A DSGE Model with Habit Formation
and Nonconvex Capital Adjustment Costs

Jonghyeon Oh*

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Abstract

The literature debates the importance of micro-level lumpy investment on macro-level economy. Khan and Thomas (2003, 2008) build general equilibrium models with nonconvex capital adjustment costs to explain lumpy investment. They conclude that lumpy investment in the disaggregate level has essentially no impact on the aggregate level of the economy. The price effects of the general equilibrium is critical for the result. However, their price implications are counterfactual given financial market data; their price dynamics imply 0.007 of the Hansen-Jagannathan upper bound of the Sharpe Ratio, while the Sharpe Ratio is 0.5 in the U.S. postwar data. I reevaluate the Khan-Thomas result under more realistic price implications. I extend Khan and Thomas’s (2003) model with slow-moving external habit by Campbell and Cochrane (1999). My model improves on the financial market data. The habit increases the Hansen-Jagannathan upper bound by an order of magnitude from 0.007 to 0.06, while it is still lower than 0.5. Given the better price implications, I find that the Khan-Thomas result is intact. In other words, my model supports the view that lumpy investment does not matter at the aggregate level of the economy.

Keywords: Lumpy investment, Nonconvex capital adjustment costs, Habit formation, Krusell-Smith algorithm

JEL Classification: C68, E22, E32

*Department of Economics, The Ohio State University, Columbus, OH 43210, oh.183@osu.edu
1 Introduction

Thomas (2002), Khan and Thomas (2003, 2008) showed that, under general equilibrium frameworks, micro-economic frictions do not matter in aggregate level mainly because of equilibrium price feedback effects. Their models, especially Khan and Thomas (2008), well explain overall business cycle moments of macro data as well as lumpy investment of micro data. Furthermore, both models with and without micro-frictions generated similar business cycle moments in aggregate level.

However, these models have a limitation. As Mehra and Prescott (1985) and many others, their models do not fit the asset market data, which is usually called equity premium puzzle, risk-free rate puzzle, and so on in the literature. In particular, the Hansen-Jagannathan bound is calculated using simulation data from Khan and Thomas’s model without labor. Hansen and Jagannathan (1991) derived the upper bounds of Sharpe ratio. The Hansen-Jagannathan bound is defined by the ratio of standard deviation to expectation of stochastic discount factor, while the Sharpe ratio is the ratio of expectation to standard deviation of equity premium.

In the postwar U.S. data, the average excess rate of return is 8% and its standard deviation is 16% so that the Sharpe ratio is about 0.5. However, an average stochastic discount factor from Khan and Thomas’s model is 0.954 and its standard deviation is only 0.0071 so that the implied Hansen-Jagannathan bound is 0.0075. This number is much below than the Sharpe ratio in the U.S. data. The appropriate Hansen-Jagannathan bound is necessary to resolve other asset pricing puzzles. This implies Khan and Thomas’s models suffer from the puzzles as RBC literature.

This limitation mainly came from the extremely low standard deviation of the stochastic discount factor. To explain the asset market behaviors, a mechanism to amplify it is necessary. One plausible way to do it is to introduce habit persistence in preferences. When we consider a constant relative risk aversion utility function, the habit makes marginal utility more rapidly change to the variation of consumption. With more variation of marginal utility, the stochastic discount factor would be more volatile than a standard preferences.

When it comes to habit formation, there are two kinds: simple and slow-moving habit. In simple habit formation models, the current level of habit depends only on the consumption in the last period. In slow-moving habits such as Campbell and Cochrane (1999), on the other hand, the current habit level depends on all the history of consumptions until the last period so that the level of habit is not sensitive to the last period consumption itself compared with simple habit. Between them, this paper considers the slow-moving habit rather than the simple habit.

With the habit formation, nonconvex capital adjustment costs are introduced to a standard RBC model without labor to reflect the lumpy investment in the establishment level. Then, the importance of lumpy investment can be addressed by comparing two models with habit persistent preferences: models with and without the fixed capital adjustment costs.
The main result is that, in a habit world, two economies in aggregate level are essentially identical. Impulse responses, simulation data and their moments from both models are indistinguishable. This result supports the implication of Khan and Thomas (2003, 2008). However, both models are only partially successful to account for asset pricing puzzles. The Hansen-Jagannathan bounds of both models are around 0.065 which is larger by an order of magnitude than that of Khan and Thomas’s model, but still much below the Sharpe ratio of 0.5. This result is consistent with asset pricing literature.

Using the slow-moving habit, Campbell and Cochrane (1999) fairly well explained the asset market behaviors. However, their results are based on an endowment economy where consumption is determined exogenously. For enough volatility of stochastic discount factor, enough consumption volatility are needed as well. In Campbell and Cochrane (1999), this is guaranteed by the exogenous consumption process. However, this is not the case in a production economy. The household with habit persistent preferences has a motive to smooth consumption. In a production economy, she would not much increase her current consumption to a positive technology shock by accumulating more capital today and deferring consumption to the future.

Jermann (1998) and Boldrin et al. (2001) attempt to explain asset market behaviors within production economies. They also introduced habit formation to a standard RBC model, but their habit is a simple habit. A common result from both papers is that only habit cannot resolve asset pricing puzzles in production economies. So, Jermann (1998) considered capital adjustment costs and Boldrin et al. (2001) constructed multisector model with inflexible mobility of factors in both capital and labor. These models explain the risk-free rate and the risk premium, but the volatility puzzle is still need to be resolved.

