The Effect of Labor Supply on Unemployment Fluctuation

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Abstract

In this paper, I investigate the role of operative labor supply margin in explaining the unemployment volatility puzzle. Whereas most search and matching models focus only on flows between employment and unemployment, I find that at least 20% of the volatility in unemployment is attributable to the change in flows between unemployment and non-labor force in U.S. data. In order to capture how and how much the operative labor supply margin contributes to unemployment fluctuation, I embed the non-labor force into the otherwise standard search model. This insertion makes the marginal cost and profit of posting a vacancy smaller than in a standard search model. Therefore, in posting vacancies, firms respond more sensitively to changes in the productivity shock. As vacancies and the probability of finding a job become more volatile, flows between employment and unemployment become more volatile, as do flows between non-labor force and unemployment; therefore, unemployment fluctuates more. In the model omitting the non-labor force, the standard deviations of unemployment and vacancies – relative to the standard deviation of labor productivity – are 0.45 and 0.58. However, in the model including the non-labor force, they increase to 3.42 and 1.81. These results show that the operative labor supply margin integrated within a search model can explain the unemployment volatility puzzle better than the canonical search model.

Keywords: Unemployment, Vacancy, Heterogeneity, Extensive Margin, Search and Matching, Labor Supply.
I. Introduction

One of the most salient and important facts in business cycles is that unemployment is very volatile. The U.S. data show that the standard deviation of unemployment is almost 10 times as large as the standard deviation of labor productivity. Unfortunately, as Shimer (2005) demonstrated, a standard search and matching model, known as the best model to describe the micro-foundation of unemployment and its dynamics, is not able to explain the cyclical fluctuation of unemployment.\(^1\) This is called the unemployment volatility puzzle, or Shimer’s puzzle. Shimer’s paper kicks off a literature on modifications to the search and matching model. In line with these efforts, several papers show that unemployment can be as volatile as the data under special conditions.\(^2\) However, these papers have not brought a consensus to the literature, so the puzzle remains. In this paper, I investigate the role of the operative labor supply margin as another factor which may explain the puzzle.

A common feature of most models based on Mortensen and Pissarides’s search and matching model is to allow only two labor statuses - employment and unemployment. This assumption has been justified by the argument that the change in the size of the labor force is trivial when it is compared to the size of the change in the employment-population ratio.\(^3\) However, when we look at the data, the standard deviation of detrended employment-population ratio is 1.21% and that of labor force participation rate is 0.37%. In addition, the correlation coefficient is 0.8, which means that approximately one third of the fluctuation in employment can be attributed to the change in participation, as shown in Figure 1. From this observation, it is not persuasive to argue that changes in participation can be ignored in business cycle models. Now let’s get back to the point of this paper - unemployment. Is the change in labor participation important or not in explaining the fluctuation of unemployment? Answering this question is necessary before we try to solve Shimer’s puzzle. If it is non-trivial, we must identify channels through which the operative labor supply affects unemployment fluctuation. Furthermore, it is important to know how much the volatility of unemployment increases when such channels are embedded into an otherwise standard search and matching model.

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1. Shimer(2005) shows that the volatility of unemployment is only 0.45 times as large as the volatility of labor productivity in a standard search and matching model when only labor productivity shocks are fed. In the same paper, he also shows that it becomes 1.5 times when stochastic but perfectly correlated labor productivity and separation rate shocks are fed. However, most literature does not assume such ad-hoc separation rate shocks. Thus, the model without separation rate shocks in Shimer (2005) paper is the benchmark model in my paper.

2. Hagedorn and Manovskii (2008) and Nakajima (2012) are successful in generating unemployment as volatile as the U.S. data by using different calibration methods. Hall (2005), Gertler and Trigari (2009), and Shimer (2011) are also successful by assuming real wage stickiness.

3. Hall (2008) argues that roughly 80% of business cycle fluctuations in employment show up as offsetting changes in unemployment rather than changes in participation. Rogerson and Shimer (2010) argue that the change in the size of the labor force is a secondary factor by showing that the size of the labor force has been little changed.
Two kinds of extensive margin exist in the real world. One is made by search and matching frictions, the flows between employment and unemployment. The other one is extensive margin by operative labor supply decisions, the flows between unemployment and non-labor force. In this paper, I assess how much the operative labor supply margin contributes to the volatility of unemployment using U.S. data. Then, I explain how the operative labor supply margin affects unemployment fluctuation. Last, I analyze how the results differ from Shimer’s (2005) findings when the operative labor supply margin is embedded into a standard search and matching model. Hereafter, $E$, $U$, and $N$ denote employment, unemployment, and non-labor force while $EU$ denotes the flow from $E$ into $U$.

A change in unemployment comes from two sources. One is the flows between $E$ and $U$ and the other is the flows between $N$ and $U$. In other words, a change in $U$ is the sum of the net flow from $E$ into $U$ and the net flow from $N$ into $U$. My findings are as follows: first, from the data, at least 20% of volatility of unemployment is directly attributable to the operative labor supply margin ($net NU$). Especially during recessions, the operative labor supply margin counts for 30~40% of increases in unemployment. Thus, it is obvious that the role of operative labor supply in explaining the volatility of unemployment is not trivial at all. However, we still have to answer why the $net NU$ increases during recessions. Related to this question, my second finding is that the existence of the non-labor force makes the job-finding probability more volatile than an otherwise standard search model. This occurs because the size of average matched rent due to frictions is reduced when the non-labor force is embedded in the model. A small rent implies a small profit to a matched firm, which means that the cost to post a vacancy is also small. That is, marginal benefit and marginal cost of posting a vacancy are small. Then, firms respond more to changes in total factor productivity, hereafter TFP. For example, responding to negative TFP shocks, fewer firms create jobs. This decrease in vacancies makes finding new jobs more difficult, therefore, fewer people can exit unemployment (high $net EU$) and fewer workers who enter the labor force find a job (high $net NU$). In summary, the existence of non-labor force, first, adds flows between non-labor force and unemployment as a new factor which changes unemployment. Second, the $net EU$ and $net NU$ are more volatile because firms respond more to TFP shocks, so unemployment fluctuates more than in a standard search model.

However, there is no way to measure how and how much the operative labor supply margin affects firms’ vacancy-posting and job-finding probability from the data. Moreover, we want to know how this insertion affects Shimer’s puzzle. Therefore I build up two models – one without operative labor supply

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4 In my paper, intensive margin of hours worked is not taken into account.  
5 Net flow from $N$ into $U$ ($net NU$) = flow from $N$ into $U$ ($NU$) – flow from $U$ into $N$ ($UN$). Thus, $\Delta U = net EU + net NU$  
6 It is well explained in Hagedorn and Manovskii (2008). “What matters for the incentives to post vacancies is the size of the percentage changes of profits in response to changes in productivity. Theses percentage changes are large if the size of profits is small and the increase in productivity is not fully absorbed by an increase in wages” (p. 1696)
margin, and the other one with it. In the model without it, the relative standard deviation of unemployment is 0.45 and the relative standard deviation of vacancies is 0.58. However, in the model with the operative labor supply margin, the values are 3.42 and 1.81. What is more interesting is the vacancy cost decreases from 0.51 to 0.28 when the operative labor supply is added. These numerical results are explained as follows: due to the existence of non-labor force, an average firm’s profit and vacancy cost decreases (0.51 to 0.28). Therefore, vacancies respond more to TFP shocks (0.58 to 1.81), which makes unemployment more volatile (0.45 to 3.42). These findings confirm that including the operative labor supply margin improves the performance of the existing standard search model in terms of Shimer’s puzzle, or we can say that what is left unexplained by the model measures the size of the puzzle more accurately.

