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**Essays in Time Series Econometrics**

My research is based on the analyses of the Hodrick-Prescott filter. The Hodrick-Prescott (HP) filter is a commonly used technique to extract a trend from a series in macroeconomics. After the trend is extracted from the original series, the remaining component is called the cyclical component and is the main focus of the real business cycle literature in macroeconomics. The statistical properties of the cyclical component can influence the results in real business cycle literature. These properties have not been analyzed rigorously in the literature and my research aims to fill this void.

My first job market paper, co-authored with Robert M. de Jong, titled “The econometrics of the Hodrick-Prescott filter,” explores the properties of the HP filter. The HP filter calculates the trend component by minimizing the distance between the original series and the trend component while penalizing for the lack of smoothness of the trend component. The magnitude of the penalization is determined by so-called the smoothing parameter which needs to be chosen by the practitioners and is typically chosen as 1600 for the quarterly series. The solution of the minimization problem implies that the HP filter calculates a trend that is the weighted average of the original series. In this paper, we first derive the weights of the HP filter that are the functions of time points, the sample size of the original series and the smoothing parameter. The weights of the HP filter allow us to analyze the asymptotic behavior of the HP filter. Second, we analyze the statistical properties of the cyclical component when the HP filter is applied to a constant plus a linear time trend plus a unit root process. It is found that the HP filter is capable of absorbing a constant and a linear time trend into the trend component and the cyclical component possesses weak dependence properties. The weak dependence result is important in the sense that it guarantees that conventional statistical inference (i.e., the weak law of large numbers and the central limit theorems) holds. Lastly, we derived a procedure for adjusting the smoothing parameter for the data frequency. There is no value for the smoothing parameter that is commonly accepted for annual or monthly data. We derive a large smoothing value approximation for adjusting the smoothing parameter for the data frequency. We find that 6.25 and 129,600 are asymptotically sensible values for the smoothing parameter for the annual and monthly series, respectively if 1600 is the appropriate value for the quarterly series.

My second job market paper, co-authored with Robert M. de Jong, titled “A property of the Hodrick-Prescott filter and its application,” proposes a remarkable property of the HP filter which has not been established in the literature since its introduction by Hodrick and Prescott (1980, 1997). The cyclical component of the HP filter is equal to the trend component of the HP filter when applied to the fourth differences, plus an additional term. Our result is remarkable due to its simplicity and its strength in explaining many aspects of the HP filter that have not been studied rigorously. The approach of this paper is completely different than the approach of my first job market paper. We first use our main result to analyze the consequence of a deterministic trend break and find that the effect of the structural break is asymptotically negligible for the points that are away from the break point. However, for the points that are close to the break point, the effect does not die out even in large samples and a characterization is provided for it. Second, we apply our result to show that the cyclical component of an integrated process that is integrated up to order 2 is typically weakly dependent, while the situation is more subtle for the series that are integrated of order 3 or more. This result contrasts with the conjecture in the literature that the HP filter extracts a stationary series when it is applied to the series that are integrated up to order 4. Third, we characterize the cyclical component when the HP filter is applied to a polynomial deterministic trend, which suggests that the HP filter decreases the order of polynomial by 4. Finally, we give a characterization of the cyclical component when the HP filter is applied to an exponential deterministic trend. This characterization shows that the HP filter is incapable of dealing with trend that increases this fast.