To account for the asset market behaviors, the model need to be extended to generate enough volatility of consumption. One possible extension is that consumption process is allowed to be exogenous as Campbell and Cochrane (1999). With this specification, the stochastic discount factor is also exogenous and we can focus on firm’s profit maximization problem. Another extension is introduce capital adjustment costs as Jermann (1998) or construct two-sector model like Boldrin et al. (2001). Both models include mechanism to reduce volatility of investment and hours worked and increase that of consumption.

This paper considers the model without labor. This is because labor is another channel for the household to achieve consumption smoothing motive. To shut down this channel, simply labor is eliminated from the model. However, labor can be introduced without the labor channel if the household does not value her leisure as Jermann (1998). Moreover, although the household values her leisure, the importance of the labor channel can be reduced by introducing labor market frictions as Boldrin et al. (2001)

As Khan and Thomas (2003, 2008), the method from Krusell and Smith (1998) is used to numerically solve the model with the fixed capital adjustment costs. The idiosyncratic fixed
adjustment costs of capital make firms heterogeneous so that a distribution of firms over capital is one of the state variables. Unfortunately, it is impossible to solve the model with this state variable. Krusell and Smith (1998) provide a method to solve heterogeneous agent problems. The distribution can be reduced to its moments, which makes state variable spaces finite.

2 Model

Four models are investigated: (1) RBC model, (2) KT model, (3) CC model, and (4) CCKT model. The RBC model is a standard RBC model without labor. The KT model is based on Khan and Thomas (2003) by adding nonconvex capital adjustment costs to the RBC model. According to the implication from Khan and Thomas (2003), results from the KT model should not much different from those of the RBC model. The CC model, on the other hand, is based on Campbell and Cochrane (1999) by introducing slow-moving habit to the RBC model. Habit formation would help fit models to asset market data by increasing the volatility of stochastic discount factor. Finally, the CCKT model considers both habit formation and fixed capital adjustment costs. This model will be compared to the CC model to see whether the main results from Khan and Thomas (2003) hold although accounting for asset market behaviors.

In the below, only the CCKT model are described since others are simply derived by eliminating one or two ingredients from the CCKT model.

2.1 The firms

In the economy, there are continuum of firms with unit measure. The firms maximize their expected discount profits. At the beginning of period \( t \), to firm \( i \in [0, 1] \), two random state variables are realized: total factor productivity, \( z_t \), and firm specific nonconvex capital adjustment cost, \( \xi_{it} \). The firm produces consumption goods using predetermined capital \( k_{it} \) with the technology

\[
y_{it} = z_t f(k_{it}) = z_t k_{it}^\theta, \quad \theta \in (0, 1).
\]

A concave production technology is assumed.

After production, the firm decides whether to invest in capital or not. If it decides to invest, then the firm must pay the fixed capital adjustment cost, \( \xi_{it} \). Without paying the adjustment cost, the firm only let its capital depreciate.

\[
\text{adjustment cost} = \begin{cases} 
\xi_{it}, & \text{if } i_{it} \neq 0 \\
0, & \text{if } i_{it} = 0 
\end{cases}
\]

The nonconvex capital adjustment cost \( \xi_{it} \) is drawn from a distribution \( G(\xi_{it}) \), which is time
invariant and common across firms. By the law of large number, the realized distribution of the fixed adjustment costs across firms is the same as $G(\xi_{it})$ for every period. Specifically, a uniform distribution is assumed, or

$$\xi_{it} \sim G(\xi_{it}) = i.i.d.U[0, B]. \tag{1}$$

Then the firm enter the next period with capital stock which is determined by the law of motion

$$\gamma k_{it+1} = (1 - \delta)k_{it} + i_{it}, \tag{2}$$

where $\gamma$ is a steady-state growth rate of output and $\delta$ is a capital depreciation rate.

Finally, the productivity follows a log-AR(1) process with identically and independently distributed innovation $\varepsilon_t$,

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t \tag{3}$$

$$\varepsilon_t \sim \tilde{F}(\varepsilon_t) = i.i.d.N(0, \sigma^2).$$

### 2.1.1 Firms’ value function

Firms’ problem is

$$v^1(k; \xi; z, \mu, S) = zf(k) + (1 - \delta)k + \max\{v^a - \xi, v^n\}$$

$$v^a(k; z, \mu, S) = \max_{k'} \left\{ -\gamma k' + \int d(z, \mu, S, z', \mu', S')v^0(k'; z', \mu', S')F(z, dz') \right\}$$

$$v^n(k; z, \mu, S) = -(1 - \delta)k + \int d(z, \mu, S, z', \mu', S')v^0 \left( \frac{(1 - \delta)}{\gamma}k; z', \mu', S' \right) F(z, dz')$$

$$v^0(k; z, \mu, S) = \int_0^B v^1(k; \xi; z, \mu, S)G(d\xi)$$

subject to

$$\mu' = \Gamma(z, \mu, S), \text{ and } (1), (2), (3),$$

and the law of motion of $S$ which is specified in the households’ problem. In the above, $\mu$ is a distribution of firms over capital, $S$ is surplus consumption ratio, $d(\cdot)$ is a stochastic discount