An implication from this paper is that excluding the non-labor force is a large shortcoming of the traditional search and matching model. Without the operative labor supply margin, the search model had to generate volatile vacancies to make the volatility of unemployment match the data. However, when including the margin, unemployment becomes more volatile, directly or indirectly, than a standard search model. This means that the burden to make market tightness volatile is reduced when integrating the operative labor supply margin into the model. Some papers are successful in generating market tightness and unemployment which fit the data, but they must overshoot the volatility of market tightness or unemployment to do so.

Related papers are Chang and Kim (2006, 2007), Krusell, Mukoyama, Rogerson, and Sashin (2012), Nakajima (2012). A common feature of these papers is that they are heterogeneous agent models with idiosyncratic individual shocks, incomplete capital market, and indivisibility in labor supply. Chang and Kim (2006, 2007) divide people only into employment and non-employment in their model. Nakajima (2012) only allows two groups – employment and unemployment by frictional margin. Krusell et al. (2012) integrate both margins by adding search frictions into the Chang and Kim (2007) model. Though Krusell et al. (2012) already built a model with two extensive margins, there are two significant differences between their paper and mine. First, the purpose of Krusell et al. (2012) is to show that both extensive margins should be included in a model in order to explain several key facts in business cycles, rather than how to generate volatile frictional shocks. Second, they assume market tightness is exogenous and feed exogenous search and matching frictions, which are perfectly correlated with TFP shocks, into

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7 Relative standard deviation of x means the standard deviation of x divided by the standard deviation of labor productivity in line with Shimer (2005)’s analysis.

8 Though Nakajima (2012) includes intensive margin during the Nash-Bargaining process, his model is a two-state model between employment and unemployment.
the model. However, the job-finding probability is determined endogenously in my paper following Bils, Chang, and Kim (2011), so that I can measure how much of Shimer’s puzzle is explained by adding the operative labor supply into a standard search model. In endogenizing market tightness, my model can be viewed as an advanced version of Krusell et al. (2012) in terms of the frictional margin.

In Section II, I measure the contribution of the operative labor supply margin to changes in unemployment from the U.S. data. In Section III, I explain the channels through which the operative labor supply may affect unemployment. In Section IV, the model and its calibration are explained. Results and implications are discussed in Section V. I conclude this paper in Section VI. A detailed computational method is explained in an Appendix.

II. Data: Measuring the Contribution of the Operative Labor Supply

In this section, I assess whether the operative labor supply affects unemployment fluctuation. To answer this question, we need flow data among three labor statuses – employment (E), unemployment (U), and non-labor force (N). The U.S. Bureau of Labor Statistics (BLS) reports estimates of the number of workers changing their labor force status from one month to the next, and these flow data series are available from 1990. The upper panel in Figure 2 shows the time series of unemployment. Shaded regions represent periods when unemployment increases. The middle panel shows inflows to unemployment from employment and non-labor force. The bottom panel shows outflows from unemployment to employment and non-labor force. The solid lines in the middle and bottom panels show the contribution of extensive margins by search and matching frictions to the change in unemployment. In the same way, the dashed lines in both figures show the contribution of the operative labor supply margin to the change in unemployment. On the middle panel, the transition rate from E to U (f_{EU}) and the transition rate from N to U (f_{NU}) show similar patterns. On the bottom panel, the transition rate from U to E (f_{UE}) and the transition rate from U to N (f_{UN}) also move together, although not as closely as f_{EU} and f_{NU}. That is, the two extensive margins have very similar cyclical properties, as described in Table 1. This implies that the operative labor supply margin raises unemployment during recessions and lowers it during expansions, as
the frictional extensive margin does. Then, the next question is how much the operative labor supply margin contributes to the volatility in unemployment. Table 2 describes monthly flows to and from unemployment from June 2007 to May 2008. According to the NBER recession chart, the recent recession began in December 2007. So, the first 6 months are before the recession and the second 6 months are the beginning of the recession. In the third row from the bottom, the average net EU (-170,000) and net NU (195,000) almost cancel out each other before the recession, so unemployment increased modestly. In column 9, the monthly average change in unemployment is just 25,000 before the recession. However, the monthly average net EU increased by 107,000 (from -170,000 to -62,000), and net NU increased by 70,000 (from 195,000 to 265,000) for the six month period before to the six month period after December 2007. Therefore, from the last row, we find that approximately 40% of the average increase in unemployment is attributed to the operative labor supply margin.

Now let’s expand this analysis to the three recent recessions since 1990. The first and second rows in Table 3 are the monthly average net NU and net EU during each period. The third row is the total net flow to unemployment (= net NU + net EU = ΔU). The fourth row is the hypothetical total net flow to unemployment by assuming that the size of net NU is fixed to its monthly average since 1990. That is, the fourth row is the change in unemployment under the assumption that the operative labor supply margin does not affect the changes in unemployment. During the first recession, unemployment increased monthly by 119,000 on average. However, when the size of the operative labor supply margin is assumed to be fixed, it would have increased by only 76,000. That is just 64% of the actual increase. In the same way, we find that when the frictional extensive margin is frozen, unemployment would have increased only by 40,000. That is just 34% of the actual increase in unemployment. Using either method, during the first recession approximately 35% of the change in unemployment is attributed to the operative labor supply margin. During the second and third periods, the contribution rates of the operative labor supply margin are 42% and 19%. By using a weighted average, due to different lengths of each shaded region, we can see approximately 70% of changes in unemployment comes from the frictional extensive margin and 30% comes from the operative labor supply margin during the three most recent recessionary periods. The final step is to find the level of contribution of each extensive margin since 1990. In the same way as before, I construct a hypothetical unemployment time series only using the operative labor supply margin and plot it on the actual unemployment time series. In Figure 3 and Table 4, we can confirm the two series are highly and positively correlated. Approximately 20% of the change in unemployment is

13 Since 1990, the monthly average of net NU is 185,000, and that of net EU is -185,000. Hypothetical total net flow to U with frozen net NU = net EU + 185,000. Hypothetical net flow to U with frozen net EU = net NU – 185,000.
14 Hypothetical $U_t$ only with the operative labor supply margin = Hypothetical $U_t$ with frozen net EU = net NU – 185,000 + Hypothetical $U_{t-1}$ with frozen net EU.
attributed to the operative labor supply margin. However, this contribution becomes clearer and stronger during recessions as we see in Figure 3.

III. Channels Through Which the Operative Labor Supply Affects Unemployment

In the previous section, I measured how much the flows between unemployment and non-labor force contribute to changes in unemployment. However, two issues take place in this step. The first is why the net NU increases during recessions. The second issue is that careful thought suggests another channel through which the operative labor supply margin affects unemployment fluctuation. In order to approach these two issues, we have to reconsider the change in unemployment as follows:

\[ \Delta U_t = \text{net NU}_t + \text{net EU}_t. \]

The previous section with the gross flow data focuses on the first term (net NU\(_t\)). Workers out of the labor force in the previous period (N\(_{t-1}\)) are divided into three statuses in the current period as follows:

\[ N_{t-1} = NN_t + NE_t + NU_t = (1 - \epsilon) \cdot N_{t-1} + \epsilon \cdot N_{t-1} \cdot p(\theta_t) + \epsilon \cdot N_{t-1} \cdot (1 - p(\theta_t)), \]

where \(\epsilon\) is the rate of entering the labor force, and \(p(\cdot)\) is the probability of finding a job. In Table 5, 92.5% of the non-labor force remains non-labor force in the next period. Though the participation rate (\(\epsilon\)) is procyclical as shown in Figure 1, I assume \(\epsilon\) is fixed at 0.075 for now. \(p(\theta_t)\) is the job-finding probability when the current market tightness is \(\theta_t\). That is, of the non-labor force workers who just decide to participate in the labor force (\(\epsilon \cdot N_{t-1}\)), only \(p(\theta_t)\) of them find a job, while the remaining (1 - \(p(\theta_t)\)) become unemployed. Then, net NU is decomposed as:

\[ \text{net NU}_t = NU_t - UN_t = \epsilon \cdot N_{t-1} \cdot (1 - p(\theta_t)) - UN_t. \]

Also, net EU is decomposed as:

\[ \text{net EU}_t = EU_t - UE_t = E_{t-1} \cdot \lambda - U_{t-1} \cdot p(\theta_t)^{15}. \]

\(\lambda\) is the job-destruction probability.\(^{16}\) Of workers who were employed, the proportion \(\lambda\) becomes unemployed. Out of workers who were unemployed, \(p(\theta_t)\) of them find a job and exit unemployment.

