1}).$$

Based on (77), (81), and (83), the moment conditions are

First stage:

$$m_1(\theta)=\begin{cases}\Delta y_{it}-\rho\Delta y_{i,t-1}-(1-\rho)\mu\\ [\Delta y_{it}-\rho\Delta y_{i,t-1}-(1-\rho)\mu]\Delta y_{i,t-1}\\ [\Delta y_{it}-\rho\Delta y_{i,t-1}-(1-\rho)\mu]\Delta y_{i,t-2}\end{cases}. \tag{84}$$

Second stage:

$$m_R(\theta) = \begin{cases} \psi_{it} \\ \psi_{it}\Delta y_{i,t-4} \\ \psi_{it}\Delta P_{i,t-4} \\ \psi_{it}\Delta N_{i,t-4} \\ \zeta_{it} \\ \zeta_{it}\Delta y_{i,t-3} \\ \zeta_{it}\Delta P_{i,t-3} \\ \zeta_{it}\Delta N_{i,t-3} \end{cases}, \tag{85}$$

where ψ_{it} and ζ_{it} are defined as

$$\psi_{it} = -(1-\rho)(1-\phi)K_1 + \Delta N_{it} - (\rho+\phi)\Delta N_{i,t-1} + \rho\phi\Delta N_{i,t-2}, \tag{86}$$

and

$$\zeta_{it} = -(1-\rho)[S_1 + \frac{\alpha + (1-\delta-\phi)c_1}{1+\eta}K_1] + \Delta P_{it} - \rho\Delta P_{i,t-1} + \frac{\alpha + (1-\delta-\phi)c_1}{1+\eta}(\Delta N_{it} - \rho\Delta N_{i,t-1}). \tag{87}$$

Appendix 6: Moment conditions for adaptive expectations

Substitute $n_{it} = N_{it} - \widehat{N}_{it}$, $p_{it} = P_{it} - \widehat{P}_{it}$ and $y_{it} = \overline{D}_i + \mu t + x_{it}$ into (2), (28) and (29), we have the three equations based on which the moment conditions will be derived. One of them are the same as (75). The other two are (after taking the first difference):

$$-\Delta P_{i,t-1} + \frac{c_1}{\lambda}\Delta N_{it} - \frac{(2-\lambda-\delta)c_1}{\lambda}\Delta N_{i,t-1} + \frac{(1-\lambda)(1-\delta)c_1}{\lambda}\Delta N_{i,t-2} = \frac{1}{\lambda}\Delta u_{i,t-1} - \frac{1-\lambda}{\lambda}\Delta u_{i,t-2}, \tag{88}$$

and

$$(1+\eta-\lambda)\Delta P_{it} - (1+\eta)(1-\lambda)\Delta P_{i,t-1} + \alpha\Delta N_{it} - \alpha(1-\lambda)\Delta N_{i,t-1} - \Delta y_{it} + (1-\lambda)\Delta y_{i,t-1} = \Delta w_{it} - (1-\lambda)\Delta w_{i,t-1}. \tag{89}$$

Therefore, the first stage moment conditions are the same as the ones in the rational expectations case. The second stage moment conditions are

$$_A(\theta) = \begin{cases} \chi_{it} \\ \chi_{it}\Delta y_{i,t-4} \\ \chi_{it}\Delta P_{i,t-4} \\ \chi_{it}\Delta N_{i,t-4} \\ \kappa_{it} \\ \kappa_{it}\Delta y_{i,t-3} \\ \kappa_{it}\Delta P_{i,t-3} \\ \kappa_{it}\Delta N_{i,t-3} \end{cases}, \tag{90}$$

where χ_{it} is defined as

$$\chi_{it} = -\Delta P_{i,t-1} + \frac{c_1}{\lambda}\Delta N_{it} - \frac{(2-\lambda-\delta)c_1}{\lambda}\Delta N_{i,t-1} + \frac{(1-\lambda)(1-\delta)c_1}{\lambda}\Delta N_{i,t-2}, \tag{91}$$

and κ_{it} is defined as

$$\kappa_{it} = (1+\eta-\lambda)\Delta P_{it} - (1+\eta)(1-\lambda)\Delta P_{i,t-1} + \alpha\Delta N_{it} - \alpha(1-\lambda)\Delta N_{i,t-1} - \Delta y_{it} + (1-\lambda)\Delta y_{i,t-1}. \tag{92}$$