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1It is assumed the adjustment cost $\xi$ is directly taken from output. This implies, under the growing economy, the upper bound of the distribution of the costs keeps growing with the same growth rate of output. To avoid this problem, the costs can be denominated as a fraction of production $zf(k)$ as Bloom (2009). If labor is introduced to the model, the costs can also be denominated as labor as Khan and Thomas (2003, 2008). Currently, I am working with this direction. But I expect this specification does not much alter the results.
factor, and $F(\cdot)$ is the conditional distribution of $z$ associated with the distribution of technology innovation $\bar{F}(\cdot)$. $v^1(\cdot)$ is the value of firm when all the state variables are realized. $v^a(\cdot)$ is the value when the firm invests in capital before paying the fixed adjustment cost. $v^o(\cdot)$ is the value of firm when it does not invest in capital. $v^0(\cdot)$ is value of firm before knowing its fixed costs drawn from a common distribution $G(\xi)$

2.2 The households

A representative household maximizes her expected present value of utilities. At period $t$, the household initially owns assets, $\varphi(k_t)$, which is the shares of firms having capital stock of $k_t$. According to the assets, she receives dividends from the firms, and then sells the shares in the asset market. With these resources, she consumes goods, $c_t$, produced by the firms, and buys assets, $\varphi'(k_t)$, for the next period.

As in Campbell and Cochrane (1999), a period utility depends on an individual consumption, $c_t$, and a habit, $X_t$, as follows:

$$u(c_t; X_t) = \frac{(c_t - X_t)^{1-\alpha} - 1}{1 - \alpha},$$

It is assumed that the utility is time separable. Campbell and Cochrane (1999) modeled the surplus consumption ratio, $S_t$, rather than the habit, $X_t$, itself. The surplus consumption ratio are defined as

$$S_t = \frac{C_t - X_t}{C_t},$$

where $C_t$ is an aggregate consumption. The habit is external since it depends only on aggregate variables. Also The habit is not predetermined since a current aggregate consumption have an impact on a current level of habit. In this sense, the habit is thought of as keeping up with the Joneses rather than catching up with the Joneses. The law of motion of the surplus consumption ratio is

$$S_{t+1} = \bar{S}^{1-\phi}S_t^{\phi}\left(\frac{C_{t+1}}{C_t}\right)^{\lambda(S_t)},$$

(4)

where $\bar{S}$ is a steady-state surplus consumption ratio.\(^2\) In the above, $\lambda(S_t)$ is called a sensitivity

\(^2\)Campbell and Cochrane used the specific form of $\bar{S} = \sigma_c\sqrt{\alpha/(1-\phi)}$, where $\sigma_c$ is a standard deviation of aggregate consumption. This form makes a risk-free rate constant in their endowment economy where consumption of household is exogenously given every period. However, when consumption is endogenously determined, a risk-free rate depends on aggregate states. For $\bar{S}$, Campbell and Cochrane’s (1999) value is used as Lettau and Uhlig (2000). This makes the implied $\sigma_c$ different from the volatility of consumption of the model.
function which has the following functional form.

$$\lambda(S_t) = \begin{cases} 
\frac{1}{\bar{S}} \sqrt{1 - 2 \log(S_t/\bar{S})} - 1, & S_t \leq S_{\text{max}} \\
0, & S_t > S_{\text{max}}
\end{cases}$$

$$S_{\text{max}} = \bar{S} \exp \left\{ \frac{1 - \bar{S}^2}{2} \right\}$$

As shown in Figure 1, the closer to zero the surplus consumption ratio is, the larger positive value the function has. As discussed below, the sensitivity function plays an important role in amplifying the volatility of stochastic discount factor.
2.2.1 Households’ value function

Household’s problem is

\[ W(\varphi; z, \mu, S) = \max_{c, \varphi'} \left\{ u(c; X) + \beta \int W(\varphi'; z', \mu', S') F(z, dz') \right\} \]

subject to

\[ c + \int \zeta(k; z, \mu, S) \varphi'(dk) \leq \int v^0(k; z, \mu, S) \varphi(dk), \]

where \( v^0(\cdot) \) is the ex-dividend price of asset, \( \zeta(\cdot) \) is the post-dividend price. The dividend should be the amount of post- minus ex-dividend prices.

2.2.2 Stochastic discount factor

From the household’s utility function, the stochastic discount factor is derived as follows.

\[ d(z, \mu, S, z', \mu', S') = \beta \frac{u_c(c'; X')}{u_c(c; X)} = \beta \left( \frac{c' - X'}{c - X} \right)^\alpha. \]

At equilibrium, an individual consumption should be the same as an aggregate consumption, or \( c_t = C_t \). Therefore, with the definition of the consumption surplus ratio,

\[ d(z, \mu, S, z', \mu', S') = \beta \left( \frac{S'C'}{SC} \right)^\alpha. \]

2.2.3 Elasticity of stochastic discount factor

A necessary mechanism to explain asset market behaviors is to amplify the volatility of stochastic discount factor. Figure 2 illustrates the elasticity of stochastic discount factor with respect to consumption with and without the slow-moving habit. In both panels, all the dashed lines are stochastic discount factors without habit. Since they do not depend on surplus consumption ratio, stochastic discount factors are flat without habit. All the solid curves are stochastic discount factors with habit. Blue lines or curves are stochastic discount factors when consumption does not grow. Red line and curve in the left panel are stochastic discount factors when consumption increases by one percent. The counterparts in the right panel are those when consumption decreases by one percent.