\(^{15}\) Strictly speaking, \(UE_t\) is \((1 - \zeta) \cdot U_{t-1} \cdot p(\theta_t)\), where \(\epsilon\) the transition rate from U to N. In Table 5, \(\zeta = 0.222\) in the steady state. (1 - \(\zeta\)) means the ratio that the unemployed remain in the labor force. Also, I assume \(\epsilon\) is fixed to 0.222 in this paper.

\(^{16}\) In this paper, I assume the separation probability is fixed. Except for several papers dealing with endogenous separation probability like Mortensen and Pissarides (1994), Haan, Ramey, and Watson (2000), and Bils, Chang, and Kim (2011), most papers assume a fixed separation probability. Otherwise, stochastic separation shocks may be criticized as an ad-hoc exogenous process designed to match the data on purpose. Thus, I also assume the separation probability is fixed through this paper.
Here, the job-finding probability, \( p(\theta_t) \), sheds light on the possibility that the operative labor supply margin affects unemployment fluctuation. In Equation 3, when the job-finding probability decreases, \( NU_t \) increases, so net \( NU_t \) increases. In the same way, when the job-finding probability decreases, \( UE_t \) decreases, so net \( EU_t \) increases in Equation 4. The total derivative of Equation 1 with respect to TFP shock \( (z_t) \) is

\[
(5) \quad \frac{d(\Delta U_t)}{dz_t} = -N_{t-1} \frac{\partial p(\theta_t)}{\partial \theta_t} \frac{d\theta_t}{dz_t} - U_{t-1} \frac{\partial p(\theta_t)}{\partial \theta_t} \frac{d\theta_t}{dz_t} ^{17}.
\]

The key part is \( \frac{d\theta}{dz} \). If market tightness \( (\theta) \) is sensitive to TFP shocks \( (z) \), then net \( EU \) and net \( NU \) change largely, so that unemployment is more volatile. This is the key problem of Shimer’s puzzle. The counterpart of Equation 5, when there is no non-labor force, is

\[
(6) \quad \frac{d(\Delta U_t)}{dz_t} = -U_{t-1} \frac{\partial p(\theta_t)}{\partial \theta_t} \frac{d\theta_t}{dz_t}.
\]

A standard search model cannot generate a large \( \frac{d(\Delta U)}{dz} \) because \( \frac{d\theta}{dz} \) in Equation 6 is too low. However, by embedding non-labor force, a new term \( (-N_{t-1} \frac{\partial p(\theta_t)}{\partial \theta_t} \frac{d\theta_t}{dz_t}) \) which can contribute to unemployment is added. However, this is not the whole story. What is important is that the existence of non-labor force makes \( \frac{d\theta}{dz} \) larger than a standard search model without it. Therefore, unemployment becomes more volatile than a standard search model without non-labor force. In the next section, I show why and how much the operative labor supply margin affects the market tightness and unemployment. In order to compare the volatility of market tightness, I build two models – one without non-labor force (Model I) and the other with it (Model II). Then, I investigate the effect of the operative labor supply margin on market tightness and unemployment by comparing the two models’ results. Last, I explain why the volatility of market tightness gets larger.

IV. Model \(^{18}\)

In this section, I describe a search model with the operative labor supply margin. To describe three labor statuses and individuals’ distributions over them, a heterogeneous agent economy is modeled. Agents are distributed on three dimensions: productivity, asset stock, and labor status.

\(^{17}\) I also assume \( UN_t \) is constant for now.

\(^{18}\) This model is based on Krusell et al. (2012). However, because their model does not endogenize market tightness, I endogenize it based on the model by Bils et al. (2011). In their model, market tightness is normalized to 1 in the Steady State. However, Bils et al. (2011) do not use capital in production and assume market interest rate is given like a small open economy. In order to achieve the general equilibrium in interest rate and to use capital in production, I use Nakajima (2012)’s method.
A. Environment

There is a continuum of workers who have identical preferences but different productivities and asset stocks. A worker’s preferences are:

\[ U = \max_{(c_t, h_t)} E_0 \left[ \sum_{t=0}^{\infty} B^t (\ln c_t - B \frac{h_t^{1+\gamma}}{1+1/\gamma}) \right] \]

s.t. \[(1 + r_t) c_t + (1 - \tau) w_{it} + t r_t + d_t = c_{it} + a_{it+1} \]

\[ a_{it+1} \geq \bar{a}. \]

Workers trade claims for physical capital, \( a_{it} \), which yield the rate of return, \( r_t \). The capital markets are incomplete in that physical capital is the only asset available to workers, and workers face a borrowing constraint. The labor supply is indivisible. If a worker is employed, she supplies \( \bar{h} \) hours and earns \((1 - \tau)w_{it}\), where \( \tau \) is the labor income tax rate. \( tr_t \) means the transfer payment from the government and \( d_t \) means the dividend. Individual productivity, \( x_{it} \), varies exogenously according to a stochastic process with a transition probability distribution function, \( F(x' | x) = \Pr(x_{it+1} \leq x' | x_t = x) \), and it represents idiosyncratic risks that agents face in this model economy. Each period, a worker belongs to one of three states – employment (E), unemployment (U), or non-labor force (N).

There is also a continuum of firms which have identical production technologies. A firm can be either matched or unmatched. A matched firm produces output according to a constant returns-to-scale Cobb-Douglas technology,

\[ y_t = F(l_t, k_t, z_t) = z_t l_t^\alpha k_t^{1-\alpha}, \]

where \( z_t \) is aggregate productivity which evolves with transition probability distribution function \( \pi_{z_t}(z') | z = \Pr(z_{t+1} \leq z' | z_t = z) \). A firm uses the effective units of labor \((l_t = x_t h_t)\) which its employee provides. It also rents capital \((k_t)\) in the competitive capital market with the rental price \( r_t \). Here I mention one of the key market structures: in this economy, there is no aggregate production function. Each matched pair is a small production unit. However, every matched pair should offer the same rate of return on capital in equilibrium because the capital market is competitive and capital is freely mobile across production units. Therefore, the capital-labor ratio will be the same across all the matched pairs when the CRS production technology is used. This means that I can derive the equilibrium marginal

\[ \text{Subscript } i \text{ denotes a worker and } t \text{ denotes a period.} \]

\[ \text{I assume all firms are owned by all people jointly. As a result, the sum of firms’ profits, net of total costs for posting vacancies, is shared by the owners equally.} \]
product of labor and capital through thinking of a stand-in aggregate production function with the aggregate capital stock $K_t$ and the aggregate labor supply $L_t$.\(^{21}\)

The number of new matches between firms with a vacancy and unemployed workers is determined by the matching function

\begin{equation}
(9) \quad m(u, v) = \eta u^\xi v^{1-\xi},
\end{equation}

where $v$ is the number of vacancies and $u$ is the number of unemployed workers. The probability that an unemployed worker meets a vacancy is $p(\theta) = m(u, v)/u = \eta \theta^{1-\xi}$, where $\theta$ is the market tightness ($\theta \equiv v/u$). The probability that a vacancy meets an unemployed worker is $q(\theta) = m(u, v)/v = \eta \theta^{-\xi}$.