As shown in the figure, without the habit, stochastic discount factors are extremely inelastic with respect to consumption growth. On the other hand, they are amplified by the slow-moving habit, especially when consumption is close to the level of habit. However, when consumption is far above the habit, elasticity of stochastic discount factors are as small as the case without the
habit. This is because of the shape of the sensitivity function. When the surplus consumption ratio is small, it amplifies the role of consumption growth in the stochastic discount factor. See Campbell and Cochrane (1999) in more detail.

2.3 Recursive competitive equilibrium

A recursive competitive equilibrium in this economy is defined by a set of functions \( \{v^1, k^f, W, c, \Phi, \zeta, d\} \). \( v^1 \) and \( k^f \) are the value and policy function solving the firms’ problem. \( W \) and \( (c, \Phi) \) are the value and policy functions solving the households problem. \( (\zeta, d) \) are pricing functions. There are two market clearing conditions: asset and goods market. Asset market clears with

\[
\Phi(k'; z, \mu, S) = \mu'(k') = \int_{\{(k, \xi) | k' = k^f(k, \xi; z, \mu, S)\}} G(d\xi) \mu(dk).
\]

Goods market clears with

\[
c(\mu; z, \mu, S) = \int_K \int_0^B \left[ z f(k) + (1 - \delta)k - \gamma k^f(k, \xi; z, \mu, S) \right] G(d\xi) \mu(dk).
\]
The law of motion $\Gamma(z, \mu, S)$ should be consistent with the firm’s policy function $k^f(k, \xi; z, \mu, S)$ given the exogenous law of motion of $z$ and $S$.

3 Solution Methods

From the household’s problem, we can derive the shadow price of consumption goods and the stochastic discount factor

$$p(z, \mu, S) = u_c(c; X)$$

$$d(z, \mu, S, z', \mu', S') = \beta \frac{u_c(c'; X')}{u_c(c; X)} = \beta \frac{p(z', \mu', S')}{p(z, \mu, S)}$$

As Khan and Thomas (2003), the household problem is simplified with the equilibrium implications. By defining $V(\cdot) \equiv p(\cdot)v(\cdot)$, the firm’s problem is transformed into constant discount factor problem.

The problem is not feasible since $\mu$ is an infinite dimensional argument. As Krusell and Smith (1998) and Khan and Thomas (2003), it is approximated by its first moment or aggregate capital, $m$, to make the problem feasible. Accordingly, the law of motion of $\mu$ also replaced with a forecasting rule for $m'$. Then the dynamic programming becomes

$$V^1(k, \xi; z, m, S) = p(\theta k^\gamma + (1 - \delta)k) + \max\{V^a - p\xi, V^n\}$$

where,

$$V^a(k; z, m, S) = \max_{k'} \left\{ -p\gamma k' + \beta \int V^0(k'; z', m', S') F(z, dz') \right\}$$

$$V^n(k; z, m, S) = -p(1 - \delta)k + \beta \int V^0 \left( \frac{1 - \delta}{\gamma} k; z', m', S' \right) F(z, dz')$$

$$V^0(k; z, m, S) \equiv \int_0^B V^1(k, \xi; z, m, S) G(d\xi)$$

with transition functions of aggregate variables.

The algorithm to solve the model consists of two stages: inner loop and outer loop.

3.1 Inner loop

At this stage, a value function is found given log-linear forecasting rules for the next period aggregate capital $m'$ and the current price $p$

$$\log m' = \beta_{10} + \beta_{11} \log m + \beta_{12} \log S + \beta_{13} \log z$$

$$\log p = \beta_{20} + \beta_{21} \log m + \beta_{22} \log S + \beta_{23} \log z.$$
The price forecasting rule (7) and the law of motion of surplus consumption ratio (4) imply

\[ S' = \left[ \bar{S}^{1-\phi} S^{\phi+\lambda} \left( \frac{p}{e^{\beta_{23} M^{\beta_{21}} z^{\beta_{23}}}} \right)^{\lambda/\alpha} \right]^{1/(1+\lambda+\beta_{22}\lambda/\alpha)} \]  

(8)

Given (6), (7), (8), and the law of motion of \( z \), value functions are iterated over finite grids for states with piecewise cubic spline interpolations. The algorithm to find the value function is as follows:

1. Set initial value function as zero.
2. Given the values of \( l \) th iteration, \( V_i^0(k; z, m, S) \), find an expected continuation value function over individual capitals conditional aggregate states \( (z, m, S) \). At this step, the integral should be evaluated. A Gauss-Hermit numerical integration is used.

\[ EV_i^0(k'|z, m, S) = \int V_i^0(k'; z', m', S') F(z, dz'). \]

3. Find a target capital, \( k_* \), using golden section search, and calculate a value when a firm adjusts its capital. The target capital does not depend on the current individual capital.

\[ V_i^a(k; z, m, S) = \max_{k'} \{ -p\gamma k' + \beta EV_i^0(k'|z, m, S) \} = -p\gamma k_* + \beta EV_i^0(k*|m, z, S) \]

4. Calculate a value when a firm does not adjust its capital.

\[ V_i^n(k; z, m, S) = -p(1 - \delta)k + \beta EV_i^0 \left( \frac{(1 - \delta)}{\gamma} k \big| z, m, S \right) \]

5. Find a threshold of capital adjustment costs, \( \hat{\xi} \).

\[ \hat{\xi} = \frac{V^a - V^n}{p} \]

\[ \hat{\xi} = \max \left[ \min \{ \xi, B \}, 0 \right] \]

6. Update a value function.

\[ TV_i^0(k; z, m, S) = p \left( zk^\theta + (1 - \delta)k \right) + \frac{\hat{\xi}}{B} \left( V^a - \frac{p\hat{\xi}}{2} \right) + \left( 1 - \frac{\hat{\xi}}{B} \right) V^n \]
7. Iterate on value functions until it converges.