The matched surplus is shared by standard Nash bargaining. There are no bargaining rigidities, such as wage rigidity. There are two kinds of separation – one by the operative labor supply margin and the other by the frictional margin. Separation by the operative labor supply margin is made when the matched surplus falls below zero. Then, the worker stays out of the labor force. Separations by search frictions occur when an exogenous separation shock arrives, regardless of the size of the surplus, so the worker becomes unemployed.

B. Timing

Figure 4 shows the time sequence.

- During period $t-1$, the employed engage in production activities. At the same time, the unemployed search for jobs and workers out of the labor force enjoy leisure. But at the end of period $t-1$, some matched pairs are separated by frictions and become unemployed. At the same time, some workers who have been unemployed or out of the labor force can be matched.

- At the beginning of period $t$, individuals observe the aggregate shock and their idiosyncratic productivity shock. Upon observing these shocks, they determine their amount of consumption and saving. Also, matched workers and firms decide whether to continue being matched. If they decide to be separated voluntarily, the worker exits the labor force. At the same time, people who are not matched decide whether to stay in the labor force.

- Matched workers and firms, who decide to continue, engage in production during period $t$. At the same time, unemployed workers and firms with vacancies search, and workers out of the labor force enjoy their leisure.

\(^{21}\) Refer to Nakajima (2012) for further details. He assumes a CRS production function which uses capital in a heterogeneous agent model with searching frictions for the first time. His paper does not differentiate unemployment and non-labor force.
C. Value functions

For a worker, $W^E$ denotes the value of being employed, $W^N$ denotes the value of not being employed, and $W$ is the maximum of $W^E$ and $W^N$. $J$ and $V$ denote the values for a matched firm and an unmatched firm, respectively. There are three measures which capture the distribution of workers. $\mu(a,x)$ measures the distribution of employed workers, and $\phi(a,x)$ measures the distribution of unemployed workers. $\psi(a,x)$ measures the distribution of workers out of the labor force. The worker’s value of being employed is

$$W^E(a,x,s) = \max_{a'_e} \{u(c_e) - B \frac{\lambda^{1+\frac{1}{r}}}{1+r} + E[(1-\lambda)W(a'_e, x', s') + \lambda W^N(a'_e, x', s')|x,s]\}$$

subject to

$$c_e = (1+r(s))a - a'_e + w(a,x,s)(1-\tau) + d + tr^{22}$$

$$a'_e \geq a.$$ 

where $s = \{z, \mu, \phi, \psi\}$. $\lambda$ is the probability of exogenous separation. $d$ and $tr$ denote the dividend and the government transfer respectively. Both are lump-sum. There is also a borrowing limit of $a$. The value of not being employed is

$$W^N(a,x,s) = \max_{a'_n} \{u(c_n) + E[(1-p(\theta(s)))W(a'_n, x', s') + p(\theta(s))W(a'_n, x', s')|x,s]\}$$

subject to

$$c_n = (1+r(s))a - a'_n + \sigma w(a,x,s) + d + tr$$

$$a'_n \geq a.$$ 

where $p(\theta(s))$ is the probability that a worker finds a job, and $\sigma$ is the replacement rate. For a firm, the value of a match is

$$f(a,x,s) = \max_{k} \{ZF(k,xh) - (r(s) + \delta)k - w(a,x,s) + \frac{1}{1+r(s)}(1-\lambda)E[\max\{f(a'_e, x', s'), 0\} |x,s]\}$$

and the value of a vacancy is

$$V(s) = -\kappa + \frac{1}{1+r(s)} \{q(\theta(s)) \int E[\max\{f(a'_n, x', s'), 0\} |s] d\phi(a'_n, x')\}$$

Dividend and transfer also vary according to state variables. Therefore, $d(s)$ and $tr(s)$ are correct expressions. However, we have to forecast both in computation procedure based on the Krusell and Smith’s bounded rationality in this case. For simplicity, I just use their steady state values in the cyclical fluctuation.
D. Wage Bargaining

The surplus from a matched firm and worker is shared through standard Nash bargaining,

\[ \arg\max_w (W^E(a, x, s) - W^N(a, x, s))^\alpha (f(a, x, s) - V(s))^{1-\alpha} \]

where \( \alpha \) is the parameter of bargaining power of workers. The first order condition is

\[ J = \frac{1-\alpha}{\alpha(1-\tau)} (W^E - W^N) \frac{1}{u'(z)} \]

So, I can say a separation by the operative labor supply is efficient for the worker-firm pair in that it happens if and only if the match surplus falls below zero. In Equation (15), a matched rent is split between a worker and a firm. From this equation, the value to a firm is derived when workers' values are known. Free entry implies \( V(s) \) is zero. Therefore Equation (13) can be rearranged as

\[ (13-1) \quad \kappa = \frac{1}{1+r(s)} q(\theta(s)) \int E[\max\{J(a_n', x', s'), 0\}] d\psi(a_n', x') \]

From Equation (13-1), the steady state vacancy cost is derived, because market tightness is normalized to one in the steady state.

E. Measures

The measures for the employed, the unemployed, and those out of the labor force - \( \mu(a, x) \), \( \phi(a, x) \), and \( \psi(a, x) \) - evolve as follows:

\[ (16) \quad \mu'(A^0, X^0) = (1 - \lambda) \int_{A^0, X^0} \int_{D, X} 1_{x^0 \leq x^*(a, sr)} dF(x'|x) d\mu(a, x) da' dx' \]

\[ + p(\theta(s)) \int_{A^0, X^0} \int_{D, X} 1_{x^0 \leq x^*(a, sr)} dF(x'|x) d\phi(a, x) da' dx' \]

\[ + p(\theta(s)) \int_{A^0, X^0} \int_{D, X} 1_{x^0 \leq x^*(a, sr)} dF(x'|x) d\psi(a, x) da' dx' \]

\[ (17) \quad \phi'(A^0, X^0) = \lambda \int_{A^0, X^0} \int_{D, X} 1_{x^0 \leq x^*(a, sr)} dF(x'|x) d\mu(a, x) da' dx' \]

\[ + (1 - p(\theta(s))) \int_{A^0, X^0} \int_{D, X} 1_{x^0 \leq x^*(a, sr)} dF(x'|x) d\phi(a, x) da' dx' \]
\[
+ \int_{A_0, X_0} \int_{A, X} 1_{[x' > x^*(a, s), a = a^*_n(a, x)]} dF(x'|x) d\psi(a, x) da'dx',
\]
and
\[
\psi'(A_0, X_0) = \int_{A_0, X_0} \int_{A, X} 1_{[x' < x^*(a, s), a = a^*_n(a, x)]} dF(x'|x) d\mu(a, x) da'dx'
+ (1 - p(\theta(s))) \int_{A_0, X_0} \int_{A, X} 1_{[x' < x^*(a, s), a = a^*_n(a, x)]} dF(x'|x) d\phi(a, x) da'dx'
+ \int_{A_0, X_0} \int_{A, X} 1_{[x' < x^*(a, s), a = a^*_n(a, x)]} dF(x'|x) d\psi(a, x) da'dx',
\]
where \(x^*\) is a threshold level of individual productivity at which the worker is indifferent between \(W^E\) and \(W^N\). At that time, \(J\) becomes zero as shown in (15).

**F. Equilibrium**

The equilibrium consists of a set of value functions, \(\{W^E(a, x, s), W^N(a, x, s), J(a, x, s)\}\), a set of decision rules for consumption, \(\{c^*_e(a, x, s), c^*_n(a, x, s)\}\), a set of decision rules for saving, \(\{a^*_e(a, x, s), a^*_n(a, x, s)\}\), a threshold for separation, \(x^*(a, x, s)\), the labor-market tightness, \(\theta(s)\), and a law of motion for the distributions \((\mu', \phi', \psi') = T(\mu, \phi, \psi, z)\).

- **(Optimal savings):** Given \(\theta, w, r, d, tr, a, \mu, \phi, \psi, \text{and } T\), \(a^*\) solves the Bellman equations for \(W^E, W^N, J\), and \(V\) in (10) ~ (13).
- **(Optimal endogenous separation):** \(x^*\) satisfies \(J(a, x^*, s) = 0\), given \(W^E, W^N, J, V, \mu, \phi, \text{and } T\).
- **(Bargaining):** Given \(W^E, W^N, J, \text{and } V, w\) satisfies (15).
- **(Free entry):** Given \(W^E, W^N, x^*, J, V, \mu, \phi, \text{and } T\), the vacancies are posted until \(V = 0\).
- **(Invariant measures):** Given \(a^*_e, a^*_n\) and \(x^*\), three measures, \(\mu, \phi\), and \(\psi\) are invariant in (16), (17), and (18) in the steady state.

**G. Calibration**

First, calibration for Model II is described. The most important thing in this calibration is to choose parameters so that the distribution of workers across the three labor states, and gross flows among these states, in the steady state are similar to their average values in the U.S. data. Therefore, the disutility parameter of working, \(B\), is set for the steady-state employment rate to be 61.1%, which is the value of the employment to population ratio for the population aged 16 and older for the period of 1990-2012(Q3). The coefficient in the matching function, \(\eta\), is chosen to match the unemployment rate of 6.1% during the same period. Gross flow data is shown in Table 5. Separation probability, \(\lambda\), is chosen to match the
transition rate from $E$ to $U$, 0.016. I assume that the log of individual productivity, $\ln x$, follows an AR(1) process: $\ln x' = \rho_x \ln x + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. $\rho_x$ and $\sigma_x$ are chosen to match the transition rate from $E$ to $E$, 0.954. The power in the matching function, $\xi$, is set based on the previous literature. Shimer (2005) estimates it to be 0.72 using the unemployment data of the BLS (Bureau of Labor Statistics) and the help-wanted advertising index constructed by the Conference Board. Hall (2005) estimates it to be 0.235 using the JOLTS (Job Openings and Labor Turnover Survey) data. Thus, I set it to 0.5. For the bargaining power of a worker over the matched rent, I set $\alpha$ to 0.5 to satisfy the Hosios (1990) condition. I assume that aggregate productivity, $z$, also follows an AR(1) process in logs: $\ln z' = \rho_z \ln z + \varepsilon_z$, where $\varepsilon_z \sim N(0, \sigma_z^2)$. $\rho_z$ and $\sigma_z$ are calibrated to yield a time series for TFP with autocorrelation of 0.84 and standard deviation of 2 percent. These parameters generate cyclicality of labor productivity similar to Shimer (2005). The discount factor, $\beta$, is set for the steady-state annual interest rate to be 6%. In this model, because the unemployed and workers out of the labor force use the same value function, $W^N$, the replacement rate is set to zero.

Second, I calibrate parameters of the model without the operative labor supply to be similar to Shimer (2005). The replacement rate for the unemployed is 0.4, and $B$ is set to zero so that everyone stays in the labor force. To make the interest rate 6% in the steady state, I change $\beta$ to 0.9948. There is no difference between Shimer (2005)’s model and Model I, except that Shimer (2005) uses a representative agent model.

V. Results

A. Comparison of Cyclical Properties

Table 7 and Figure 5 summarize key cyclical properties. We can easily find the severity of the unemployment volatility puzzle by comparing the first two columns of Table 7. The relative standard deviation of unemployment to labor productivity in a standard search model is only one 20th of the size in the data. In a search and matching model with a fixed separation probability, the only way to make unemployment volatile is to make market tightness voluntary. However, Shimer (2005) reports that the volatility of market tightness in his model is too weak compared to the data. From the third column of Table 7, we find that adding heterogeneities into an otherwise standard search and matching model does not seem to improve anything, which means that Model I is a successful replication of Shimer’s model, which suffers from a large volatility puzzle. In other words, without the operative labor supply margin,
there is little difference between a representative agent model and a heterogeneous agent model in terms of cyclical properties of key variables. However, when the operative labor supply margin is embedded, the results are starkly different. The relative standard deviation of unemployment jumps from 0.45 to 3.42. This increase accounts for 35% of the volatility in unemployment that was not explained by a standard search and matching model, by simply embedding the operative labor supply margin. In addition, the correlation coefficients come closer to the values in the U.S. data than Model I or Shimer (2005) does.\(^{23}\)

Now, let’s review the results of Sections II and III. While Section II shows that at least 20% of volatility in unemployment is explained by the direct effect of the operative labor supply margin, Section III argues that the existence of the operative labor supply margin raises the volatility of unemployment by making market tightness more volatile, which happens in Model II. By adding non-labor force, the relative standard deviation of market tightness and job-finding probability increase by 3.5 times those in Model I. This is the numerical evidence that the existence of non-labor force increases the volatility of market tightness \(\frac{d\theta}{dz}\) in Equation 5. In Equation 3 and Equation 4, \(UE_t\) is \(U_{t-1} \cdot p(\theta_t)\) and \(NU_t\) is \(\varepsilon \cdot N_{t-1} \cdot (1 - p(\theta_t))\).

\[
\begin{align*}
(3) \quad \text{net } NU_t &= NU_t - UN_t = \varepsilon \cdot N_{t-1} \cdot (1 - p(\theta_t)) - UN_t \\
(4) \quad \text{net } EU_t &= EU_t - UE_t = E_{t-1} \cdot \lambda - U_{t-1} \cdot p(\theta_t)
\end{align*}
\]

Let’s compare Figures 5 and 6. When the market tightness decreases during recessions, the job-finding rate \(f_{UE}\) decreases in the both models.\(^{24}\) Thus, net \(EU\) increases, which means unemployment increases. However, \(f_{UE}\) in Model II is much more volatile than it is in Model I, because the market tightness is more cyclical in Model II. In addition, \(NU\) exists only in Model II and \(f_{NU}\) increases when the market tightness decreases, which makes the cyclicity of unemployment stronger. The standard deviation of \(f_{UE}\) in Model I is 0.005, and it is 0.014 in Model II as shown in Table 8. That is, including the non-labor force makes the transition rate between \(U\) and \(E\) more than 2.5 times more volatile. The standard deviation of \(f_{NU}\) is 0.004. When we compare Table 1 from the actual data and Table 8 from the simulated data, the sizes of standard deviation of \(f_{NU}\) and \(f_{UE}\) in Model II are smaller than the actual standard deviations from the actual data, so Model II still suffers from the Shimer’s puzzle. However, the standard deviations of Model II are vastly improved from Shimer’s baseline or from Model I.

\(^{23}\) I simulate the economy for 9,000 periods and discard the initial 3,000 periods. The statics results are derived from the latter 6,000 periods.