The converged value function is passed to the next stage.

### 3.2 Outer loop

At this stage, the forecasting rules are updated by generating simulation data and estimating the coefficients of the forecasting rules. To do that, the converged value function and two forecasting rules are brought to the outer loop from the inner loop. The model is simulated for 2,500 periods with steady-state values for the initial period. The Ordinary Least Square estimation is used to update the forecasting rules. Unless the new forecasting rules are not close enough to the old ones, then the inner loop is conducted again with the updated forecasting rule. In every simulation stage, the same aggregate productivity series, $z_t's$, are used.

More specifically, the simulation data are generated as follows:

1. Once completing simulation until period $t - 1$, we have an actual end-of-period distribution of firms over capital stock and its first moment $m_t$ as well as the past period consumption $C_{t-1}$, surplus consumption ratio $S_{t-1}$.

2. Enter into period $t$ with $z_t$, $m_t$, $C_{t-1}$, $S_{t-1}$, and the distribution of firms.

3. Solve the following three equations for $\hat{p}$, $\hat{C}$, and $\hat{S}$. The hat-sign is used to distinguish them from equilibrium quantities, $p$, $C$, and $S$.

   - Forecasting rule for price
     \[
     \hat{p}_t = e^{\beta_{20} m_t^{\beta_{21}} S_t^{\beta_{22}} z_t^{\beta_{23}}}
     \] (9)

   - Law of motion of $S$
     \[
     \hat{S}_t = \bar{S}^{1-\phi} S_{t-1}^\phi \left( \frac{\hat{C}_t}{C_{t-1}} \right)^{\lambda(S_{t-1})}
     \]

   - Equilibrium condition
     \[
     \hat{p}_t = \left( \frac{\hat{C}_t}{\hat{S}_t} \right)^{-\alpha}
     \] (10)

4. Now, all the aggregate states, $(z_t, m_t, \hat{S}_t)$, are realized and the implied external habit is $\hat{X}_t = \hat{C}_t (1 - \hat{S}_t)$. 

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5. Find a fixed point $p^* \in [\text{plow}, \text{phigh}]$ of price mapping using bisection search. The price mapping $T(p)$ is this:

- Fix $p \in [\text{plow}, \text{phigh}]$.
- Given $p$, the current distribution of firms, and the value function from the inner loop, solve the firm’s profit maximization problem.
- Find a distribution of firms over the next period capital stock and aggregate consumption that the firm’s problem implies.

$$C_t = C(p|z_t, m_t, \hat{S}_t)$$

- Then,

$$T(p) = u_c(C_t; \hat{X}_t)$$

6. With the equilibrium price $p_t = p^*$, the actual distribution of firms and its first moments, $m_{t+1}$, and equilibrium consumption $C_t$ are determined, and these are passed to period $t + 1$.

7. Also, the equilibrium $p_t$ and $C_t$ imply

$$S_t = \frac{1}{C_t p_t^{1/\alpha}},$$

which is also passed to period $t + 1$.

4 Parameter choices

Table 1 describes the parameter values chosen to solve the models. Most of the parameter values are chosen to be consistent with Khan and Thomas (2003). The length of a period is assumed to be one year. The household’s period preferences are represented by log-utility function. In addition, Khan and Thomas (2003) chose the steady state growth rate $\gamma$ to imply an 1.6 percent average annual growth rate of real output per capita, the subjective discount factor $\beta$ to match an average interest rate of 6.5 percent, the capital depreciation rate $\delta$ to fit an average investment-to-capital ratio of 7.6 percent, and the capital share of output $\theta$ to yield an average capital-to-output ratio of 2.6. They selected the persistence $\rho$ of the exogenous productivity process and the standard deviation $\sigma$ of its innovations to be consistent with measured Solow residuals from the 1953-1997 U.S. data set by Stock and Watson (1999). The persistence of surplus consumption ratio $\phi$ and the steady state surplus consumption ratio $\bar{S}$ are set to be the same values as those in Campbell and Cochrane (1999).
Table 1: Parameter choices

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>description</th>
<th>source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>relative risk aversion</td>
<td>KT (2003)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.954</td>
<td>subjective discount factor</td>
<td>KT (2003)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.016</td>
<td>growth rate</td>
<td>KT (2003)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.325</td>
<td>capital share of output</td>
<td>KT (2003)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.06</td>
<td>depreciation rate of capital</td>
<td>KT (2003)</td>
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<tr>
<td>$B$</td>
<td>0.025</td>
<td>upper bound of fixed costs</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9225</td>
<td>persistence of agg. shock</td>
<td>KT (2003)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0134</td>
<td>std. dev. of agg. shock</td>
<td>KT (2003)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.87</td>
<td>persistence of S</td>
<td>CC (1999)</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>0.057</td>
<td>steady state value of S</td>
<td>CC (1999)</td>
</tr>
</tbody>
</table>

Table 2: Target for the upper bound of the distribution of capital adjustment costs

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>model</th>
</tr>
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<tbody>
<tr>
<td>fraction of lumpy investment</td>
<td>0.25</td>
<td>0.566</td>
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<tr>
<td>fraction of lumpy investors</td>
<td>0.08</td>
<td>0.132</td>
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<tr>
<td>fraction of low-level investors</td>
<td>0.80</td>
<td>0.715</td>
</tr>
</tbody>
</table>

Note: lumpy investment: $|i/k| > 0.3$, low-level investment: $|i/k| < 0.1$

Given other parameter values, the upper bound of the uniform distribution of the nonconvex capital adjustment costs, $B$, is calibrated to match three facts in micro level from Doms and Dunne (1998) as in Table 2. At the steady-state, the model implies that the lumpy investment among total investment is 56.6 percent, lumpy investors is 13.2 percent, and low-level investors is 71.5 percent, while they are 25, 8, and 80 percent, respectively, in U.S. data. Figure 3 shows steady state hazard rate of adjusting capital and firms’ distribution over capital.

5 Results

There are two main ingredients in the CCKT model: habit formation and nonconvex capital adjustment costs. To investigate the role of each ingredient, four models are compared: RBC, KT, CC, and CCKT model. Each model is solved separately using value function iteration, Krusell-Smith algorithm, policy function iteration, and Krusell-Smith algorithm, respectively.
5.1 Forecasting rules

Table 3 are the converged forecasting rules for the future aggregate capital and the current price. As the standard errors and the adjusted $R^2$’s indicate, agents precisely forecast aggregate variables’ behavior only with the first moment of the firm’s distribution.

5.2 Impulse responses

Figure 4 shows impulse responses of aggregate variables to one standard deviation of technology shock. As the figure clearly displays, the impulse responses of RBC and KT model are essentially identical, and those of CC and CCKT model are also indistinguishable. This implies that the fixed capital adjustment costs play a negligible role in aggregate level. The adjustment costs are introduced to match the facts from micro level of data, lumpy investment. Therefore, this result implies that lumpy investment in disaggregate level does not matter in aggregate level.

In particular, positive and negative exogenous shock are given at period 0 and 200, respectively, to see whether there are nonlinearities in the responses of the aggregate variables. The responses are symmetric although we considered the lumpy investment. This results are consistent with
Table 3: Forecasting rules

<table>
<thead>
<tr>
<th></th>
<th>cons.</th>
<th>log (m)</th>
<th>log (S)</th>
<th>log (z)</th>
<th>s.e.</th>
<th>adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>log ($m'$)</td>
<td>0.1112</td>
<td>0.9660</td>
<td>0.0215</td>
<td>0.1992</td>
<td>8.985E-4</td>
<td>0.999965</td>
</tr>
<tr>
<td>log (p)</td>
<td>0.3394</td>
<td>-0.2746</td>
<td>-0.9361</td>
<td>-0.5199</td>
<td>4.443E-4</td>
<td>0.999997</td>
</tr>
</tbody>
</table>

Note: The forecasting rules are estimated by ordinary least squared estimation with 2,500 simulation data.

Khan and Thomas (2003).

On the other hand, the habit plays an important role. Since the household has more consumption smoothing motive in the habit world (CC and CCKT model) than the no-habit world (RBC and KT model), it takes more time for each variable to converge to its steady state level in the habit world although the magnitude of the shocks are the same. Moreover, overshooting is observed in the habit world. This is because the habit moves slowly than other aggregate variables do. For example, although consumption converged to its steady state level after positive shocks, the habit is still above its steady-state level. Therefore, the consumption further decreases, and then again increases to its steady state level.

5.3 Simulations and business cycle moments

Figure 5 shows simulation data from all the models, generated by the same aggregate productivity series across the models. This figure displays the first 1,000 data among 2,500. Basically the simulations confirm that the capital adjustment costs do not play a role both in the habit world and in the no-habit world. In addition, the figure shows that consumption moves more smoothly and investment moves more aggressively in the habit world than in the no-habit world.

Table 4 shows the business cycle moments, calculated from the 2,500 simulation data. To calculate the moments, for output ($y$), consumption ($c$), investment ($i$), and capital ($k$) panel, each variable takes log and hp-filtered with the smoothing parameter of 100. In risk-free rate ($r$) and stochastic discount factor ($sdf$) panel, the variables are hp-filtered with the same smoothing parameter value without taking log. The last panel are calculated by the original simulation data of stochastic discount factor.

In the table, the patterns of business cycle moments are similar between RBC and KT model, and between CC and CCKT model. This supports again the main result of Khan and Thomas (2003). When it comes to the Hansen-Jagannathan bound (the last row of the table), the slow-moving habit increases it by an order of magnitude from 0.007 to 0.06, but it is still much below the Sharpe ratio of 0.5 in postwar U.S. data. This is mainly because the habit reduces the volatility of consumption and raises the volatility of investment. As shown in the table, consumption volatility is smaller in the habit world than in the no-habit world, and investment volatility is larger in the
habit world than in the no-habit world.

To investigate the importance of lumpy investment within a model accounting for asset market behaviors, the Hansen-Jagannathan bound need to be at least the Sharpe ratio of U.S. data. Although Campbell and Cochrane (1999) well explain the asset markets in endowment economies, the results from CC model implies that simply introducing production technology to their model is only partially successful. This implication is consistent with that from Jermann (1998) and Boldrin et al. (2001).