\(^{24}\) The job-finding rate is the transition rate from \(U\) to \(E\) and it is denoted \(f_{UE}\).
B. A Cause Making Market Tightness More Volatile

In this part, I investigate the reason why the operative labor supply margin makes market tightness more volatile. This is the key part in this paper. An important clue is found in Hagedorn and Manovskii (2008). The reason why a standard search model fails to generate enough volatility in market tightness is that the incentive for a firm to post vacancy responds very sluggishly to productivity shocks. The first group of researchers including Shimer (2005) and Hall (2005) attribute such insensitiveness of firms to the Nash Bargaining process. According to them, wages set by Nash Bargaining absorb most of the productivity increase, reducing the incentive for a firm to create a vacancy. However, Hagedorn and Manovskii (2008) argue that the problem does not lie in the model itself but in the way the model is calibrated and also argue that the costs for posting vacancies in the data is small. They target this small vacancy cost when they determine the replacement rate for the unemployed.\(^{25}\) Equation 13 is rearranged using free entry condition.

\[
(13-1) \quad \kappa = \frac{1}{1+r(s)} q(\theta(s)) \left[ \int E[\max\{J(a_n', x', s'), 0\}] d\phi(a_n', x') \right. \\
+ \left. \int E[\max\{J(a_n', x', s'), 0\}] d\psi(a_n', x') \right],
\]

When operative labor supply margin is omitted, it is

\[
(13-2) \quad \kappa = \frac{1}{1+r(s)} q(\theta(s)) E[J(s), s']
\]

Even when there is no heterogeneity, it is

\[
(13-3) \quad \kappa = \frac{1}{1+r(s)} q(\theta(s)) E[J(s), s']
\]

Therefore, targeting small vacancy cost (\(\kappa\)) means targeting small value of a firm (\(J\)). Equation 15, the result of Nash Bargaining, is

\[
(15) \quad J = \frac{1-\alpha}{\alpha(1-r)} (W^E - W^N) \frac{1}{u'(c_p)}
\]

Now, small \(J\) means the gap between value of working and value of not-working is small. In order to make the gap small, Hagedorn and Manovskii (2008) set the replacement rate to 95%. Using this calibration method, they successfully generate very volatile unemployment and market tightness. The rationale is as follows: when the matched rent is small, the profit to a firm is also small as shown in Equation (15). This small value to a firm in turn means the vacancy cost is also small, as in Equation (13-3). Then, the size of the percentage changes in profits in response to changes in TFP shocks becomes

\(^{25}\) Replacement rate is the ratio of unemployment insurance benefit to average compensation before layoff.
larger, because the marginal benefit and marginal cost are small now. Therefore, the incentive to post vacancies becomes strongly cyclical. However, Hall and Milgrom (2008) and Pissarides (2009) criticize Hagedorn and Manovskii (2008), by arguing that their calibration implies labor supply elasticity is too high compared to the findings from other research on labor supply. Moreover, making all workers almost indifferent to working or searching by guaranteeing 95% of replacement rate is also unrealistic, because such indifference is true only to some people located around the operative labor supply margin. Figure 7 shows the distribution of people in each labor status. The line shown on Panel (A) is the threshold of the operative labor supply margin. Brighter color means denser population. Flows between Panel (B) and Panel (C) are flows between N and U. Flows between Panel (B) and Panel (D) are flows between E and U. Thus, indifference between working and not working is valid only for people located around the threshold.

However, Hagedorn and Manovoski (2008) lower the profit of a representative matched firm by making the difference between working and not working be almost zero for all people on Panel (A). Whereas Hagedorn and Manovoski (2008) determine parameters by targeting small J, Model II does not target small J in calibration. Instead, I determine the disutility parameter, B, to target 61.1% of employment-population ratio. This is a very natural target for calibration. Such calibration makes the threshold penetrate through Panel (A) so as to obtain 61.1% of employment-population ratio. In the case of Model I, the threshold does not exist because everyone wants to work, since \( W^E \) is always higher than \( W^N \). However, in Model II, the gap between \( W^E \) and \( W^N \) of many workers near the threshold is nearly zero. Therefore, the weighted average of firms’ values (J) is lower than that of Model I. Figure 8 shows \( W^E \) and \( W^N \) at a specific asset stock of 50. Only the area from A to B where \( W^E \) is higher than \( W^N \) counts in determining J. At a glance, we can say that J in Model II is smaller than J in Model I when taking account of the distribution of workers. Indeed, vacancy costs in Models I and II are 0.51 and 0.28. This result means that the vacancy cost is lowered by embedding the operative labor supply margin even without any disputable calibration method. This is the key to why the market tightness or job-finding probability are more volatile in Model II. This mechanism affects both terms on the right hand side of Equation (5) by making \( \frac{d\theta}{dz} \) more volatile.

\[
\frac{d(\Delta U_t)}{dz_t} = -N_{t-1} \frac{\partial p(\theta_t)}{\partial \theta_t} \frac{d\theta_t}{dz_t} - U_{t-1} \frac{\partial p(\theta_t)}{\partial \theta_t} \frac{d\theta_t}{dz_t}
\]

Therefore, unemployment becomes more cyclical than in a standard search model.
VI. Conclusion

The role of operative labor supply in explaining the volatility of unemployment is not trivial. From the data, at least 20% of the volatility of unemployment can be attributed to the operative labor supply margin. However, this number does not measure the role of the operative labor supply margin accurately. When the operative labor supply margin is embedded in a standard search model, the standard deviation of market tightness increases from 0.98 to 3.40, and the standard deviation of unemployment increases from 0.45 to 3.42. Though this increase in volatility of unemployment is not big enough to match the data, 35% of volatility in unemployment is now explained by a search and matching model embedded with the operative labor supply margin. The increase in the volatility of market tightness is the key point in explaining how and why the operative labor supply margin affects unemployment fluctuation.

The reason why market tightness becomes more volatile is that embedding the operative labor supply into a standard search model lowers the average value of a matched firm. This low value of a matched job implies that the cost of posting a vacancy is also small. In turn, this makes firms’ incentives to post vacancies respond strongly to productivity shocks. Though this mechanism is similar to Hagedorn and Manovskii (2008), my model avoids a critique of a very unusual calibration. These findings confirm that including the operative labor supply margin improves the performance of existing standard search and matching models in terms of Shimer’s puzzle, or we can say that these findings allow us to measure the size of the puzzle more accurately.

An implication from this paper is that excluding the non-labor force is a large shortcoming of the traditional search and matching model. Without the operative labor supply margin, the search model had to generate volatile vacancies to make the volatility of unemployment match the data. However, when including the margin, unemployment becomes more volatile, directly or indirectly, than a standard search model. This means that the burden to make market tightness volatile is reduced when integrating the operative labor supply margin into the model. Some papers are successful in generating market tightness and unemployment which fit the data, but they must overshoot the volatility of market tightness or unemployment to do so.
Figure 1. Employment-population ratio and labor force participation rate

Notes: All variables are logged and detrended by the H-P filter.

Table 1. Cyclical properties of inflows to unemployment and outflows from unemployment

<table>
<thead>
<tr>
<th></th>
<th>Inflows to $U$</th>
<th>Outflows from $U$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{EU}$</td>
<td>$f_{NU}$</td>
</tr>
<tr>
<td>$\text{std}(x)$</td>
<td>.092</td>
<td>.093</td>
</tr>
<tr>
<td>$\text{Corr}(x, U)$</td>
<td>.81</td>
<td>.86</td>
</tr>
</tbody>
</table>

Notes: All monthly variables are logged and detrended by the H-P filter. Shimer (2005) detrendes the quarterly US data by the H-P filter with smoothing parameter of $10^5$. A period in this data is one month. To match Shimer’s quarterly statistics, $9\times10^5$ of smoothing parameter is used here.
Figure 2. Unemployment, inflow to unemployment and outflow from unemployment

Notes: All monthly variables are logged and detrended by the H-P filter. A period in this data is one month. To match Shimer’s quarterly statistics, $9 \times 10^5$ of smoothing parameter is used here. Three month moving average is applied on figures.

Some of fluctuations are certainly due to demographic and other factors unrelated to business cycle. To highlight business-cycle-frequency fluctuation, Shimer (2005) detrends quarterly data by using HP filter with smoothing parameter $10^5$. By doing this, an extremely low frequency trend is filtered out. If data is quarterly, I use $10^5$ in this paper. If data is monthly, I use $9 \times 10^5$ in order to match Shimer’s quarterly statistics.
Table 2. Monthly flows to and from unemployment and average flows

<table>
<thead>
<tr>
<th>Year (1)</th>
<th>Month (2)</th>
<th>Extensive margin by frictions</th>
<th>Extensive margin by operative labor supply</th>
<th>Total net flows = change in unemployment (ΔU) (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Extensive margin by frictions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>EU (3)</td>
<td>UE (4)</td>
<td>Net EU27 (5)</td>
</tr>
<tr>
<td>2007</td>
<td>5</td>
<td>1743</td>
<td>1805</td>
<td>-62</td>
</tr>
<tr>
<td>2007</td>
<td>6</td>
<td>1765</td>
<td>1888</td>
<td>-123</td>
</tr>
<tr>
<td>2007</td>
<td>7</td>
<td>1723</td>
<td>1935</td>
<td>-212</td>
</tr>
<tr>
<td>2007</td>
<td>8</td>
<td>1794</td>
<td>1945</td>
<td>-151</td>
</tr>
<tr>
<td>2007</td>
<td>9</td>
<td>1779</td>
<td>1937</td>
<td>-158</td>
</tr>
<tr>
<td>2007</td>
<td>10</td>
<td>1814</td>
<td>2025</td>
<td>-311</td>
</tr>
<tr>
<td>2007</td>
<td>11</td>
<td>1924</td>
<td>1906</td>
<td>18</td>
</tr>
<tr>
<td>2008</td>
<td>1</td>
<td>1768</td>
<td>2030</td>
<td>-262</td>
</tr>
<tr>
<td>2008</td>
<td>2</td>
<td>1789</td>
<td>1916</td>
<td>-127</td>
</tr>
<tr>
<td>2008</td>
<td>3</td>
<td>2057</td>
<td>1974</td>
<td>83</td>
</tr>
<tr>
<td>2008</td>
<td>4</td>
<td>1852</td>
<td>2129</td>
<td>-277</td>
</tr>
<tr>
<td>2008</td>
<td>5</td>
<td>2045</td>
<td>1854</td>
<td>191</td>
</tr>
<tr>
<td>Monthly average during the first six months (before the recession)</td>
<td>1753</td>
<td>1923</td>
<td>-170</td>
<td>1810</td>
</tr>
<tr>
<td>Monthly average during the second six months (during the recession)</td>
<td>1902</td>
<td>1968</td>
<td>-62</td>
<td>1955</td>
</tr>
<tr>
<td>Difference between two periods</td>
<td>149</td>
<td>46</td>
<td>107</td>
<td>145</td>
</tr>
</tbody>
</table>

Notes: This table is based on ‘Table 1’ in ‘Issues in Labor Statistics’ published by BLS in June 2008.

27 Net EU = Net flow from E to U = EU - UE
Table 3. Contribution of each extensive margin to changes in unemployment (Numbers in thousands)

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net NU</td>
<td>226</td>
<td>283</td>
<td>250</td>
</tr>
<tr>
<td>Net EU</td>
<td>-107</td>
<td>-74</td>
<td>90</td>
</tr>
<tr>
<td>Total net flow to U</td>
<td>119</td>
<td>195</td>
<td>339</td>
</tr>
<tr>
<td>( \text{Hypo. total net flow to U} )\footnotesize{( \text{only with frictional margin (2)} )}</td>
<td>76</td>
<td>110</td>
<td>273</td>
</tr>
<tr>
<td>Contribution of net EU</td>
<td>0.64</td>
<td>0.56</td>
<td>0.80</td>
</tr>
<tr>
<td>Hypo. total net flow to U ( \text{only with operative labor supply margin (3)} )</td>
<td>40</td>
<td>82</td>
<td>63</td>
</tr>
<tr>
<td>Contribution of net NU</td>
<td>0.34</td>
<td>0.42</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Figure 3. Actual unemployment and unemployment only with the operative labor supply

Notes: All variables are logged and detrended by the H-P filter. To match Shimer’s quarterly statistics, \(9 \times 10^5\) smoothing parameter is used. Three month moving average is applied on figures.

Table 4. Comparison between actual unemployment and hypothetical unemployment

<table>
<thead>
<tr>
<th></th>
<th>( U )</th>
<th>( U \text{ only from net NU} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{std}(x) )</td>
<td>0.146</td>
<td>0.033</td>
</tr>
<tr>
<td>( \text{Corr} )</td>
<td></td>
<td>0.65</td>
</tr>
</tbody>
</table>

Notes: All variables are logged and detrended by the H-P filter. To match Shimer’s quarterly statistics, \(9 \times 10^5\) smoothing parameter is used.
Table 5. Flows among three states (Monthly)

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>E</th>
<th>U</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td></td>
<td>0.954</td>
<td>0.016</td>
<td>0.030</td>
</tr>
<tr>
<td>U</td>
<td></td>
<td>0.270</td>
<td>0.508</td>
<td>0.222</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td>0.048</td>
<td>0.027</td>
<td>0.925</td>
</tr>
</tbody>
</table>

Table 6. Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Nash Bargaining parameter of workers</td>
<td>same</td>
<td>0.5</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Matching technology power parameter</td>
<td>same</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Matching technology coefficient parameter</td>
<td>same</td>
<td>0.51</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Labor supply elasticity parameter</td>
<td>same</td>
<td>0.4</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Production function parameter</td>
<td>same</td>
<td>0.64</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9948</td>
<td>0.9933</td>
</tr>
<tr>
<td>$B$</td>
<td>Disutility from working</td>
<td>0.0</td>
<td>103</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Separation probability</td>
<td>0.033</td>
<td>0.015</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Replacement rate (unemployment benefit)</td>
<td>0.4</td>
<td>0.0</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Persistence of idiosyncratic productivity ln x</td>
<td>same</td>
<td>0.990</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>Standard deviation of innovation to ln x</td>
<td>same</td>
<td>0.101</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of aggregate productivity ln z</td>
<td>same</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Standard deviation of innovation to ln z</td>
<td>same</td>
<td>0.0077</td>
</tr>
</tbody>
</table>
Figure 4. Time sequence in the benchmark model.

Figure 5. Comparison of cyclical properties in Model I (upper) and Model II (lower).

Note: All variables are logged and detrended by the H-P filter. The interval I selected for these figure is Period 101 to Period 250 out of the 6,000 simulated periods.
Table 7. Comparison of cyclical properties

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Shimer (2005)</th>
<th>Model I</th>
<th>Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviation (relative to productivity) for</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>9.50</td>
<td>0.45</td>
<td>0.45</td>
<td>3.42</td>
</tr>
<tr>
<td>Vacancy</td>
<td>10.1</td>
<td>1.35</td>
<td>0.58</td>
<td>1.81</td>
</tr>
<tr>
<td>Market tightness (θ)</td>
<td>19.1</td>
<td>1.75</td>
<td>0.98</td>
<td>3.40</td>
</tr>
<tr>
<td>Job-finding probability</td>
<td>5.90</td>
<td>0.50</td>
<td>0.49</td>
<td>1.70</td>
</tr>
<tr>
<td><strong>Correlation with productivity for</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.41</td>
<td>-0.96</td>
<td>-0.92</td>
<td>-0.50</td>
</tr>
<tr>
<td>Vacancy</td>
<td>0.36</td>
<td>1.00</td>
<td>0.93</td>
<td>0.58</td>
</tr>
<tr>
<td>Market tightness (θ)</td>
<td>0.40</td>
<td>1.00</td>
<td>0.97</td>
<td>0.72</td>
</tr>
<tr>
<td>Job-finding probability</td>
<td>0.40</td>
<td>1.00</td>
<td>0.97</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Notes: All variables are logged and detrended by the H-P filter. Statistics for US data are from Shimer (2005), in which quarterly data are detrended by the H-P filter with smoothing parameter of 10^5 for 1951 to 2003. A period in the models of this paper is one month. To match Shimer’s quarterly statistics, 9×10^5 smoothing parameter is used. Model I is the model without non-labor force. Model II is the model with it. The statistics in my models are derived from the last 6,000 periods’ sample.