6 Extensions

The model need to be modified in order to reflect asset market moments. Without it, it cannot be said that the role of lumpy is investigated in the context. The problem of the current model comes mainly from the low volatility of consumption and the high volatility of investment. In
Figure 5: Simulations

Note: Each panel shows the first 1,000 data among 2,500.

In this section, three extensions are discussed separately to fit asset market moments. In addition, introducing labor to the model is discussed.

Up to the point, only the Hansen-Jagannathan bound is considered for asset market behaviors since it is necessary condition for resolving other asset pricing puzzles. However, there are other asset market moments to be considered in the literature such as risk-free rate, equity premium, and Sharpe ratio. If the appropriate Hansen-Jagannathan bound is obtained, other moments should be considered.
Table 4: Business cycle moments

<table>
<thead>
<tr>
<th></th>
<th>RBC</th>
<th>Adj. Costs</th>
<th>Habit</th>
<th>Full</th>
</tr>
</thead>
<tbody>
<tr>
<td>std(x)</td>
<td>1.2798</td>
<td>1.2787</td>
<td>1.2421</td>
<td>1.2351</td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>std(x)</td>
<td>0.6826</td>
<td>0.7431</td>
<td>0.3174</td>
<td>0.3146</td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>0.5334</td>
<td>0.5812</td>
<td>0.2555</td>
<td>0.2547</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>0.9489</td>
<td>0.9568</td>
<td>0.9680</td>
<td>0.9634</td>
</tr>
<tr>
<td>std(x)</td>
<td>3.9557</td>
<td>3.7437</td>
<td>4.9594</td>
<td>5.0699</td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>3.0909</td>
<td>2.9278</td>
<td>3.9928</td>
<td>4.1048</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>0.9744</td>
<td>0.9706</td>
<td>0.9928</td>
<td>0.9868</td>
</tr>
<tr>
<td>std(x)</td>
<td>0.5355</td>
<td>0.5044</td>
<td>0.6949</td>
<td>0.7032</td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>0.4184</td>
<td>0.3945</td>
<td>0.5595</td>
<td>0.5693</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>0.0637</td>
<td>0.0782</td>
<td>-0.0819</td>
<td>-0.0812</td>
</tr>
<tr>
<td>std(x)</td>
<td>0.6392</td>
<td>0.7072</td>
<td>6.5624</td>
<td>6.0758</td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>0.4994</td>
<td>0.5531</td>
<td>5.2833</td>
<td>4.9192</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>0.6449</td>
<td>0.6304</td>
<td>0.4813</td>
<td>0.5038</td>
</tr>
<tr>
<td>std(x)</td>
<td>0.5724</td>
<td>0.6334</td>
<td>5.8461</td>
<td>5.4091</td>
</tr>
<tr>
<td>std(x)/std(y)</td>
<td>0.4472</td>
<td>0.4953</td>
<td>4.7066</td>
<td>4.3794</td>
</tr>
<tr>
<td>corr(x,y)</td>
<td>-0.6450</td>
<td>-0.6305</td>
<td>-0.4890</td>
<td>-0.5080</td>
</tr>
<tr>
<td>E(sdf)</td>
<td>0.9541</td>
<td>0.9541</td>
<td>0.9564</td>
<td>0.9561</td>
</tr>
<tr>
<td>std(sdf)</td>
<td>0.0067</td>
<td>0.0073</td>
<td>0.0636</td>
<td>0.0593</td>
</tr>
<tr>
<td>std/E</td>
<td>0.0070</td>
<td>0.0077</td>
<td>0.0665</td>
<td>0.0620</td>
</tr>
</tbody>
</table>

Note: In output (y), consumption (c), investment (i), and capital (k) panel, each variable takes log and hp-filtered with the smoothing parameter of 100. In risk-free rate (r) and stochastic discount factor (sdf) panel, the variables are hp-filtered with the same smooth parameter without taking log. The last panel are calculated by the original simulation data of stochastic discount factor.

6.1 Exogenous consumption growth

To increase the volatility of consumption, the consumption process is allowed to be exogenous. As Campbell and Cochrane (1999), it can be assumed that

\[ C' = C e^\nu \quad \text{or} \quad \frac{C'}{C} = e^\nu. \]
With the exogenous consumption growth, the stochastic discount factor in the CC and CCKT model becomes

\[ d(z, S, z', S') = \beta \left( \frac{C' S'}{CS} \right)^{-\alpha} \]

\[ = \beta \left\{ \frac{S'}{S} \right\}^{1-\phi} e^{(1+\lambda(S))\nu} \right\}^{-\alpha}. \]

Then, the volatility of the stochastic discount factor would increase in the volatility of the consumption growth innovations, \( \nu \), which would help raise the Hansen-Jagannathan bound. Furthermore, a low risk-free rate can be obtained by appropriate parameter choices. The risk-free rate is defined as

\[ R_{t+1}^f = \frac{\gamma}{Ed(z_t, S_t, z_{t+1}, S_{t+1})}. \]

Within this framework, the risk-free rate is derived as

\[ r_{t+1}^f = \log(R_{t+1}^f) = \log\left( \frac{\gamma}{\beta} \right) - \frac{1}{2} \alpha (1 - \phi), \]

which is constant over states with the specific sensitivity function defined before as in Campbell and Cochrane (1999).