Figure 6. Two transition rates in Model I (upper) and Model II (lower)

Note: All variables are logged and detrended by the H-P filter. The interval I selected for these figure is Period 101 to Period 250 out of the 6,000 simulated periods.
Table 8. Cyclical properties of transition rate of $UE$ and $NU$ in each model.

<table>
<thead>
<tr>
<th></th>
<th>Model I</th>
<th></th>
<th>Model II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{UE}$</td>
<td>$f_{SU}$</td>
<td>$f_{UE}$</td>
<td>$f_{SU}$</td>
</tr>
<tr>
<td>$std(x)$</td>
<td>.005</td>
<td></td>
<td>.014</td>
<td>.004</td>
</tr>
<tr>
<td>$Corr(x, U)$</td>
<td>-.97</td>
<td></td>
<td>-.69</td>
<td>.86</td>
</tr>
</tbody>
</table>

Note: All variables are logged and detrended by the H-P filter. The interval I selected for these figure is Period 101 to Period 250 out of the 6,000 simulated periods.
Figure 7. Distribution on each labor status in the steady state.

Figure 8. Values of being employed and not being employed in Model I (left) and Model II (right)
VII. References


VIII. Appendix – Computation

In order to approximate the equilibrium with aggregate fluctuations, the aggregate productivity, $z$, and the measure of workers in each labor statuses, $\mu, \phi, \psi$ are required. Using this aggregate state, $s = \{z, \mu, \phi, \psi\}$, an individual forecasts wage, interest rate, and the matching probability to make decisions of saving, consumption, and labor force participation in each period. However, current measures of workers also depends on the next period’s aggregate state, $s'$, and it is impossible to keep track of the evolution of these measures. Thus, this model is computed using Krusell and Smith’s (1998) limited information approach. I restrict $s$ to a few variables that can easily be kept track of. Individuals now make decisions based on simple forecasting rules which again are based on this limited information. Then, I check whether these forecasts are consistent with what actually happen. Here, I choose the current aggregate capital stock, $k$, the market tightness in the previous period, $\theta_{-1}$, and the aggregate productivity, $z$. The computational procedure is as followed.

1. The value functions are now $W^E(a, x, z, k, \theta_{-1})$, $W^N(a, x, z, k, \theta_{-1})$, $J(a, x, z, k, \theta_{-1})$, and $V(z, k, \theta_{-1})$.

   \begin{align}
   W^E(a, x, z, k, \theta_{-1}) &= \max_{a'} \{u(c_e) - B \frac{r^{1+\rho}}{1+\rho} + \beta E[(1-\lambda) W(a'_e, x', z', k', \theta) \\
   &+ \lambda W^N(a'_e, x', z', k', \theta)]_{x, z, k, \theta_{-1}}\} \\
   \text{subject to} \quad c_e &= (1 + r(z, k, \theta_{-1}))a - a'_e + w(a, x, z, k, \theta_{-1})(1 - r) + d + tr^{28} \\
   a'_e &\geq a
   \end{align}

   \begin{align}
   W^N(a, x, z, k, \theta_{-1}) &= \max_{a'_n} \{u(c_n) + \beta E[(1 - p(\theta(z, k, \theta_{-1}))) W^N(a'_n, x', z', k', \theta) \\
   &+ p(\theta(z, k, \theta_{-1})) W(a_n', x', z', k', \theta)]_{x, z, k, \theta_{-1}}\} \\
   \text{subject to} \quad c_n &= (1 + r(z, k, \theta_{-1}))a - a'_n + aw(a, x, z, k, \theta_{-1}) + d + tr \\
   a'_n &\geq a
   \end{align}

   \begin{align}
   J(a, x, z, k, \theta_{-1}) &= \max_k \{zF(k, xh) - (r(z, k, \theta_{-1}) + \delta)k - w(a, x, z, k, \theta_{-1}) + \\
   \frac{1}{1 + r(z, k, \theta_{-1})}(1 - \lambda)E[\max\{J(a'_e, x', z', k', \theta), 0\}]_{x, z, k, \theta_{-1}}\}
   \end{align}

28 Dividend and transfer also vary according to state variables. Therefore, $d(s)$ and $tr(s)$ are correct expressions. However, we have to forecast both in computation procedure based on the Krusell and Smith’s bounded rationality in this case. For simplicity, I just use their steady state values in the cyclical fluctuation.
2. In order to solve the right hand sides of the Bellman equations (A-1) ~ (A-4), individuals have to forecast the prices and job-finding probability in this period as well as form an expectation on the future aggregate state variables. Now, we guess a set of prediction rules for the equilibrium market tightness ($\theta$) in the current period, the aggregate asset of the economy ($k'$), and the marginal product of capital ($mpk'$). The equilibrium real interest rate, $r = mpk' - \delta$. I adopt a set of log-linear forecasting rules.

\[
\log k' = b_{k,0}^0 + b_{k,1}^0 \log k + b_{k,2}^0 \log \theta_{-1} + b_{k,3}^0 \log z
\]
\[
\log \theta = b_{\theta,0}^0 + b_{\theta,1}^0 \log k + b_{\theta,2}^0 \log \theta_{-1} + b_{\theta,3}^0 \log z
\]
\[
\log mpk = b_{mpk,0}^0 + b_{mpk,1}^0 \log k + b_{mpk,2}^0 \log \theta_{-1} + b_{mpk,3}^0 \log z
\]

The superscript on coefficients denotes the number of iteration. At the first iteration, I make a guess for the coefficients.

3. Given these prediction rules, we solve the individual optimization and wage bargaining problems, (A-1) ~ (A-5). This step is called the inner loop. Then, we have the value functions, the policy functions for the asset choice, and the wage functions.

4. Then we simulated the economy to generate a set of artificial time-series data \{\theta_t, k_t, mpk_t\} of the length of 9,000 periods. The number of people is 500,000. The aggregate capital, $k = \int ad\mu(a, x) + \int ad\phi(a, x) + \int ad\psi(a, x)$. The number of employed workers, $e = \int d\mu(a, x)$. This is needed to calculate $mpk$.

5. Once the simulation is done, we run the above OLS regressions. Then, we obtain the new values for the coefficients ($b^1$). This is called the outer loop.

6. We repeat the same steps (3) ~ (5) until the coefficients converge.
The converged forecasting equations for Model II are

\[ \log k' = 0.059841 + 0.984165 \log k + 0.001218 \log \theta + 0.032008 \log z, \quad R^2 = 99.9996 \]
\[ \log \theta = -0.461274 + 0.122069 \log k + 0.576541 \log \theta + 0.800242 \log z, \quad R^2 = 74.4075 \]
\[ \log mpk = -1.548558 - 0.735440 \log k + 0.020385 \log \theta + 1.320712 \log z, \quad R^2 = 99.8382 \]

The converged forecasting equations for Model I are

\[ \log k' = 0.055162 + 0.986093 \log k + 0.000429 \log \theta + 0.027446 \log z, \quad R^2 = 99.9999 \]
\[ \log \theta = -1.391310 + 0.351303 \log k + 0.079346 \log \theta + 0.870953 \log z, \quad R^2 = 97.6566 \]
\[ \log mpk = -1.806919 - 0.636423 \log k + 0.021138 \log \theta + 1.002971 \log z, \quad R^2 = 99.9839 \]