### 6.1.1 Dynamic programming of CC model

With exogenous consumption process, the stochastic discount factor is also exogenous. Then, we can solve the firm’s profit maximization problem with the exogenous stochastic discount factor. In the case of CC model, it is

\[ v(z, k, S) = \max_{k'} \left[ zf(k) + (1 - \delta)k - \gamma k' \right. \]

\[ + \beta \left( \frac{S}{\bar{S}} \right)^{-\alpha(1-\phi)} E_0 \left\{ e^{-\alpha(1+\lambda)\nu} v(z', k', S') \right\} \]

subject to

\[ z' = z^\rho e^\epsilon \]

\[ S' = \bar{S}^{-\phi} S^{\phi} e^{\lambda \nu} \]

\[ \lambda = \frac{1}{S} \sqrt{1 - 2 \log \left( \frac{S}{\bar{S}} \right)} - 1 \]

\[ (\epsilon, \nu)' \sim N.i.i.d.(0_{2x1}, \Sigma_{2x2}). \]
v(z, k, S) is the ex-dividend stock price of the firm. However, it is not guaranteed that this firm’s problem is a contraction mapping. Applying value function iteration directly to the Bellmann equation might make the program diverge. To avoid this problem, both sides of the Bellmann equation is multiplied by

\[ p(S) = (\bar{S}/S)^\alpha. \]

\( p(S) \) is no longer shadow price here. Let \( V(z, k, S) \equiv p(S)v(z, k, S) \). Then, the Bellmann equation becomes

\[
V(z, k, S) = \max_{k'} \left\{ p \left[ zf(k) + (1 - \delta)k - \gamma k' \right] + \beta E_0 \left[ e^{-\alpha \nu} V(z', k', S') \right] \right\}.
\]

It is still not guaranteed that this problem is a contraction mapping. However, depending on the parameter values, a fixed point might be found.

If this model match the asset market moments, the role of lumpy investment can be investigated by introducing fixed capital adjustment costs as the CCKT model.

### 6.2 Capital adjustment costs (Jermann, 1998)

Jermann (1998) attempt to explain asset market behaviors within a production economy. The main ingredients of his model are a simple habit and capital adjustment costs. With this model, Jermann (1998) explained the low risk-free rate and the risk premium. However, the model cannot resolve the volatility puzzle. It is interesting to see if the slow-moving habit rather than the simple habit can also explain the volatility puzzle with other asset market moments.

Jermann (1998) used the following capital accumulation technology with adjustment costs,

\[ \gamma k' = (1 - \delta)k + g \left( \frac{i}{k} \right) k, \]

where \( g(\cdot) \) is concave function. The concave function \( g(\cdot) \) makes changing capital stock rapidly costly so that the volatility of investment would decrease and the volatility of consumption would increase.

### 6.3 Two sector model (Boldrin et al., 2001)

Boldrin et al. (2001) addressed the similar problem to Jermann (1998). They used a simple habit. However, their baseline model do not consider the capital adjustment costs. Instead, they constructed two sector model: consumption goods and investment goods sector. In the model, both capital and labor are predetermined and inflexible to move between two sectors. The household
values her leisure. However, both capital and labor market frictions might make the volatility of consumption increase.

6.4 Introducing labor

From the models, labor is abstracted because the representative household can smooth her consumption profile by adjusting leisure or labor. However, low consumption volatility would make asset pricing puzzle more puzzling. In the current stage, therefore, this channel is simply removed by eliminating labor.

Another way to shut down this channel is that the representative household supply her labor to earn wage but she does not value leisure. In this framework, labor supply is fixed at 1 which is time endowment of the household for each period, and wage will be determined to clear labor market. Unlike Khan and Thomas (2003), this environment would require a wage forecasting rule in Krusell-Smith algorithms which are used as solution methods for KT and CCKT model. The consumer in Khan and Thomas (2003) value leisure so that wage can be derived by a consumption-leisure euler equation with a price forecast. However, without consumer’s valuing leisure, the relationship between price and wage cannot be exploited. Therefore, wage forecasting rules are necessary.

Other way to introduce labor is that the household is allowed to value her leisure and introduce labor market frictions to reduce volatility of labor and increase that of consumption. This direction does not require wage forecasting rules in Krusell-Smith algorithms.

7 Conclusion

The importance of lumpy investment is addressed in the case of habit persistence in preferences. A slow-moving habit of Campbell and Cochrane (1999) was introduced to a standard RBC model without labor. To fit the lumpiness of investment in disaggregate level, firm specific nonconvex capital adjustment costs as in Khan and Thomas (2003) are also considered. The main result is that the lumpy investment play a negligible role in aggregate level, which supports the implication from Khan and Thomas (2003).

The reason to introduce habit persistence in preferences is to account for asset market moments. In the paper, only the Hansen-Jagannathan bound is considered for asset market behaviors. However, there are many asset market puzzles such as risk-free rate puzzle, risk premium puzzle, and volatility puzzle. At this stage, the Hansen-Jagannathan bound is not enough to explain the asset market data so that other puzzles cannot be resolved as well. The main problem is that consumption volatility is smaller than the U.S. data. Therefore, the mechanism to amplify the consumption volatility should be considered and then other asset market behaviors as well.

To fit the asset market data, several extensions are considered: exogenous consumption growth,
capital adjustment costs, two sector model with predetermined inputs. All the models attempt to introduce a mechanism to reduce the volatility of investment and raise that of consumption.
References


